

# Package ‘ChainLadder’

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**Type** Package

**Title** Statistical Methods and Models for Claims Reserving in General Insurance

**Version** 0.2.21

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**Description** Various statistical methods and models which are typically used for the estimation of outstanding claims reserves in general insurance, including those to estimate the claims development result as required under Solvency II.

**Imports** Matrix, actuar, methods, stats, lattice, grid, tweedie, utils, systemfit, statmod, cplm ( $\geq 0.7-3$ ), ggplot2, MASS

**Suggests** RUnit, knitr, rmarkdown

**VignetteBuilder** knitr

**License** GPL ( $\geq 2$ )

**URL** <https://mages.github.io/ChainLadder/>

**BugReports** <https://github.com/mages/ChainLadder/issues>

**LazyLoad** yes

**LazyData** yes

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**Author** Markus Gesmann [aut, cre],  
Daniel Murphy [aut],  
Yanwei (Wayne) Zhang [aut],  
Alessandro Carrato [aut],  
Giuseppe Crupi [ctb],  
Christophe Dutang [ctb],  
Arnaud Lacoume [ctb],  
Arthur Charpentier [ctb],  
Mario Wuthrich [aut],  
Fabio Concina [aut],  
Eric Dal Moro [aut],

Yuriy Krvavych [ctb],  
 Vincent Goulet [ctb],  
 Marco De Virgilis [ctb],  
 Marco Spina [ctb]

**Maintainer** Markus Gesmann <markus.gesmann@googlemail.com>

**Repository** CRAN

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## Description

ChainLadder provides methods and models which are typically used in insurance claims reserving.

The package grew out of presentations given at the Stochastic Reserving Seminar at the Institute of Actuaries in 2007 and 2008 and followed by talks at CAS meetings in 2008 and 2010.

More information is available on the project web site <https://github.com/mages/ChainLadder>

For more financial packages see also CRAN Task View 'Emperical Finance' at <https://CRAN.R-project.org/view=Finance>.

## Author(s)

Maintainer: Markus Gesmann <markus.gesmann@gmail.com>

## References

Thomas Mack. *Distribution-free calculation of the standard error of chain ladder reserve estimates*. Astin Bulletin. Vol. 23. No 2. 1993. pp.213:225

Thomas Mack. *The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor*. Astin Bulletin. Vol. 29. No 2. 1999. pp.361:366

Gerhard Quarg and Thomas Mack. *Munich Chain Ladder*. Blatter DGVM 26. Munich. 2004.

England, PD and Verrall, RJ. *Stochastic Claims Reserving in General Insurance (with discussion)*. British Actuarial Journal 8. III. 2002

B. Zehnwirth and G. Barnett. *Best Estimates for Reserves*. Proceedings of the CAS. Volume LXXXVII. Number 167. November 2000.

Clark, David R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach," CAS Forum, Fall 2003.

Zhang Y. *A general multivariate chain ladder model*. Insurance: Mathematics and Economics, 46, pp. 588:599, 2010.

Zhang, Y. *Likelihood-based and Bayesian Methods for Tweedie Compound Poisson Linear Mixed Models*, Statistics and Computing, forthcoming.

Bardis, Majidi, Murphy. *A Family of Chain-Ladder Factor Models for Selected Link Ratios*. Variance. Pending. Variance 6:2, 2012, pp. 143-160.

*Modelling the claims development result for solvency purposes*. Michael Merz, Mario V. Wüthrich. Casualty Actuarial Society E-Forum, Fall 2008.

*Claims Run-Off Uncertainty: The Full Picture*. Michael Merz, Mario V. Wüthrich. Swiss Finance Institute Research Paper No. 14-69. 2014

Markus Gesmann. *Claims Reserving and IBNR*. Computational Actuarial Science with R. Chapman and Hall/CRC. 2014

**Examples**

```
## Not run:  
  demo(ChainLadder)  
  
## End(Not run)
```

---

ABC	<i>Run off triangle of accumulated claims data</i>
-----	--

---

**Description**

Run-off triangle of a worker's compensation portfolio of a large company

**Usage**

```
data(ABC)
```

**Format**

A matrix with 11 accident years and 11 development years.

**Source**

*B. Zehnwirth and G. Barnett. Best Estimates for Reserves. Proceedings of the CAS. Volume LXXXVII. Number 167. November 2000.*

**Examples**

```
ABC  
plot(ABC)  
plot(ABC, lattice=TRUE)
```

---

as.LongTriangle	<i>Convert Triangle from wide to long</i>
-----------------	---

---

**Description**

Given a Triangle in matrix ("wide") format, convert to data.frame ("long") format.

**Usage**

```
as.LongTriangle(Triangle, varnames = names(dimnames(Triangle)),  
                value.name = "value", na.rm = TRUE)
```

**Arguments**

Triangle	a loss "triangle". Must be a matrix.
varnames	character names for the columns that will store the rownames and colnames of matrix Triangle. Defaults to <code>names(dimnames(Triangle))</code> if available. If not provided, uses <code>c("origin", "dev")</code> .
value.name	column name to be given to the matrix values that will be stored in the data.frame. Defaults to "value".
na.rm	should NA values be excluded from the data.frame? Defaults to TRUE.

**Details**

Unlike the `as.data.frame.triangle` method, and Unlike the 'melt' method in the 'reshape2' package, this function returns a data.frame where the rownames and colnames of Triangle are stored as *factors*. This can be a critical feature when the order of the levels of the columns is important. For example, when a Triangle is plotted, the order of the origin and dev dimensions is important. See Examples section.

**Value**

A data.frame.

**Author(s)**

Daniel Murphy

**See Also**

`as.data.frame.triangle`

**Examples**

```
as.LongTriangle(GenIns)
## Not run:
ggplot(as.LongTriangle(GenIns),
       aes(x = dev, y = value, group = origin, color = origin)) + geom_line()

## End(Not run)
```

---

 ata

---

*Calculate Age-to-Age Factors*


---

**Description**

Calculate the matrix of age-to-age factors (also called "report-to-report" factors, or "link ratios") for an object of class `triangle`.

**Usage**

```
ata(Triangle, NArw.rm = TRUE, colname.sep = "-",
    colname.order=c("ascending", "descending"))
```

**Arguments**

Triangle	a loss "triangle". Must be a matrix.
NArw.rm	logical indicating if rows of age-to-age (ata) factors that are all NA should be removed. "All-NA" rows typically occur for the most recent origin year of a loss triangle.
colname.sep	a character indicating the separator character to place between the column names of Triangle that will be used to label the columns of the resulting matrix of ata factors
colname.order	"ascending" indicates that the less mature age comes first in the column labels of the ata matrix

**Details**

ata constructs a matrix of age-to-age (ata) factors resulting from a loss "triangle" or a matrix. Simple averages and volume weighted averages are saved as "smpl" and "vwtd" attributes, respectively.

**Value**

A matrix with "smpl" and "vwtd" attributes.

**Author(s)**

Daniel Murphy

**See Also**

[summary.ata](#), [print.ata](#) and [chainladder](#)

**Examples**

```
ata(GenIns)

# Volume weighted average age-to-age factor of the "RAA" data
y <- attr(ata(RAA), "vwtd")
y
# "To ultimate" factors with a 10% tail
y <- rev(cumprod(rev(c(y, 1.1))))
names(y) <- paste(colnames(RAA), "Ult", sep="-")
y

## Label the development columns in "ratio-type" format
ata(RAA, colname.sep=":", colname.order="desc")
```

---

auto	<i>Run off triangle of accumulated claim data</i>
------	---

---

**Description**

Run-off triangles of Personal Auto and Commercial Auto insurance.

**Usage**

data(auto)

**Format**

A list of three matrices, paid Personal Auto, incurred Personal Auto and paid Commercial Auto respectively.

**Source**

Zhang (2010). *A general multivariate chain ladder model*. Insurance: Mathematics and Economics, 46, pp. 588-599.

**Examples**

data(auto)  
names(auto)

---

AutoBI	<i>Run off triangles of accumulated claim data</i>
--------	--

---

**Description**

Run-off triangles of Automobile Bodily Injury Liability.

**Usage**

data(AutoBI)

**Format**

Portfolio of automobile bodily injury liability for an experience period of 1969 to 1976. Paid Claims, Closed Claims and Reported Claim Counts respectively

**Source**

Berquist, J.R. and Sherman, R.E., *Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach*, Proceedings of the Casualty Actuarial Society, LXIV, 1977, pp.123-184.

**Examples**

```

data(AutoBI)
names(AutoBI)
AutoBI$AutoBIPaid
AutoBI$AutoBIClosed
AutoBI$AutoBIReportedCounts

```

---

BootChainLadder	<i>Bootstrap-Chain-Ladder Model</i>
-----------------	-------------------------------------

---

**Description**

The `BootChainLadder` procedure provides a predictive distribution of reserves or IBNRs for a cumulative claims development triangle.

**Usage**

```
BootChainLadder(Triangle, R = 999, process.distr=c("gamma", "od.pois"), seed = NULL)
```

**Arguments**

Triangle	cumulative claims triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix $C_{ik}$ which is filled for $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$ . See <code>qpaid</code> for how to use (mxn)-development triangles with $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g accident years).
R	the number of bootstrap replicates.
process.distr	character string indicating which process distribution to be assumed. One of "gamma" (default), or "od.pois" (over-dispersed Poisson), can be abbreviated
seed	optional seed for the random generator

**Details**

The `BootChainLadder` function uses a two-stage bootstrapping/simulation approach. In the first stage an ordinary chain-ladder methods is applied to the cumulative claims triangle. From this we calculate the scaled Pearson residuals which we bootstrap  $R$  times to forecast future incremental claims payments via the standard chain-ladder method. In the second stage we simulate the process error with the bootstrap value as the mean and using the process distribution assumed. The set of reserves obtained in this way forms the predictive distribution, from which summary statistics such as mean, prediction error or quantiles can be derived.

**Value**

BootChainLadder gives a list with the following elements back:

call	matched call
Triangle	input triangle
f	chain-ladder factors
simClaims	array of dimension $c(m, n, R)$ with the simulated claims
IBNR.ByOrigin	array of dimension $c(m, 1, R)$ with the modeled IBNRs by origin period
IBNR.Triangles	array of dimension $c(m, n, R)$ with the modeled IBNR development triangles
IBNR.Totals	vector of $R$ samples of the total IBNRs
ChainLadder.Residuals	adjusted Pearson chain-ladder residuals
process.distr	assumed process distribution
R	the number of bootstrap replicates

**Note**

The implementation of BootChainLadder follows closely the discussion of the bootstrap model in section 8 and appendix 3 of the paper by England and Verrall (2002).

**Author(s)**

Markus Gesmann, <markus.gesmann@gmail.com>

**References**

*England, PD and Verrall, RJ. Stochastic Claims Reserving in General Insurance (with discussion), British Actuarial Journal 8, III. 2002*

*Barnett and Zehnwrith. The need for diagnostic assessment of bootstrap predictive models, Insureware technical report. 2007*

**See Also**

See also [summary.BootChainLadder](#), [plot.BootChainLadder](#) displaying results and finally [CDR.BootChainLadder](#) for the one year claims development result.

**Examples**

# See also the example in section 8 of England & Verrall (2002) on page 55.

```
B <- BootChainLadder(RAA, R=999, process.distr="gamma")
B
plot(B)
# Compare to MackChainLadder
MackChainLadder(RAA)
quantile(B, c(0.75,0.95,0.99, 0.995))
```

```

# fit a distribution to the IBNR
library(MASS)
plot(ecdf(B$IBNR.Totals))
# fit a log-normal distribution
fit <- fitdistr(B$IBNR.Totals[B$IBNR.Totals>0], "lognormal")
fit
curve(plnorm(x,fit$estimate["meanlog"], fit$estimate["sdlog"]), col="red", add=TRUE)

# See also the ABC example in Barnett and Zehnwirth (2007)
A <- BootChainLadder(ABC, R=999, process.distr="gamma")
A
plot(A, log=TRUE)

## One year claims development result
CDR(A)

```

---

BS.paid.adj

*Berquist-Sherman Paid Claim Development Adjustment*


---

## Description

The B-S Paid Claim Development Adjustment methods adjusts paid claims based on the underlying relation between paid and closed claims.

## Usage

```
BS.paid.adj(Triangle.rep.counts = NULL, Triangle.closed, Triangle.paid,
            ult.counts = NULL, regression.type = "exponential")
```

## Arguments

**Triangle.rep.counts** cumulative reported claim counts triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix  $C_{ik}$  which is filled for  $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$ , see [qpaid](#).

**Triangle.closed** cumulative closed claim counts triangle. Assume columns are the development period, use transpose otherwise.

**Triangle.paid** cumulative paid claims triangle. Assume columns are the development period, use transpose otherwise.

**ult.counts** vector of ultimate claim counts.

**regression.type** Default = "exponential". Type of regression used in the model, it can take 'exponential' or 'linear'. See also 'Details'

## Details

The importance of recognizing the impact of shifts in the rate of settlement of claims upon historical paid loss data can materially affect the ultimate projections.

This functions adjusts the paid claims based on the numerical method described in the B-S paper.

Berquist and Sherman presented a technique to adjust the paid claim development method for changes in settlement rates. The first step of the paid claims adjustment is to determine the disposal rates by accident year and maturity.

The disposal rate is defined as as the cumulative closed claim counts for each accident year-maturity age cell divided by the selected ultimate claim count for the particular accident year.

If ultimate claim counts have been provided, they will be used to calculate the disposal rates, otherwise ultimate claim counts will be estimated from the cumulative reported claim counts triangle with a standard development method.

The disposal rates along the latest diagonal will be selected as the basis for adjusting the closed claim count triangle, The selected disposal rate for each maturity are multiplied by the ultimate number of claims to determine the adjusted triangle of closed claim counts.

Berquist and Sherman then use regression analysis to identify a mathematical formula that approximates the relationship between the cumulative number of closed claims (X) and cumulative paid claims (Y). The algorithm gives the possibility, through the choice of the 'regression.type' field, to fit an exponential model,  $Y = a * e^{(bX)}$ , or a linear model,  $Y = a + b * X$ .

The relation is estimated based on unadjusted closed claim counts and unadjusted paid claims. Once the regression coefficients are estimated, they will be used to adjust paid claims based on such coefficients and the adjusted closed claim counts triangle.

## Value

BS.paid.adj returns the adjusted paid claim triangle

## Author(s)

Marco De Virgilis <devirgilis.marco@gmail.com>

## References

*Berquist, J.R. and Sherman, R.E., Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach, Proceedings of the Casualty Actuarial Society, LXIV, 1977, pp.123-184.*

## See Also

See also [qpaid](#) for dealing with non-square triangles, [inflateTriangle](#) to inflate a triangle based on an inflation rate,

## Examples

```
# Adjust the Triangle of Paid Claims based on Reported Claim Counts
adj_paid <- BS.paid.adj( Triangle.rep.counts = AutoBI$AutoBIReportedCounts,
                        Triangle.closed = AutoBI$AutoBIClosed,
```

```

Triangle.paid = AutoBI$AutoBIPaid,
regression.type = 'exponential' )

# Calculate the IBNR from the standard unadjusted Paid Triangle

std_ibnr <- summary(MackChainLadder(AutoBI$AutoBIPaid))$Totals[4, 1]

# Calculate the IBNR from the adjusted Paid Triangle

adj_ibnr <- summary(MackChainLadder(adj_paid))$Totals[4, 1]

# Compare the two

adj_ibnr
std_ibnr

## For more examples see:
## Not run:
demo(BS.paid.adj)

## End(Not run)

```

---

CDR

*One year claims development result*


---

## Description

Standard deviation of the claims development result after one year for the distribution-free chain-ladder model (Mack) and Bootstrap model.

## Usage

```

CDR(x, ...)
## S3 method for class 'MackChainLadder'
CDR(x, dev=1, ...)
## S3 method for class 'BootChainLadder'
CDR(x, probs=c(0.75, 0.95), ...)
## Default S3 method:
CDR(x, ...)

```

## Arguments

x	output of either <a href="#">MackChainLadder</a> or <a href="#">BootChainLadder</a>
dev	vector of development periods or "all". Currently only applicable for <a href="#">MackChainLadder</a> output. Defines the years for which the run off claims development result should be returned.
probs	only applicable for <a href="#">BootChainLadder</a> output. Define quantiles to be returned.
...	other arguments

**Details**

Merz & Wüthrich (2008) derived analytic formulae for the mean square error of prediction of the claims development result for the Mack chain-ladder model after one year assuming:

- The opening reserves were set using the pure chain-ladder model (no tail)
- Claims develop in the year according to the assumptions underlying Mack's model
- Reserves are set after one year using the pure chain-ladder model (no tail)

**Value**

A data.frame with various IBNR/reserves and one-year statistics of the claims development result.

**Note**

Tail factors are currently not supported.

**Author(s)**

Mario Wüthrich and Markus Gesmann with contributions from Arthur Charpentier and Arnaud Lacoume for CDR.MackChainLadder and Giuseppe Crupi and Markus Gesmann for CDR.BootChainLadder.

**References**

*Michael Merz, Mario V. Wüthrich. Modelling the claims development result for solvency purposes. Casualty Actuarial Society E-Forum, Fall 2008.*

*Michael Merz, Mario V. Wüthrich. Claims Run-Off Uncertainty: The Full Picture. Swiss Finance Institute Research Paper No. 14-69. 2014*

**See Also**

See also [MackChainLadder](#) and [BootChainLadder](#)

**Examples**

```
# Example from the 2008 Merz, Wuthrich paper mentioned above
MW2008
M <- MackChainLadder(MW2008, est.sigma="Mack")
plot(M)
CDR(M)
# Return all run-off result developments
CDR(M, dev="all")
```

```
# Example from the 2014 Merz, Wuthrich paper mentioned above
MW2014
W <- MackChainLadder(MW2014, est.sigma="Mack")
plot(W)
CDR(W)
```

```
# Example with the BootChainLadder function, assuming overdispersed Poisson model
B <- BootChainLadder(MW2008, process.distr=c("od.pois"))
```

B  
CDR(B)

---

chainladder

*Estimate age-to-age factors*

---

## Description

Basic chain-ladder function to estimate age-to-age factors for a given cumulative run-off triangle. This function is used by Mack- and MunichChainLadder.

## Usage

```
chainladder(Triangle, weights = 1, delta = 1)
```

## Arguments

Triangle	cumulative claims triangle. A (mxn)-matrix $C_{ik}$ which is filled for $k \leq n + 1 - i$ ; $i = 1, \dots, m$ ; $m \geq n$ , see <a href="#">qpaid</a> for how to use (mxn)-development triangles with $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g annual).
weights	weights. Default: 1, which sets the weights for all triangle entries to 1. Otherwise specify weights as a matrix of the same dimension as Triangle with all weight entries in [0; 1], where entry $w_{i,k}$ corresponds to the point $C_{i,k+1}/C_{i,k}$ . Hence, any entry set to 0 or NA eliminates that age-to-age factor from inclusion in the model. See also 'Details'.
delta	'weighting' parameters. Default: 1; delta=1 gives the historical chain-ladder age-to-age factors, delta=2 gives the straight average of the observed individual development factors and delta=0 is the result of an ordinary regression of $C_{i,k+1}$ against $C_{i,k}$ with intercept 0, see Barnett & Zehnwirth (2000). Please note that <a href="#">MackChainLadder</a> uses the argument alpha, with $\alpha = 2 - \text{delta}$ , following the original paper Mack (1999)

## Details

The key idea is to see the chain-ladder algorithm as a special form of a weighted linear regression through the origin, applied to each development period.

Suppose  $y$  is the vector of cumulative claims at development period  $i+1$ , and  $x$  at development period  $i$ ,  $weights$  are weighting factors and  $F$  the individual age-to-age factors  $F=y/x$ . Then we get the various age-to-age factors:

- Basic (unweighted) linear regression through the origin:  $\text{lm}(y \sim x + 0)$
- Basic weighted linear regression through the origin:  $\text{lm}(y \sim x + 0, \text{weights}=\text{weights})$
- Volume weighted chain-ladder age-to-age factors:  $\text{lm}(y \sim x + 0, \text{weights}=1/x)$
- Simple average of age-to-age factors:  $\text{lm}(y \sim x + 0, \text{weights}=1/x^2)$

Barnett & Zehnwirth (2000) use  $\delta = 0, 1, 2$  to distinguish between the above three different regression approaches: `lm(y~x + 0, weights=weights/x^delta)`.

Thomas Mack uses the notation  $\alpha = 2 - \delta$  to achieve the same result: `sum(weights*x^alpha*F)/sum(weights*x^alpha*F)`  
 # Mack (1999) notation

## Value

chainladder returns a list with the following elements:

Models	linear regression models for each development period
Triangle	input triangle of cumulative claims
weights	weights used
delta	deltas used

## Author(s)

Markus Gesmann <markus.gesmann@gmail.com>

## References

Thomas Mack. *The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor*. Astin Bulletin. Vol. 29. No 2. 1999. pp.361:366

G. Barnett and B. Zehnwirth. *Best Estimates for Reserves*. Proceedings of the CAS. Volume LXXXVII. Number 167. November 2000.

## See Also

See also [ata](#), [predict.ChainLadder](#) [MackChainLadder](#),

## Examples

```
## Concept of different chain-ladder age-to-age factors.
## Compare Mack's and Barnett & Zehnwirth's papers.
x <- RAA[1:9,1]
y <- RAA[1:9,2]

F <- y/x
## wtd. average chain-ladder age-to-age factors
alpha <- 1 ## Mack notation
delta <- 2 - alpha ## Barnett & Zehnwirth notation

sum(x^alpha*F)/sum(x^alpha)
lm(y~x + 0 ,weights=1/x^delta)
summary(chainladder(RAA, delta=delta)$Models[[1]])$coef

## straight average age-to-age factors
alpha <- 0
delta <- 2 - alpha
sum(x^alpha*F)/sum(x^alpha)
lm(y~x + 0, weights=1/x^(2-alpha))
```

```

summary(chainladder(RAA, delta=delta)$Models[[1]])$coef

## ordinary regression age-to-age factors
alpha=2
delta <- 2-alpha
sum(x^alpha*F)/sum(x^alpha)
lm(y~x + 0, weights=1/x^delta)
summary(chainladder(RAA, delta=delta)$Models[[1]])$coef

## Compare different models
CL0 <- chainladder(RAA)
## age-to-age factors
sapply(CL0$Models, function(x) summary(x)$coef["x","Estimate"])
## f.se
sapply(CL0$Models, function(x) summary(x)$coef["x","Std. Error"])
## sigma
sapply(CL0$Models, function(x) summary(x)$sigma)
predict(CL0)

CL1 <- chainladder(RAA, delta=1)
## age-to-age factors
sapply(CL1$Models, function(x) summary(x)$coef["x","Estimate"])
## f.se
sapply(CL1$Models, function(x) summary(x)$coef["x","Std. Error"])
## sigma
sapply(CL1$Models, function(x) summary(x)$sigma)
predict(CL1)

CL2 <- chainladder(RAA, delta=2)
## age-to-age factors
sapply(CL2$Models, function(x) summary(x)$coef["x","Estimate"])
## f.se
sapply(CL2$Models, function(x) summary(x)$coef["x","Std. Error"])
## sigma
sapply(CL2$Models, function(x) summary(x)$sigma)
predict(CL2)

## Set 'weights' parameter to use only the last 5 diagonals,
## i.e. the last 5 calendar years
calPeriods <- (row(RAA) + col(RAA) - 1)
(weights <- ifelse(calPeriods <= 5, 0, ifelse(calPeriods > 10, NA, 1)))
CL3 <- chainladder(RAA, weights=weights)
summary(CL3$Models[[1]])$coef
predict(CL3)

```

---

checkTriangleInflation

*Check Y-o-Y Triangle Inflation Rates*


---

**Description**

Check for Year-on-Year Inflation rates down the columns of a run-off triangle

**Usage**

```
checkTriangleInflation(Triangle)
```

**Arguments**

**Triangle** average claim amounts triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix  $C_{ik}$  which is filled for  $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$ , see [qpaid](#) for how to use (mxn)-development triangles with  $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g accident years).

**Details**

The sensitivity of projections of ultimate losses based on incurred loss development factors to changes in the adequacy level of case reserves increases significantly for the long-tail lines. In particular, if the adequacy of the case reserve is changing, the estimates of ultimate losses based on reported claims could be severely distorted.

The function fits an exponential inflation model that takes the form of:

$$Y = a * (1 + b)^x$$

where  $Y$  represents the inflated claim amount,  $a$  represents the claim amount at the beginning of each period (e.g.  $AY=0$ ),  $b$  is the inflation rate and  $x$  is the time (e.g.  $AY$ ).

Fitting such a model on the average level of the case outstanding (or any other average claim amount) for each development period, it is possible to appreciate the inflation rate that has affected the average case reserve.

It is necessary to check the inflation on average amounts, otherwise the estimates may be distorted due to an increase in the number of claims rather than an actual increase in the inflation level.

If the level of inflation is material, it would be necessary to adjust each cell in the triangle.

This is to to have each diagonal in the triangle at the same level as the latest diagonal (i.e. latest valuation). This adjustment would prevent distortions in the estimates caused by inflation and not by actual variations in the claim experience.

**Value**

checkTriangleInflation returns a list with the following elements

Triangle	Input triangle
summ_table	summary table showing the inflation rate, the $R^2$ of the regression and the number of points used

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

## References

*Berquist, J.R. and Sherman, R.E., Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach, Proceedings of the Casualty Actuarial Society, LXIV, 1977, pp.123-184.*

## See Also

See also [qpaid](#) for dealing with non-square triangles, [inflateTriangle](#) to inflate a triangle based on an inflation rate,

## Examples

```
# Create a triangle of average outstanding claims as the ratio between O/S Claims
# and Open Claims (i.e. the number of outstanding claims)
avg <- MedMal$MedMalOutstanding / MedMal$MedMalOpen

# Check the level of average inflation Y-o-Y
test<-checkTriangleInflation(avg)

# Plot the results
# A model of exponential inflation fits quite well the level of average O/S claims
# This is particularly evident for DP 1,2,3
plot(test)

# Get the summary in an analytical way to observe the ratios and the number of points used
summary(test)

# Print the output
print(test)
# There is an inflation level equal to .15 at the first development period. It would be
# appropriate to adjust the triangle before proceeding with any estimate method.
```

---

ClarkCapeCod

*Clark Cape Cod method*

---

## Description

Analyze loss triangle using Clark's Cape Cod method.

## Usage

```
ClarkCapeCod(Triangle, Premium, cumulative = TRUE, maxage = Inf,
             adol = TRUE, adol.age = NULL, origin.width = NULL,
             G = "loglogistic")
```

## Arguments

Triangle	A loss triangle in the form of a matrix. The number of columns must be at least four; the number of rows may be as few as 1. The column names of the matrix should be able to be interpreted as the "age" of the losses in that column. The row names of the matrix should uniquely define the year of origin of the losses in that row. Losses may be inception-to-date or incremental.
Premium	The vector of premium to use in the method. If a scalar (vector of length 1) is given, that value will be used for all origin periods. (See "Examples" below.) If the length is greater than 1 but does not equal the number of rows of Triangle the Premium values will be "recycled" with a warning.
cumulative	If TRUE (the default), values in Triangle are inception to date. If FALSE, Triangle holds incremental losses.
maxage	The "ultimate" age to which losses should be projected.
adol	If TRUE (the default), the growth function should be applied to the length of time from the average date of loss ("adol") of losses in the origin year. If FALSE, the growth function should be applied to the length of time since the beginning of the origin year.
adol.age	Only pertinent if adol is TRUE. The age of the average date of losses within an origin period in the same units as the "ages" of the Triangle matrix. If NULL (the default) it will be assumed to be half the width of an origin period (which would be the case if losses can be assumed to occur uniformly over an origin period).
origin.width	Only pertinent if adol is TRUE. The width of an origin period in the same units as the "ages" of the Triangle matrix. If NULL (the default) it will be assumed to be the mean difference in the "ages" of the triangle, with a warning if not all differences are equal.
G	A character scalar identifying the "growth function." The two growth functions defined at this time are "loglogistic" (the default) and "weibull".

## Details

Clark's "Cape Cod" method assumes that the incremental losses across development periods in a loss triangle are independent. He assumes that the expected value of an incremental loss is equal to the *theoretical* expected loss ratio (**ELR**) times the on-level premium for the origin year times the change in the *theoretical* underlying growth function over the development period. Clark models the growth function, also called the percent of ultimate, by either the loglogistic function (a.k.a., "the inverse power curve") or the weibull function. Clark completes his incremental loss model by wrapping the expected values within an overdispersed poisson (ODP) process where the "scale factor"  $\sigma^2$  is assumed to be a known constant for all development periods.

The parameters of Clark's "Cape Cod" method are therefore: ELR, and omega and theta (the parameters of the **loglogistic** and **weibull** growth functions). Finally, Clark uses maximum likelihood to parameterize his model, uses the ODP process to estimate process risk, and uses the Cramer-Rao theorem and the "delta method" to estimate parameter risk.

Clark recommends inspecting the residuals to help assess the reasonableness of the model relative to the actual data (see [plot.clark](#) below).

**Value**

A list of class "ClarkLDF" with the components listed below. ("Key" to naming convention: all caps represent parameters; mixed case represent origin-level amounts; all-lower-case represent observation-level (origin, development age) results.)

method	"CapeCod"
growthFunction	name of the growth function
Origin	names of the rows of the triangle
Premium	Premium amount for each origin year
CurrentValue	the most mature value for each row
CurrentAge	the most mature "age" for each row
CurrentAge.used	the most mature age used; differs from "CurrentAge" when <code>adol=TRUE</code>
MAXAGE	same as 'maxage' argument
MAXAGE.USED	the maximum age for development from the average date of loss; differs from MAXAGE when <code>adol=TRUE</code>
FutureValue	the projected loss amounts ("Reserves" in Clark's paper)
ProcessSE	the process standard error of the FutureValue
ParameterSE	the parameter standard error of the FutureValue
StdError	the total standard error (process + parameter) of the FutureValue
Total	a list with amounts that appear on the "Total" row for components "Origin" (= "Total"), "CurrentValue", "FutureValue", "ProcessSE", "ParameterSE", and "StdError"
PAR	the estimated parameters
ELR	the estimated loss ratio parameter
THETAG	the estimated parameters of the growth function
GrowthFunction	value of the growth function as of the CurrentAge.used
GrowthFunctionMAXAGE	value of the growth function as of the MAXAGE.used
FutureGrowthFactor	the ("unreported" or "unpaid") percent of ultimate loss that has yet to be recorded
SIGMA2	the estimate of the $\sigma^2$ parameter
Ldf	the "to-ultimate" loss development factor (sometimes called the "cumulative development factor") as defined in Clark's paper for each origin year
LdfMAXAGE	the "to-ultimate" loss development factor as of the maximum age used in the model
TruncatedLdf	the "truncated" loss development factor for developing the current diagonal to the maximum age used in the model
FutureValueGradient	the gradient of the FutureValue function
origin	the origin year corresponding to each observed value of incremental loss

age	the age of each observed value of incremental loss
fitted	the expected value of each observed value of incremental loss (the "mu's" of Clark's paper)
residuals	the actual minus fitted value for each observed incremental loss
stdresid	the standardized residuals for each observed incremental loss (= residuals/sqrt(sigma2*fitted), referred to as "normalized residuals" in Clark's paper; see p. 62)
FI	the "Fisher Information" matrix as defined in Clark's paper (i.e., without the sigma^2 value)
value	the value of the loglikelihood function at the solution point
counts	the number of calls to the loglikelihood function and its gradient function when numerical convergence was achieved

**Author(s)**

Daniel Murphy

**References**

Clark, David R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach", *Casualty Actuarial Society Forum*, Fall, 2003 [https://www.casact.org/sites/default/files/database/forum\\_03fforum\\_03ff041.pdf](https://www.casact.org/sites/default/files/database/forum_03fforum_03ff041.pdf)

**See Also**

[ClarkLDF](#)

**Examples**

```
X <- GenIns
colnames(X) <- 12*as.numeric(colnames(X))
CC.loglogistic <- ClarkCapeCod(X, Premium=10000000+400000*0:9, maxage=240)
CC.loglogistic

# Clark's "CapeCod method" also works with triangles that have
# more development periods than origin periods. The Premium
# is a contrived match to the "made up" 'qincurred' Triangle.
ClarkCapeCod(qincurred, Premium=1250+150*0:11, G="loglogistic")

# Method also works for a "triangle" with only one row:
# 1st row of GenIns; need "drop=FALSE" to avoid becoming a vector.
ClarkCapeCod(GenIns[1, , drop=FALSE], Premium=1000000, maxage=20)

# If one value of Premium is appropriate for all origin years
# (e.g., losses are on-level and adjusted for exposure)
# then only a single value for Premium need be provided.
ClarkCapeCod(GenIns, Premium=1000000, maxage=20)

# Use of the weibull function generates a warning that the parameter risk
# approximation results in some negative variances. This may be of small
```

```

# concern since it happens only for older years with near-zero
# estimated reserves, but the warning should not be disregarded
# if it occurs with real data.
Y <- ClarkCapeCod(qincurred, Premium=1250+150*0:11, G="weibull")

# The plot of the standardized residuals by age indicates that the more
# mature observations are more loosely grouped than the less mature, just
# the opposite of the behavior under the loglogistic curve.
# This suggests that the model might be improved by analyzing the Triangle
# in two different "blocks": less mature vs. more mature.
# The QQ-plot shows that the tails of the empirical distribution of
# standardized residuals are "fatter" than a standard normal.
# The fact that the p-value is essentially zero says that there is
# virtually no chance that the standardized residuals could be
# considered draws from a standard normal random variable.
# The overall conclusion is that Clark's ODP-based CapeCod model with
# the weibull growth function does not match up well with the qincurred
# triangle and these premiums.
plot(Y)

```

ClarkLDF

*Clark LDF method***Description**

Analyze loss triangle using Clark's LDF (loss development factor) method.

**Usage**

```

ClarkLDF(Triangle, cumulative = TRUE, maxage = Inf,
         adol = TRUE, adol.age = NULL, origin.width = NULL,
         G = "loglogistic")

```

**Arguments**

Triangle	A loss triangle in the form of a matrix. The number of columns must be at least four; the number of rows may be as few as 1. The column names of the matrix should be able to be interpreted as the "age" of the losses in that column. The row names of the matrix should uniquely define the year of origin of the losses in that row. Losses may be inception-to-date or incremental. The "ages" of the triangle can be "phase shifted" – i.e., the first age need not be as at the end of the origin period. (See the Examples section.) Nor need the "ages" be uniformly spaced. However, when the ages are not uniformly spaced, it would be prudent to specify the <code>origin.width</code> argument.
cumulative	If TRUE (the default), values in <code>Triangle</code> are inception to date. If FALSE, <code>Triangle</code> holds incremental losses.
maxage	The "ultimate" age to which losses should be projected.

adol	If TRUE (the default), the growth function should be applied to the length of time from the average date of loss ("adol") of losses in the origin year. If FALSE, the growth function should be applied to the length of time since the beginning of the origin year.
adol.age	Only pertinent if adol is TRUE. The age of the average date of losses within an origin period in the same units as the "ages" of the Triangle matrix. If NULL (the default) it will be assumed to be half the width of an origin period (which would be the case if losses can be assumed to occur uniformly over an origin period).
origin.width	Only pertinent if adol is TRUE. The width of an origin period in the same units as the "ages" of the Triangle matrix. If NULL (the default) it will be assumed to be the mean difference in the "ages" of the triangle, with a warning if not all differences are equal.
G	A character scalar identifying the "growth function." The two growth functions defined at this time are "loglogistic" (the default) and "weibull".

### Details

Clark's "LDF method" assumes that the incremental losses across development periods in a loss triangle are independent. He assumes that the expected value of an incremental loss is equal to the *theoretical* expected ultimate loss (U) (by origin year) times the change in the *theoretical* underlying growth function over the development period. Clark models the growth function, also called the percent of ultimate, by either the loglogistic function (a.k.a., "the inverse power curve") or the weibull function. Clark completes his incremental loss model by wrapping the expected values within an overdispersed poisson (ODP) process where the "scale factor"  $\sigma^2$  is assumed to be a known constant for all development periods.

The parameters of Clark's "LDF method" are therefore: U, and omega and theta (the parameters of the **loglogistic** and **weibull** growth functions). Finally, Clark uses maximum likelihood to parameterize his model, uses the ODP process to estimate process risk, and uses the Cramer-Rao theorem and the "delta method" to estimate parameter risk.

Clark recommends inspecting the residuals to help assess the reasonableness of the model relative to the actual data (see [plot.clark](#) below).

### Value

A list of class "ClarkLDF" with the components listed below. ("Key" to naming convention: all caps represent parameters; mixed case represent origin-level amounts; all-lower-case represent observation-level (origin, development age) results.)

method	"LDF"
growthFunction	name of the growth function
Origin	names of the rows of the triangle
CurrentValue	the most mature value for each row
CurrentAge	the most mature "age" for each row
CurrentAge.used	the most mature age used; differs from "CurrentAge" when adol=TRUE

MAXAGE	same as 'maxage' argument
MAXAGE.USED	the maximum age for development from the average date of loss; differs from MAXAGE when adol=TRUE
FutureValue	the projected loss amounts ("Reserves" in Clark's paper)
ProcessSE	the process standard error of the FutureValue
ParameterSE	the parameter standard error of the FutureValue
StdError	the total standard error (process + parameter) of the FutureValue
Total	a list with amounts that appear on the "Total" row for components "Origin" (= "Total"), "CurrentValue", "FutureValue", "ProcessSE", "ParameterSE", and "StdError"
PAR	the estimated parameters
THETAU	the estimated parameters for the "ultimate loss" by origin year ("U" in Clark's notation)
THETAG	the estimated parameters of the growth function
GrowthFunction	value of the growth function as of the CurrentAge.used
GrowthFunctionMAXAGE	value of the growth function as of the MAXAGE.used
SIGMA2	the estimate of the $\sigma^2$ parameter
Ldf	the "to-ultimate" loss development factor (sometimes called the "cumulative development factor") as defined in Clark's paper for each origin year
LdfMAXAGE	the "to-ultimate" loss development factor as of the maximum age used in the model
TruncatedLdf	the "truncated" loss development factor for developing the current diagonal to the maximum age used in the model
FutureValueGradient	the gradient of the FutureValue function
origin	the origin year corresponding to each observed value of incremental loss
age	the age of each observed value of incremental loss
fitted	the expected value of each observed value of incremental loss (the "mu's" of Clark's paper)
residuals	the actual minus fitted value for each observed incremental loss
stdresid	the standardized residuals for each observed incremental loss (= residuals/sqrt( $\sigma^2$ *fitted), referred to as "normalized residuals" in Clark's paper; see p. 62)
FI	the "Fisher Information" matrix as defined in Clark's paper (i.e., without the $\sigma^2$ value)
value	the value of the loglikelihood function at the solution point
counts	the number of calls to the loglikelihood function and its gradient function when numerical convergence was achieved

**Author(s)**

Daniel Murphy

## References

Clark, David R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach", *Casualty Actuarial Society Forum*, Fall, 2003 [https://www.casact.org/sites/default/files/database/forum\\_03fforum\\_03ff041.pdf](https://www.casact.org/sites/default/files/database/forum_03fforum_03ff041.pdf)

## See Also

[ClarkCapeCod](#)

## Examples

```
X <- GenIns
ClarkLDF(X, maxage=20)

# Clark's "LDF method" also works with triangles that have
# more development periods than origin periods
ClarkLDF(qincurred, G="loglogistic")

# Method also works for a "triangle" with only one row:
# 1st row of GenIns; need "drop=FALSE" to avoid becoming a vector.
ClarkLDF(GenIns[1, , drop=FALSE], maxage=20)

# The age of the first evaluation may be prior to the end of the origin period.
# Here the ages are in units of "months" and the first evaluation
# is at the end of the third quarter.
X <- GenIns
colnames(X) <- 12 * as.numeric(colnames(X)) - 3
# The indicated liability increases from 1st example above,
# but not significantly.
ClarkLDF(X, maxage=240)
# When maxage is infinite, the phase shift has a more noticeable impact:
# a 4-5% increase of the overall CV.
x <- ClarkLDF(GenIns, maxage=Inf)
y <- ClarkLDF(X, maxage=Inf)
# Percent change in the bottom line CV:
(tail(y$Table65$TotalCV, 1) - tail(x$Table65$TotalCV, 1)) / tail(x$Table65$TotalCV, 1)
```

---

CLFMdelta

*Find "selection consistent" values of delta*

---

## Description

This function finds the values of delta, one for each development period, such that the model coefficients resulting from the 'chainladder' function with delta = solution are consistent with an input vector of 'selected' development age-to-age factors.

## Usage

```
CLFMdelta(Triangle, selected, tolerance = .0005, ...)
```

**Arguments**

Triangle	cumulative claims triangle. A (mxn)-matrix $C_{ik}$ which is filled for $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$ , see <a href="#">qpaid</a> for how to use (mxn)-development triangles with $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g accident years).
selected	a vector of selected age-to-age factors or "link ratios", one for each development period of 'Triangle'
tolerance	a 'tolerance' parameters. Default: .0005; for each element of 'selected' a solution 'delta' will be found – if possible – so that the chainladder model indexed by 'delta' results in a multiplicative coefficient within 'tolerance' of the 'selected' factor.
...	not in use

**Details**

For a given input Triangle and vector of selected factors, a search is conducted for chainladder models that are "consistent with" the selected factors. By "consistent with" is meant that the coefficients of the [chainladder](#) function are equivalent to the selected factors. Not all vectors of selected factors can be considered consistent with a given Triangle; feasibility is subject to restrictions on the 'selected' factors relative to the input 'Triangle'. See the References.

The default average produced by the chainladder function is the volume weighted average and corresponds to `delta = 1` in the call to that function; `delta = 2` produces the simple average; and `delta = 0` produces the "regression average", i.e., the slope of a regression line fit to the data and running through the origin. By convention, if the selected value corresponds to the volume-weighted average, the simple average, or the regression average, then the value returned will be 1, 2, and 0, respectively.

Other real-number values for `delta` will produce a different average. The point of this function is to see if there exists a model as determined by `delta` whose average is consistent with the value in the selected vector. That is not always possible. See the References.

It can be the case that one or more of the above three "standard averages" will be quite close to each other (indistinguishable within tolerance). In that case, the value returned will be, in the following priority order by convention, 1 (volume weighted average), 2 (simple average), or 0 (regression average).

**Value**

A vector of real numbers, say `delta0`, such that `coef(chainladder(Triangle, delta = delta0)) = selected` within tolerance. A logical attribute 'foundSolution' indicates if a solution was found for each element of selected.

**Author(s)**

Dan Murphy

**References**

*Bardis, Majidi, Murphy. A Family of Chain-Ladder Factor Models for Selected Link Ratios. Variance. Pending. Variance 6:2, 2012, pp. 143-160.*

**Examples**

```

x <- RAA[1:9,1]
y <- RAA[1:9,2]
F <- y/x
CLFMdelta(RAA[1:9, 1:2], selected = mean(F)) # value is 2, 'foundSolution' is TRUE
CLFMdelta(RAA[1:9, 1:2], selected = sum(y) / sum(x)) # value is 1, 'foundSolution' is TRUE

selected <- mean(c(mean(F), sum(y) / sum(x))) # an average of averages
CLFMdelta(RAA[1:9, 1:2], selected) # about 1.725
CLFMdelta(RAA[1:9, 1:2], selected = 2) # negative solutions are possible

# Demonstrating an "unreasonable" selected factor.
CLFMdelta(RAA[1:9, 1:2], selected = 1.9) # NA solution, with warning

```

---

coef.ChainLadder

*Extract residuals of a ChainLadder model*


---

**Description**

Extract residuals of a [MackChainLadder](#) model by origin-, calendar- and development period.

**Usage**

```

## S3 method for class 'ChainLadder'
coef(object, ...)

```

**Arguments**

object            output of the [chainladder](#) function  
...                optional arguments which may become named attributes of the resulting vector

**Value**

The function returns a vector of the single-parameter coefficients – also called age-to-age (ATA) or report-to-report (RTR) factors – of the models produced by running the 'chainladder' function.

**Author(s)**

Dan Murphy

**See Also**

See Also [chainladder](#)

**Examples**

```
coef(chainladder(RAA))
```

---

Cumulative and incremental triangles

*Cumulative and incremental triangles*

---

## Description

Functions to convert between cumulative and incremental triangles

## Usage

```
incr2cum(Triangle, na.rm=FALSE)
cum2incr(Triangle)
```

## Arguments

Triangle	triangle. Assume columns are the development period, use transpose otherwise.
na.rm	logical. Should missing values be removed?

## Details

`incr2cum` transforms an incremental triangle into a cumulative triangle, `cum2incr` provides the reverse operation.

## Value

Both functions return a triangle.

## Author(s)

Markus Gesmann, Christophe Dutang

## See Also

See also [as.triangle](#)

## Examples

```
# See the Taylor/Ashe example in Mack's 1993 paper

#original triangle
GenIns

#incremental triangle
cum2incr(GenIns)

#original triangle
incr2cum(cum2incr(GenIns))

# See the example in Mack's 1999 paper
```

```

#original triangle
Mortgage
incMortgage <- cum2incr(Mortgage)
#add missing values
incMortgage[1,1] <- NA
incMortgage[2,1] <- NA
incMortgage[1,2] <- NA

#with missing values argument
incr2cum(incMortgage, na.rm=TRUE)

#compared to
incr2cum(Mortgage)

```

---

cyEffTest

*Testing for Calendar Year Effect*


---

### Description

One of the three basic assumptions underlying the chain ladder method is the independence of the accident years. The function tests this assumption.

### Usage

```
cyEffTest(Triangle, ci = 0.95)
```

### Arguments

Triangle	cumulative claims triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix $C_{ik}$ which is filled for $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$ , see <a href="#">qpaid</a> for how to use (mxn)-development triangles with $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g accident years).
ci	confidence interval. Default: .95. A confidence interval is a type of interval estimate, computed from the statistics of the observed data, that might contain the true value of an unknown population parameter. The interval has an associated confidence level that quantifies the level of confidence that the parameter lies in the interval.

### Details

The main reason why this independence can be violated in practice is the fact that there could be certain calendar year effects such as major changes in claims handling or in case reserving or external influences such as substantial changes in court decisions or inflation.

As described by the Mack's 1994 paper a procedure is designed to test for calendar year influences.

The procedure returns a summary statistic  $Z$  which is assumed to be Normally Distributed. It is therefore possible to define a confidence interval threshold in order to evaluate the outcome of the test.

### Value

cyEffTest returns a list with the following elements

test_table	complete table of results
Z	summary statistic
E	expected value of the resulting distribution
Var	variance of the resulting distribution
Range	vector of the range corresponding the confidence interval threshold selected
ci	confidence interval

### Note

Additional references for further reading:

*Thomas Mack. Distribution-free calculation of the standard error of chain ladder reserve estimates. Astin Bulletin. Vol. 23. No 2. 1993. pp.213:225*

*Thomas Mack. The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor. Astin Bulletin. Vol. 29. No 2. 1999. pp.361:366*

### Author(s)

Marco De Virgilis <devirgilis.marco@gmail.com>

### References

*Mack, T., Measuring the Variability of Chain Ladder Reserve Estimates, Casualty Actuarial Society Forum, Spring 1994*

### See Also

See also [qpaid](#) for dealing with non-square triangles, [dfCorTest](#) for the test for correlations between subsequent development factors, [chainladder](#) for the chain-ladder method, [summary.cyEffTest](#), [plot.cyEffTest](#)

### Examples

```
# Before actually applying the Chain Ladder technique it is necessary to check
# whether the triangle has Calendar Year Effect

# Apply the function to the triangle and save the output into the variable test
test <- cyEffTest(RAA)

# Plot the confidence interval and the test metric
plot(test)
```

```
# The metric is within the confidence interval, therefore the triangle doesn't
# have Calendar Year Effect

# Print the summary table
summary(test)

# Print only the main outcomes
print(test)
# The test has returned a negative outcome. This means that the triangle is
# not affected by Calendar Year Effect and therefore the chain ladder method
# can be applied.
```

---

dfCorTest

*Testing for Correlations between Subsequent Development Factors*


---

### Description

One of the main assumptions underlying the chain ladder method is the uncorrelation of subsequent development factor. The function tests this assumption.

### Usage

```
dfCorTest(Triangle, ci = .5)
```

### Arguments

Triangle	cumulative claims triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix $C_{ik}$ which is filled for $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$ , see <a href="#">qpaid</a> for how to use (mxn)-development triangles with $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g accident years).
ci	confidence interval. Default: .5. A confidence interval is a type of interval estimate, computed from the statistics of the observed data, that might contain the true value of an unknown population parameter. The interval has an associated confidence level that quantifies the level of confidence that the parameter lies in the interval.

### Details

As described by the Mack's 1994 paper a procedure is designed to test for calendar year influences.

The usual test for uncorrelatedness requires that we have identically distributed pairs of observations which come from a Normal distribution. Both conditions are usually not fulfilled for adjacent columns of development factors. Spearman's correlation coefficient is therefore used.

The metric calculated by the procedure described return a statistic  $T$  that it is assumed to be Normally Distributed. It is therefore possible to define a confidence interval threshold in order to evaluate the outcome of the test.

**Value**

dfCorTest returns a list with the following elements

T_stat	summary statistic
Var	variance of the resulting distribution
Range	vector of the range corresponding the confidence interval threshold selected
ci	confidence interval

**Note**

Additional references for further reading:

*Thomas Mack. Distribution-free calculation of the standard error of chain ladder reserve estimates. Astin Bulletin. Vol. 23. No 2. 1993. pp.213:225*

*Thomas Mack. The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor. Astin Bulletin. Vol. 29. No 2. 1999. pp.361:366*

*Venter, G.G., Testing the Assumptions of Age-to-Age Factors, Proceedings of the Casualty Actuarial Society LXXXV, 1998, pp. 807-847*

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**References**

*Mack, T., Measuring the Variability of Chain Ladder Reserve Estimates, Casualty Actuarial Society Forum, Spring 1994*

**See Also**

See also [qpaid](#) for dealing with non-square triangles, [cyEffTest](#) for the test for calendar year effect, [chainladder](#) for the chain-ladder method, [summary.dfCorTest](#), [plot.dfCorTest](#)

**Examples**

```
# Before actually applying the Chain Ladder technique it is necessary to check
# whether the Development Factors are correlated

# Apply the function to the triangle and save the output into the variable test
test <- dfCorTest(RAA)

# Plot the confidence interval and the test metric
plot(test)

# The metric is within the confidence interval, therefore the Development Factors are nor correlated

# Print the summary table
summary(test)

# Print only the main outcomes
```

```
print(test)
# The test has returned a negative outcome. This means that the triangle is
# not affected by Development Factor Correlation and therefore the chain ladder method
# can be applied.
```

---

 GenIns

*Run off triangle of claims data.*


---

### Description

Run off triangle of accumulated general insurance claims data. GenInsLong provides the same data in a 'long' format.

### Usage

```
GenIns
```

### Format

A matrix with 10 accident years and 10 development years.

### Source

TAYLOR, G.C. and ASHE, F.R. (1983) *Second Moments of Estimates of Outstanding Claims*. Journal of Econometrics **23**, 37-61.

### References

See table 1 in: *Distribution-free Calculation of the Standard Error of Chain Ladder Reserve Estimates*, Thomas Mack, 1993, ASTIN Bulletin **23**, 213 - 225

### Examples

```
GenIns
plot(GenIns)

plot(GenIns, lattice=TRUE)

head(GenInsLong)

## Convert long format into triangle
## Triangles are usually stored as 'long' tables in data bases
as.triangle(GenInsLong, origin="accyear", dev="devyear", "incurred claims")
```

---

getLatestCumulative     *Triangle information for most recent calendar period.*

---

## Description

Return most recent values for all origin periods of a cumulative development triangle.

## Usage

```
getLatestCumulative(Triangle, na.values = NULL)
```

## Arguments

Triangle	a Triangle in matrix format.
na.values	a vector specifying values that should be considered synonymous with NA when searching for the rightmost non-NA.

## Value

A vector of most recent non-'NA' (and synonyms, if appropriate) values of a triangle for all origin periods. The names of the vector equal the origin names of the Triangle. The vector will have additional attributes: "latestcol" equalling the index of the column in Triangle corresponding to the row's rightmost entry; "rowsname" equalling the name of the row dimension of Triangle, if any; "colnames" equalling the corresponding column name of Triangle, if any; "colsname" equalling the name of the column dimension of Triangle, if any.

## Author(s)

Ben Escoto, Markus Gesmann, Dan Murphy

## See Also

See also [as.triangle](#).

## Examples

```
RAA
getLatestCumulative(RAA)
Y <- matrix(c(1, 2, 3,
              4, 5, 0,
              6, NA, NA), byrow=TRUE, nrow=3)
getLatestCumulative(Y) # c(3, 0, 6)
getLatestCumulative(Y, na.values = 0) # c(3, 5, 6)
```

glmReserve

*GLM-based Reserving Model***Description**

This function implements loss reserving models within the generalized linear model framework. It takes accident year and development lag as mean predictors in estimating the ultimate loss reserves, and provides both analytical and bootstrapping methods to compute the associated prediction errors. The bootstrapping approach also generates the full predictive distribution for loss reserves.

**Usage**

```
glmReserve(triangle, var.power = 1, link.power = 0, cum = TRUE,
           mse.method = c("formula", "bootstrap"), nsim = 1000, nb = FALSE, ...)
```

**Arguments**

triangle	An object of class <a href="#">triangle</a> .
var.power	The index ( $p$ ) of the power variance function $V(\mu) = \mu^p$ . Default to $p = 1$ , which is the over-dispersed Poisson model. If NULL, it will be assumed to be in (1, 2) and estimated using the <a href="#">cplm</a> package. See <a href="#">tweedie</a> .
link.power	The index of power link function. The default <code>link.power = 0</code> produces a log link. See <a href="#">tweedie</a> .
cum	A logical value indicating whether the input triangle is on the cumulative or the incremental scale. If TRUE, then <code>triangle</code> is assumed to be on the cumulative scale, and it will be converted to incremental losses internally before a GLM is fitted.
mse.method	A character indicating whether the prediction error should be computed analytically ( <code>mse.method = "formula"</code> ) or via bootstrapping ( <code>mse.method = "bootstrap"</code> ). Partial match is supported.
nsim	Number of simulations to be performed in the bootstrapping, with a default value of 1000.
nb	Whether the negative binomial distribution is used. If true, the arguments <code>var.power</code> and <code>link.power</code> are ignored and a negative binomial GLM is fitted using <code>glm.nb</code> .
...	Arguments to be passed onto the function <a href="#">glm</a> or <a href="#">cpglm</a> such as <code>contrasts</code> or <code>control</code> . It is important that <code>offset</code> and <code>weight</code> should not be specified. Otherwise, an error will be reported and the program will quit.

**Details**

This function takes an insurance loss triangle, converts it to incremental losses internally if necessary, transforms it to the long format (see `as.data.frame`) and fits the resulting loss data with a generalized linear model where the mean structure includes both the accident year and the development lag effects. The distributions allowed are the exponential family that admits a power

variance function, that is,  $V(\mu) = \mu^p$ . This subclass of distributions is usually called the Tweedie distribution and includes many commonly used distributions as special cases.

This function does not allow the user to specify the GLM options through the usual `family` argument, but instead, it uses the `tweedie` family internally and takes two arguments, `var.power` and `link.power`, through which the user still has full control of the distribution forms and link functions. The argument `var.power` determines which specific distribution is to be used, and `link.power` determines the form of the link function.

When the Tweedie compound Poisson distribution  $1 < p < 2$  is to be used, the user has the option to specify `var.power = NULL`, where the variance power  $p$  will be estimated from the data using the `cplm` package. The `bcplm` function in the `cplm` package also has an example for the Bayesian compound Poisson loss reserving model. See details in [tweedie](#), [cpglm](#) and [bcplm](#).

`glmReserve` allows certain measures of exposures to be used in an offset term in the underlying GLM. To do this, the user should not use the usual `offset` argument in `glm`. Instead, one specifies the exposure measure for each accident year through the exposure attribute of `triangle`. Make sure that these exposures are in the original scale (no log transformations for example). If the vector is named, make sure the names coincide with the `rownames`/origin of the triangle. If the vector is unnamed, make sure the exposures are in the order consistent with the accident years, **and the character rownames of the Triangle must be convertible to numeric**. If the exposure attribute is not `NULL`, the `glmReserve` function will use these exposures, link-function-transformed, in the offset term of the GLM. For example, if the link function is `log`, then the log of the exposure is used as the offset, not the original exposure. See the examples below. Moreover, the user **MUST NOT** supply the typical `offset` or `weight` as arguments in the list of additional arguments `...` `offset` should be specified as above, while `weight` is not implemented (due to prediction reasons).

Two methods are available to assess the prediction error of the estimated loss reserves. One is using the analytical formula (`mse.method = "formula"`) derived from the first-order Taylor approximation. The other is using bootstrapping (`mse.method = "bootstrap"`) that reconstructs the triangle `nsim` times by sampling with replacement from the GLM (Pearson) residuals. Each time a new triangle is formed, GLM is fitted and corresponding loss reserves are generated. Based on these predicted mean loss reserves, and the model assumption about the distribution forms, realizations of the predicted values are generated via the `rtweedie` function. Prediction errors as well as other uncertainty measures such as quantiles and predictive intervals can be calculated based on these samples.

## Value

The output is an object of class `"glmReserve"` that has the following components:

<code>call</code>	the matched call.
<code>summary</code>	A data frame containing the predicted loss reserve statistics. Similar to the summary statistics from <code>MackChainLadder</code> .
<code>Triangle</code>	The input triangle.
<code>FullTriangle</code>	The completed triangle, where empty cells in the original triangle are filled with model predictions.
<code>model</code>	The fitted GLM, a class of <code>"glm"</code> or <code>"cpglm"</code> . It is most convenient to work with this component when model fit information is wanted.
<code>sims.par</code>	a matrix of the simulated parameter values in the bootstrapping.

<code>sims.reserve.mean</code>	a matrix of the simulated mean loss reserves (without the process variance) for each year in the bootstrapping.
<code>sims.par</code>	a matrix of the simulated realizations of the loss reserves (with the process variance) for each year in the bootstrapping. This can be used to summarize the predictive uncertainty of the loss reserves.

**Note**

The use of GLM in insurance loss reserving has many compelling aspects, e.g.,

- when over-dispersed Poisson model is used, it reproduces the estimates from Chain Ladder;
- it provides a more coherent modeling framework than the Mack method;
- all the relevant established statistical theory can be directly applied to perform hypothesis testing and diagnostic checking;

However, the user should be cautious of some of the key assumptions that underlie the GLM model, in order to determine whether this model is appropriate for the problem considered:

- the GLM model assumes no tail development, and it only projects losses to the latest time point of the observed data. To use a model that enables tail extrapolation, please consider the growth curve model [ClarkLDF](#) or [ClarkCapeCod](#);
- the model assumes that each incremental loss is independent of all the others. This assumption may not be valid in that cells from the same calendar year are usually correlated due to inflation or business operating factors;
- the model tends to be over-parameterized, which may lead to inferior predictive performance.

To solve these potential problems, many variants of the current basic GLM model have been proposed in the actuarial literature. Some of these may be included in the future release.

Support of the negative binomial GLM was added since version 0.2.3.

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**References**

England P. and Verrall R. (1999). Analytic and bootstrap estimates of prediction errors in claims reserving. *Insurance: Mathematics and Economics*, 25, 281-293.

**See Also**

See also [glm](#), [glm.nb](#), [tweedie](#), [cpglm](#) and [MackChainLadder](#).

**Examples**

```
data(GenIns)
GenIns <- GenIns / 1000

# over-dispersed Poisson: reproduce ChainLadder estimates
(fit1 <- glmReserve(GenIns))
summary(fit1, type = "model") # extract the underlying glm

# which:
# 1 Original triangle
# 2 Full triangle
# 3 Reserve distribution
# 4 Residual plot
# 5 QQ-plot

# plot original triangle
plot(fit1, which = 1, xlab = "dev year", ylab = "cum loss")

# plot residuals
plot(fit1, which = 4, xlab = "fitted values", ylab = "residuals")

# Gamma GLM:
(fit2 <- glmReserve(GenIns, var.power = 2))

# compound Poisson GLM (variance function estimated from the data):
(fit3 <- glmReserve(GenIns, var.power = NULL))

# Now suppose we have an exposure measure
# we can put it as an offset term in the model
# to do this, use the "exposure" attribute of the 'triangle'
expos <- (7 + 1:10 * 0.4) * 100
GenIns2 <- GenIns
attr(GenIns2, "exposure") <- expos
(fit4 <- glmReserve(GenIns2))
# If the triangle's rownames are not convertible to numeric,
# supply names to the exposures
GenIns3 <- GenIns2
rownames(GenIns3) <- paste0(2007:2016, "-01-01")
names(expos) <- rownames(GenIns3)
attr(GenIns3, "exposure") <- expos
(fit4b <- glmReserve(GenIns3))

# use bootstrapping to compute prediction error
## Not run:
set.seed(11)
(fit5 <- glmReserve(GenIns, mse.method = "boot"))

# compute the quantiles of the predicted loss reserves
t(apply(fit5$sims.reserve.pred, 2, quantile,
        c(0.025, 0.25, 0.5, 0.75, 0.975)))

# plot distribution of reserve
```

```

plot(fit5, which = 3)

## End(Not run)

# alternative over-dispersed Poisson: negative binomial GLM
(fit6 <- glmReserve(GenIns, nb = TRUE))

```

---

inflateTriangle      *Inflate a Triangle based on an Inflation Rate*

---

### Description

Inflate the amounts of a Triangle from the latest diagonal based on an Inflation Rate

### Usage

```
inflateTriangle(Triangle, rate)
```

### Arguments

Triangle	claim triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix $C_{ik}$ which is filled for $k \leq n+1-i; i = 1, \dots, m; m \geq n$ , see <a href="#">qpaid</a> for how to use (mxn)-development triangles with $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g accident years).
rate	Inflation rate to be applied to the triangle according to an exponential model

### Details

The sensitivity of projections of ultimate losses based on incurred loss development factors to changes in the adequacy level of case reserves increases significantly for the long-tail lines. In particular, if the adequacy of the case reserve is changing, the estimates of ultimate losses based on reported claims could be severely distorted. The function deflates the amounts of latest diagonal to each diagonal of the triangle according to the inflation rate provided, considering an exponential model. The purpose of restating the amounts is to have each diagonal in the triangle at the same level as the latest diagonal (i.e. latest valuation). Ideally the metrics that should be restated are average O/S or average claim paid.

### Value

inflateTriangle returns the inflated triangle according to the provided rate

### Author(s)

Marco De Virgilis <devirgilis.marco@gmail.com>

## References

*Berquist, J.R. and Sherman, R.E., Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach, Proceedings of the Casualty Actuarial Society, LXIV, 1977, pp.123-184.*

## See Also

See also [qpaid](#) for dealing with non-square triangles, [checkTriangleInflation](#) to check Y-o-Y Triangle Inflation Rates,

## Examples

```
# Create a Triangle of Average Case O/S

avg <- MedMal$MedMalOutstanding / MedMal$MedMalOpen

# Select a rate of 15% and inflate the average =/S Triangle

inflated_tr <- inflateTriangle(Triangle = avg, rate = .15)

# Multiply it by open claims and add paymnets to calculate the adjusted Reported Claims Trinagle

adj_reported <- inflated_tr * MedMal$MedMalOpen + MedMal$MedMalPaid

# Calculate the IBNR from the unadjusted Triangle

std_ibnr <- summary(MackChainLadder(MedMal$MedMalReported))$Totals[4, 1]

# Calculate the IBNR from the adjusted Triangle

adj_reported_ibnr <- summary(MackChainLadder(adj_reported))$Totals[4, 1]

# Compare the two

std_ibnr - adj_reported_ibnr
```

---

Join2Fits

*Join Two Fitted MultiChainLadder Models*

---

## Description

This function is created to facilitate the fitting of the multivariate functions when specifying different models in two different development periods, especially when separate chain-ladder is used in later periods.

## Usage

```
Join2Fits(object1, object2)
```

**Arguments**

object1            An object of class "MultiChainLadder"  
object2            An object of class "MultiChainLadder"

**Details**

The inputs must be of class "MultiChainLadder" because this function depends on the model slot to determine what kind of object is to be created and returned. If both objects have "MCL", then an object of class "MCLFit" is created; if one has "GMCL" and one has "MCL", then an object of class "GMCLFit" is created, where the one with "GMCL" is assumed to come from the first development periods; if both have "GMCL", then an object of class "GMCLFit" is created.

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**See Also**

See also [MultiChainLadder](#)

---

JoinFitMse

*Join Model Fit and Mse Estimation*

---

**Description**

This function combines first moment estimation from fitted regression models and second moment estimation from Mse method to construct an object of class "MultiChainLadder", for which a variety of methods are defined, such as summary and plot.

**Usage**

```
JoinFitMse(models, mse.models)
```

**Arguments**

models            fitted regression models, either of class "MCLFit" or "GMCLFit".  
mse.models        output from a call to Mse, which is of class "MultiChainLadderMse".

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**See Also**

See also [MultiChainLadder](#).

---

liab	<i>Run off triangle of accumulated claim data</i>
------	---

---

**Description**

Run-off triangles of General Liability and Auto Liability.

**Usage**

```
data(auto)
```

**Format**

A list of two matrices, General Liability and Auto Liability respectively.

**Source**

*Braun C (2004). The prediction error of the chain ladder method applied to correlated run off triangles. ASTIN Bulletin 34(2): 399-423*

**Examples**

```
data(liab)
names(liab)
```

---

LRfunction	<i>Calculate the Link Ratio Function</i>
------------	--

---

**Description**

This calculates the link ratio function per the CLFM paper.

**Usage**

```
LRfunction(x, y, delta)
```

**Arguments**

x	beginning value of loss during a development period
y	ending value of loss during a development period
delta	numeric

**Details**

Calculated the link ratios resulting from a chainladder model over a development period indexed by (possibly vector valued) real number delta. See formula (5) in the References.

**Value**

A vector of link ratios.

**Author(s)**

Dan Murphy

**References**

*Bardis, Majidi, Murphy. A Family of Chain-Ladder Factor Models for Selected Link Ratios. Variance. Pending. 2013. pp.tbd:tbd*

**Examples**

```
x <- RAA[1:9,1]
y <- RAA[1:9,2]
delta <- seq(-2, 2, by = .1)
plot(delta, LRfunction(x, y, delta), type = "l")
```

---

M3IR5

*Run off triangle of claims data*

---

**Description**

Run off triangle of simulated incremental claims data

**Usage**

```
data(M3IR5)
```

**Format**

A matrix with simulated incremental claims of 14 accident years and 14 development years.

**Source**

Appendix A7 in *B. Zehnwirth. Probabilistic Development Factor Models with Applications to Loss Reserve Variability, Prediction Intervals, and Risk Based Capital. Casualty Actuarial Science Forum. Spring 1994. Vol. 2.*

**Examples**

```
M3IR5
plot(M3IR5)
plot(incr2cum(M3IR5), lattice=TRUE)
```

---

MackChainLadder	<i>Mack Chain-Ladder Model</i>
-----------------	--------------------------------

---

## Description

The Mack chain-ladder model forecasts future claims developments based on a historical cumulative claims development triangle and estimates the standard error around those.

## Usage

```
MackChainLadder(Triangle, weights = 1, alpha=1, est.sigma="log-linear",
tail=FALSE, tail.se=NULL, tail.sigma=NULL, mse.method="Mack")
```

## Arguments

Triangle	cumulative claims triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix $C_{ik}$ which is filled for $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$ , see <a href="#">qpaid</a> for how to use (mxn)-development triangles with $m < n$ , say higher development period frequency (e.g quarterly) than origin period frequency (e.g accident years).
weights	weights. Default: 1, which sets the weights for all triangle entries to 1. Otherwise specify weights as a matrix of the same dimension as Triangle with all weight entries in [0; 1]. Hence, any entry set to 0 or NA eliminates that age-to-age factor from inclusion in the model. See also 'Details'
alpha	'weighting' parameters. Default: 1 for all development periods; alpha=1 gives the historical chain-ladder age-to-age factors, alpha=0 gives the straight average of the observed individual development factors and alpha=2 is the result of an ordinary regression of $C_{i,k+1}$ against $C_{i,k}$ with intercept 0, see also 'Details' below, <a href="#">chainladder</a> and Mack's 1999 paper
est.sigma	defines how to estimate $\sigma_{n-1}$ , the variability of the individual age-to-age factors at development time $n - 1$ . Default is "log-linear" for a log-linear regression, "Mack" for Mack's approximation from his 1999 paper. Alternatively the user can provide a numeric value. If the log-linear model appears to be inappropriate (p-value > 0.05) the 'Mack' method will be used instead and a warning message printed. Similarly, if Triangle is so small that log-linear regression is being attempted on a vector of only one non-NA average link ratio, the 'Mack' method will be used instead and a warning message printed.
tail	can be logical or a numeric value. If tail=FALSE no tail factor will be applied, if tail=TRUE a tail factor will be estimated via a linear extrapolation of $\log(\text{chain} - \text{ladder factors} - 1)$ , if tail is a numeric value than this value will be used instead.
tail.se	defines how the standard error of the tail factor is estimated. Only needed if a tail factor > 1 is provided. Default is NULL. If tail.se is NULL, tail.se is estimated via "log-linear" regression, if tail.se is a numeric value than this value will be used instead.

<code>tail.sigma</code>	defines how to estimate individual tail variability. Only needed if a tail factor > 1 is provided. Default is NULL. If <code>tail.sigma</code> is NULL, <code>tail.sigma</code> is estimated via "log-linear" regression, if <code>tail.sigma</code> is a numeric value than this value will be used instead
<code>mse.method</code>	method used for the recursive estimate of the parameter risk component of the mean square error. Value "Mack" (default) coincides with Mack's formula; "Independence" includes the additional cross-product term as in Murphy and BMW. Refer to References below.

### Details

Following Mack's 1999 paper let  $C_{ik}$  denote the cumulative loss amounts of origin period (e.g. accident year)  $i = 1, \dots, m$ , with losses known for development period (e.g. development year)  $k \leq n + 1 - i$ . In order to forecast the amounts  $C_{ik}$  for  $k > n + 1 - i$  the Mack chain-ladder-model assumes:

$$\text{CL1: } E[F_{ik} | C_{i1}, C_{i2}, \dots, C_{ik}] = f_k \text{ with } F_{ik} = \frac{C_{i,k+1}}{C_{ik}}$$

$$\text{CL2: } \text{Var}\left(\frac{C_{i,k+1}}{C_{ik}} | C_{i1}, C_{i2}, \dots, C_{ik}\right) = \frac{\sigma_k^2}{w_{ik} C_{ik}^\alpha}$$

$$\text{CL3: } \{C_{i1}, \dots, C_{in}\}, \{C_{j1}, \dots, C_{jn}\}, \text{ are independent for origin period } i \neq j$$

with  $w_{ik} \in [0; 1]$ ,  $\alpha \in \{0, 1, 2\}$ . If these assumptions hold, the Mack chain-ladder gives an unbiased estimator for IBNR (Incurred But Not Reported) claims.

Here  $w_{ik}$  are the weights from above.

The Mack chain-ladder model can be regarded as a special form of a weighted linear regression through the origin for each development period:  $\text{lm}(y \sim x + 0, \text{weights} = \text{weights}/x^{(2-\alpha)})$ , where  $y$  is the vector of claims at development period  $k + 1$  and  $x$  is the vector of claims at development period  $k$ .

It is necessary, before actually applying the model, to check if the main assumptions behind the model (i.e. Calendar Year Effect and Correlation between subsequent Accident Years, see [dfCorTest](#), [cyEffTest](#)) are verified.

### Value

MackChainLadder returns a list with the following elements

<code>call</code>	matched call
<code>Triangle</code>	input triangle of cumulative claims
<code>FullTriangle</code>	forecasted full triangle
<code>Models</code>	linear regression models for each development period
<code>f</code>	chain-ladder age-to-age factors
<code>f.se</code>	standard errors of the chain-ladder age-to-age factors $f$ (assumption CL1)
<code>F.se</code>	standard errors of the true chain-ladder age-to-age factors $F_{ik}$ (square root of the variance in assumption CL2)
<code>sigma</code>	sigma parameter in CL2

Mack.ProcessRisk	variability in the projection of future losses not explained by the variability of the link ratio estimators (unexplained variation)
Mack.ParameterRisk	variability in the projection of future losses explained by the variability of the link-ratio estimators alone (explained variation)
Mack.S.E	total variability in the projection of future losses by the chain-ladder method; the square root of the mean square error of the chain-ladder estimate: $Mack.S.E.^2 = Mack.ProcessRisk^2 + Mack.ParameterRisk^2$
Total.Mack.S.E	total variability of projected loss for all origin years combined
Total.ProcessRisk	vector of process risk estimate of the total of projected loss for all origin years combined by development period
Total.ParameterRisk	vector of parameter risk estimate of the total of projected loss for all origin years combined by development period
weights	weights used
alpha	alphas used
tail	tail factor used. If tail was set to TRUE the output will include the linear model used to estimate the tail factor

**Note**

Additional references for further reading:

*England, PD and Verrall, RJ. Stochastic Claims Reserving in General Insurance (with discussion), British Actuarial Journal 8, III. 2002*

*Barnett and Zehnwirth. Best estimates for reserves. Proceedings of the CAS, LXXXVI I(167), November 2000.*

**Author(s)**

Markus Gesmann <markus.gesmann@gmail.com>

**References**

*Thomas Mack. Distribution-free calculation of the standard error of chain ladder reserve estimates. Astin Bulletin. Vol. 23. No 2. 1993. pp.213:225*

*Thomas Mack. The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor. Astin Bulletin. Vol. 29. No 2. 1999. pp.361:366*

*Murphy, Daniel M. Unbiased Loss Development Factors. Proceedings of the Casualty Actuarial Society Casualty Actuarial Society - Arlington, Virginia 1994; LXXXI 154-222*

*Buchwalder, Bühlmann, Merz, and Wüthrich. The Mean Square Error of Prediction in the Chain Ladder Reserving Method (Mack and Murphy Revisited). Astin Bulletin Vol. 36. 2006. pp.521:542*

**See Also**

See also [qpaid](#) for dealing with non-square triangles, [chainladder](#) for the underlying chain-ladder method, [dfCorTest](#) to check for Calendar Year Effect, [cyEffTest](#) to check for Development Factor Correlation, [summary.MackChainLadder](#), [quantile.MackChainLadder](#), [plot.MackChainLadder](#) and [residuals.MackChainLadder](#) displaying results, [CDR.MackChainLadder](#) for the one year claims development result.

**Examples**

```
## See the Taylor/Ashe example in Mack's 1993 paper
GenIns
plot(GenIns)
plot(GenIns, lattice=TRUE)
GNI <- MackChainLadder(GenIns, est.sigma="Mack")
GNI$f
GNI$sigma^2
GNI # compare to table 2 and 3 in Mack's 1993 paper
plot(GNI)
plot(GNI, lattice=TRUE)

## Different weights
## Using alpha=0 will use straight average age-to-age factors
MackChainLadder(GenIns, alpha=0)$f
# You get the same result via:
apply(GenIns[,-1]/GenIns[,-10],2, mean, na.rm=TRUE)

## Only use the last 5 diagonals, i.e. the last 5 calendar years
calPeriods <- (row(GenIns) + col(GenIns) - 1)
(weights <- ifelse(calPeriods <= 5, 0, ifelse(calPeriods > 10, NA, 1)))
MackChainLadder(GenIns, weights=weights, est.sigma = "Mack")

## Tail
## See the example in Mack's 1999 paper
Mortgage
m <- MackChainLadder(Mortgage)
round(summary(m)$Totals["CV(IBNR)",], 2) ## 26% in Table 6 of paper
plot(Mortgage)
# Specifying the tail and its associated uncertainty parameters
MRT <- MackChainLadder(Mortgage, tail=1.05, tail.sigma=71, tail.se=0.02, est.sigma="Mack")
MRT
plot(MRT, lattice=TRUE)
# Specify just the tail and the uncertainty parameters will be estimated
MRT <- MackChainLadder(Mortgage, tail=1.05)
MRT$f.se[9] # close to the 0.02 specified above
MRT$sigma[9] # less than the 71 specified above
# Note that the overall CV dropped slightly
round(summary(MRT)$Totals["CV(IBNR)",], 2) ## 24%
# tail parameter uncertainty equal to expected value
MRT <- MackChainLadder(Mortgage, tail=1.05, tail.se = .05)
round(summary(MRT)$Totals["CV(IBNR)",], 2) ## 27%

## Parameter-risk (only) estimate of the total reserve = 3142387
```

```

tail(MRT$Total.ParameterRisk, 1) # located in last (ultimate) element
# Parameter-risk (only) CV is about 19%
tail(MRT$Total.ParameterRisk, 1) / summary(MRT)$Totals["IBNR", ]

## Three terms in the parameter risk estimate
## First, the default (Mack) without the tail
m <- MackChainLadder(RAA, mse.method = "Mack")
summary(m)$Totals["Mack S.E.",]
## Then, with the third term
m <- MackChainLadder(RAA, mse.method = "Independence")
summary(m)$Totals["Mack S.E.",] ## Not significantly greater

## One year claims development results
M <- MackChainLadder(MW2014, est.sigma="Mack")
CDR(M)

## For more examples see:
## Not run:
demo(MackChainLadder)

## End(Not run)

```

---

MCLpaid

*Run off triangles of accumulated paid and incurred claims data.*


---

## Description

Run-off triangles based on a fire portfolio

## Usage

```

data(MCLpaid)
data(MCLincurred)

```

## Format

A matrix with 7 origin years and 7 development years.

## Source

*Gerhard Quarg and Thomas Mack. Munich Chain Ladder. Blatter DGVM. 26, Munich, 2004.*

## Examples

```

MCLpaid
MCLincurred
op=par(mfrow=c(2,1))
plot(MCLpaid)
plot(MCLincurred)
par(op)

```

---

MedMal

*Run off triangles of accumulated claim data*

---

**Description**

Run-off triangles of Medical Malpractice Data insurance.

**Usage**

```
data(MedMal)
```

**Format**

U.S. medical malpractice insurance for an experience period of 1969 to 1976. Reported Claims, Paid Claims, Case Outstanding and Open Claims (i.e. the number of outstanding claims) respectively

**Source**

*Berquist, J.R. and Sherman, R.E., Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach, Proceedings of the Casualty Actuarial Society, LXIV, 1977, pp.123-184.*

**Examples**

```
data(MedMal)
names(MedMal)
MedMal$MedMalReported
MedMal$MedMalPaid
MedMal$MedMalOutstanding
MedMal$MedMalOpen
```

---

Mortgage

*Run off triangle of accumulated claims data*

---

**Description**

Development triangle of a mortgage guarantee business

**Usage**

```
data(Mortgage)
```

**Format**

A matrix with 9 accident years and 9 development years.

**Source**

*Competition Presented at a London Market Actuaries Dinner, D.E.A. Sanders, 1990*

**References**

See table 4 in: *Distribution-free Calculation of the Standard Error of Chain Ladder Reserve Estimates*, Thomas Mack, 1993, ASTIN Bulletin **23**, 213 - 225

**Examples**

```
Mortgage
Mortgage
plot(Mortgage)
plot(Mortgage, lattice=TRUE)
```

---

Mse-methods

*Methods for Generic Function Mse*


---

**Description**

Mse is a generic function to calculate mean square error estimations in the chain-ladder framework.

**Usage**

```
Mse(ModelFit, FullTriangles, ...)

## S4 method for signature 'GMCLFit,triangles'
Mse(ModelFit, FullTriangles, ...)
## S4 method for signature 'MCLFit,triangles'
Mse(ModelFit, FullTriangles, mse.method="Mack", ...)
```

**Arguments**

ModelFit	An object of class "GMCLFit" or "MCLFit".
FullTriangles	An object of class "triangles". Should be the output from a call of predict.
mse.method	Character strings that specify the MSE estimation method. Only works for "MCLFit". Use "Mack" for the generalization of the Mack (1993) approach, and "Independence" for the conditional resampling approach in Merz and Wuthrich (2008).
...	Currently not used.

### Details

These functions calculate the conditional mean square errors using the recursive formulas in Zhang (2010), which is a generalization of the Mack (1993, 1999) formulas. In the GMCL model, the conditional mean square error for single accident years and aggregated accident years are calculated as:

$$\hat{m}se(\hat{Y}_{i,k+1}|D) = \hat{B}_k \hat{m}se(\hat{Y}_{i,k}|D) \hat{B}_k + (\hat{Y}'_{i,k} \otimes I) \hat{\Sigma}_{B_k} (\hat{Y}_{i,k} \otimes I) + \hat{\Sigma}_{\epsilon_{i_k}}.$$

$$\hat{m}se\left(\sum_{i=a_k}^I \hat{Y}_{i,k+1}|D\right) = \hat{B}_k \hat{m}se\left(\sum_{i=a_k+1}^I \hat{Y}_{i,k}|D\right) \hat{B}_k + \left(\sum_{i=a_k}^I \hat{Y}'_{i,k} \otimes I\right) \hat{\Sigma}_{B_k} \left(\sum_{i=a_k}^I \hat{Y}_{i,k} \otimes I\right) + \sum_{i=a_k}^I \hat{\Sigma}_{\epsilon_{i_k}}.$$

In the MCL model, the conditional mean square error from Merz and Wüthrich (2008) is also available, which can be shown to be equivalent as the following:

$$\hat{m}se(\hat{Y}_{i,k+1}|D) = (\hat{\beta}_k \hat{\beta}'_k) \odot \hat{m}se(\hat{Y}_{i,k}|D) + \hat{\Sigma}_{\beta_k} \odot (\hat{Y}_{i,k} \hat{Y}'_{i,k}) + \hat{\Sigma}_{\epsilon_{i_k}} + \hat{\Sigma}_{\beta_k} \odot \hat{m}se^E(\hat{Y}_{i,k}|D).$$

$$\hat{m}se\left(\sum_{i=a_k}^I \hat{Y}_{i,k+1}|D\right) = (\hat{\beta}_k \hat{\beta}'_k) \odot \sum_{i=a_k+1}^I \hat{m}se(\hat{Y}_{i,k}|D) + \hat{\Sigma}_{\beta_k} \odot \left(\sum_{i=a_k}^I \hat{Y}_{i,k} \sum_{i=a_k}^I \hat{Y}'_{i,k}\right) + \sum_{i=a_k}^I \hat{\Sigma}_{\epsilon_{i_k}} + \hat{\Sigma}_{\beta_k} \odot \sum_{i=a_k}^I \hat{m}se^E(\hat{Y}_{i,k}|D).$$

For the Mack approach in the MCL model, the cross-product term  $\hat{\Sigma}_{\beta_k} \odot \hat{m}se^E(\hat{Y}_{i,k}|D)$  in the above two formulas will drop out.

### Value

Mse returns an object of class "MultiChainLadderMse" that has the following elements:

mse.ay	conditional mse for each accdient year
mse.ay.est	conditional estimation mse for each accdient year
mse.ay.proc	conditional process mse for each accdient year
mse.total	conditional mse for aggregated accdient years
mse.total.est	conditional estimation mse for aggregated accdient years
mse.total.proc	conditional process mse for aggregated accdient years
FullTriangles	completed triangles

### Author(s)

Wayne Zhang <actuary\_zhang@hotmail.com>

### References

- Zhang Y (2010). *A general multivariate chain ladder model*. Insurance: Mathematics and Economics, 46, pp. 588-599.
- Zhang Y (2010). *Prediction error of the general multivariate chain ladder model*.

**See Also**

See also [MultiChainLadder](#).

---

MultiChainLadder

*Multivariate Chain-Ladder Models*


---

**Description**

The function `MultiChainLadder` implements multivariate methods to forecast insurance loss payments based on several cumulative claims development triangles. These methods are multivariate extensions of the chain-ladder technique, which develop several correlated triangles simultaneously in a way that both contemporaneous correlations and structural relationships can be accounted for. The estimated conditional Mean Square Errors (MSE) are also produced.

**Usage**

```
MultiChainLadder(Triangles, fit.method = "SUR", delta = 1,
  int = NULL, restrict.regMat = NULL, extrap = TRUE,
  mse.method = "Mack", model = "MCL", ...)
```

```
MultiChainLadder2(Triangles, mse.method = "Mack", last = 3,
  type = c("MCL", "MCL+int", "GMCL-int", "GMCL"), ...)
```

**Arguments**

<code>Triangles</code>	a list of cumulative claims triangles of the same dimensions.
<code>fit.method</code>	the method used to fit the multivariate regression in each development period. The default is "SUR" - seemingly unrelated regressions. When "OLS" (Ordinary Least Squares) is used, this is the same as developing each triangle separately.
<code>delta</code>	parameter for controlling weights. It is used to determine the covariance structure $D(Y_{i,k}^{\delta/2})\Sigma_k D(Y_{i,k}^{\delta/2})$ . The default value 1 means that the variance is proportional to the cumulative loss from the previous period.
<code>int</code>	a numeric vector that indicates which development periods have intercepts specified. This only takes effect for <code>model = "GMCL"</code> . The default NULL means that no intercepts are specified.
<code>restrict.regMat</code>	a list of matrix specifying parameter restriction matrix for each period. This is only used for <code>model = "GMCL"</code> . The default value NULL means no restriction is imposed on the development matrix. For example, if there are 3 triangles, there will be 9 parameters in the development matrix for each period if <code>restrict.regMat = NULL</code> . See <code>systemfit</code> for how to specify the appropriate parameter constraints.
<code>extrap</code>	a logical value indicating whether to use Mack's extrapolation method for the last period to get the residual variance estimation. It only takes effect for <code>model = "MCL"</code> . If the data are trapezoids, it is set to be FALSE automatically and a warning message is given.

mse.method	method to estimate the mean square error. It can be either "Mack" or "Independence", which are the multivariate generalization of Mack's formulas and the conditional re-sampling approach, respectively.
model	the structure of the model to be fitted. It is either "MCL" or "GMCL". See details below.
last	an integer. The MultiChainLadder2 function splits the triangles into 2 parts internally (see details below), and the last argument indicates how many of the development periods in the tail go into the second part of the split. The default is 3.
type	the type of the model structure to be specified for the first part of the split model in MultiChainLadder2. "MCL" - the multivariate chain-ladder with diagonal development matrix; "MCL+int" - the multivariate chain-ladder with additional intercepts; "GMCL-int" - the general multivariate chain-ladder without intercepts; and "GMCL" - the full general multivariate chain-ladder with intercepts and non-diagonal development matrix.
...	arguments passed to systemfit.

## Details

This function implements multivariate loss reserving models within the chain-ladder framework. Two major models are included. One is the Multivariate chain-ladder (MCL) model proposed by Prohl and Schmidt (2005). This is a direct multivariate generalization of the univariate chain-ladder model in that losses from different triangles are assumed to be correlated but the mean development in one triangle only depends on its past values, not on the observed values from other triangles. In contrast, the other model, the General Multivariate chain-ladder (GMCL) model outlined in Zhang (2010), extends the MCL model by allowing development dependencies among different triangles as well as the inclusion of regression intercepts. As a result, structurally related triangles, such as the paid and incurred loss triangles or the paid loss and case reserve triangles, can be developed together while still accounting for the potential contemporaneous correlations. While the MCL model is a special case of the GMCL model, it is programmed and listed separately because: a) it is an important model for its own sake; b) different MSE methods are only available for the MCL model; c) extrapolation of the residual variance estimation can be implemented for the MCL model, which is considerably difficult for the GMCL model.

We introduce some details of the GMCL model in the following. Assume  $N$  triangles are available. Denote  $Y_{i,k} = (Y_{i,k}^{(1)}, \dots, Y_{i,k}^{(N)})$  as an  $N \times 1$  vector of cumulative losses at accident year  $i$  and development year  $k$ , where  $(n)$  refers to the  $n$ -th triangle. The GMCL model in development period  $k$  (from development year  $k$  to year  $k+1$ ) is:

$$Y_{i,k+1} = A_k + B_k \cdot Y_{i,k} + \epsilon_{i,k},$$

where  $A_k$  is a column of intercepts and  $B_k$  is the  $N \times N$  development matrix. By default, MultiChainLadder sets  $A_k$  to be zero. This behavior can be changed by appropriately specifying the `int` argument. Assumptions for this model are:

$$E(\epsilon_{i,k} | Y_{i,1}, \dots, Y_{i,I+1-k}) = 0.$$

$$cov(\epsilon_{i,k} | Y_{i,1}, \dots, Y_{i,I+1-k}) = \Sigma_{\epsilon_{i,k}} = D(Y_{i,k}^{\delta/2}) \Sigma_k D(Y_{i,k}^{\delta/2}).$$

losses of different accident years are independent.

$\epsilon_{i,k}$  are symmetrically distributed.

The GMCL model structure is generally over-parameterized. Parameter restrictions are usually necessary for the estimation to be feasible, which can be specified through the `restrict.regMat` argument. We refer the users to the documentation for `systemfit` for details and the demo of the present function for examples.

In particular, if one restricts the development matrix to be diagonal, the GMCL model will reduce to the MCL model. When non-diagonal development matrix is used and the GMCL model is applied to paid and incurred loss triangles, it can reflect the development relationship between the two triangles, as described in Quarg and Mack (2004). The full bivariate model is identical to the "double regression" model described by Mack (2003), which is argued by him to be very similar to the Munich chain-ladder (MuCL) model. The GMCL model with intercepts can also help improve model adequacy as described in Barnett and Zehnwirth (2000).

Currently, the GMCL model only works for trapezoid data, and only implements `mse.method = "Mack"`. The MCL model allows an additional mse estimation method that assumes independence among the estimated parameters. Further, the MCL model using `fit.method = "OLS"` will be equivalent to running univariate chain-ladders separately on each triangle. Indeed, when only one triangle is specified (as a list), the MCL model is equivalent to `MackChainLadder`.

The GMCL model allows different model structures to be specified in each development period. This is generally achieved through the combination of the `int` argument, which specifies the periods that have intercepts, and the `restrict.regMat` argument, which imposes parameter restrictions on the development matrix.

In using the multivariate methods, we often specify separate univariate chain-ladders for the tail periods to stabilize the estimation - there are few data points in the tail and running a multivariate model often produces extremely volatile estimates or even fails. In this case, we can use the subset operator `"["` defined for class `triangles` to split the input data into two parts. We can specify a multivariate model with rich structures on the first part to reflect the multivariate dependencies, and simply apply multiple univariate chain-ladders on the second part. The two models are subsequently joined together using the `Join2Fits` function. We can then invoke the `predict` and `Mse` methods to produce loss predictions and mean square error estimations. They can further be combined via the `JoinFitMse` function to construct an object of class `MultiChainLadder`. See the demo for such examples.

To facilitate such a split-and-join process for most applications, we have created the function `MultiChainLadder2`. This function splits the data according to the last argument (e.g., if `last = 3`, the last three periods go into the second part), and fits the first part according to the structure indicated in the type argument. See the 'Arguments' section for details.

## Value

`MultiChainLadder` returns an object of class `MultiChainLadder` with the following slots:

<code>model</code>	the model structure used, either "MCL" or "GMCL"
<code>Triangles</code>	input triangles of cumulative claims that are converted to class <code>triangles</code> internally.
<code>models</code>	fitted models for each development period. This is the output from the call of <code>systemfit</code> .

<code>coefficients</code>	estimated regression coefficients or development parameters. They are put into the matrix format for the GMCL model.
<code>coefCov</code>	estimated variance-covariance matrix for the regression coefficients.
<code>residCov</code>	estimated residual covariance matrix.
<code>fit.method</code>	multivariate regression estimation method
<code>delta</code>	the value of delta
<code>mse.ay</code>	mean square error matrix for each accident year
<code>mse.ay.est</code>	estimation error matrix for each accident year
<code>mse.ay.proc</code>	process error matrix for each accident year
<code>mse.total</code>	mean square error matrix for all accident years combined
<code>mse.total.est</code>	estimation error matrix for all accident years combined
<code>mse.total.proc</code>	process error matrix for all accident years combined
<code>FullTriangles</code>	the forecasted full triangles of class triangles
<code>int</code>	intercept indicators

### Note

When MultiChainLadder or MultiChainLadder2 fails, the most possible reason is that there is little or no development in the tail periods. That is, the development factor is 1 or almost equal to 1. In this case, the `systemfit` function may fail even for `fit.method = "OLS"`, because the residual covariance matrix  $\Sigma_k$  is singular. The simplest solution is to remove these columns using the "[" operator and fit the model on the remaining part.

Also, we recommend the use of MultiChainLadder2 over MultiChainLadder. The function MultiChainLadder2 meets the need for most applications, is relatively easy to use and produces more stable but very similar results to MultiChainLadder. Use MultiChainLadder only when non-standard situation arises, e.g., when different parameter restrictions are needed for different periods. See the demo for such examples.

### Author(s)

Wayne Zhang <actuary\_zhang@hotmail.com>

### References

- Buchwalder M, Bühlmann H, Merz M, Wüthrich M.V (2006). The mean square error of prediction in the chain-ladder reserving method (Mack and Murphy revisited), ASTIN Bulletin, 36(2), 521-542.*
- Prohl C, Schmidt K.D (2005). Multivariate chain-ladder, Dresdner Schriften zur Versicherungsmathematik.*
- Mack T (1993). Distribution-free calculation of the standard error, ASTIN Bulletin, 23, No.2.*
- Mack T (1999). The standard error of chain-ladder reserve estimates: recursive calculation and inclusion of a tail factor, ASTIN Bulletin, 29, No.2, 361-366.*
- Merz M, Wüthrich M (2008). Prediction error of the multivariate chain ladder reserving method, North American Actuarial Journal, 12, No.2, 175-197.*

Zhang Y (2010). *A general multivariate chain-ladder model*. Insurance: Mathematics and Economics, 46, pp. 588-599.

Zhang Y (2010). *Prediction error of the general multivariate chain ladder model*.

### See Also

See also [MackChainLadder](#), [MunichChainLadder](#), [triangles](#), [MultiChainLadder](#), [summary](#), [MultiChainLadder-method](#) and [plot](#), [MultiChainLadder](#), [missing-method](#).

### Examples

```
# This shows that the MCL model using "OLS" is equivalent to
# the MackChainLadder when applied to one triangle

data(GenIns)
(U1 <- MackChainLadder(GenIns, est.sigma = "Mack"))
(U2 <- MultiChainLadder(list(GenIns), fit.method = "OLS"))

# show plots
parold <- par(mfrow = c(2, 2))
plot(U2, which.plot = 1:4)
plot(U2, which.plot = 5)
par(parold)

# For mse.method = "Independence", the model is equivalent
# to that in Buchwalder et al. (2006)

(B1 <- MultiChainLadder(list(GenIns), fit.method = "OLS",
  mse.method = "Independence"))

# use the unbiased residual covariance estimator
# in Merz and Wuthrich (2008)
(W1 <- MultiChainLadder2(liab, mse.method = "Independence",
  control = systemfit::systemfit.control(methodResidCov = "Theil"))))

## Not run:
# use the iterative residual covariance estimator
for (i in 1:5){
  W2 <- MultiChainLadder2(liab, mse.method = "Independence",
    control = systemfit::systemfit.control(
      methodResidCov = "Theil", maxiter = i))
  print(format(summary(W2)@report.summary[[3]][15, 4:5],
    digits = 6, big.mark = ","))
}

# The following fits an MCL model with intercepts for years 1:7
# and separate chain-ladder models for the rest periods
f1 <- MultiChainLadder2(auto, type = "MCL+int")

# compare with the model without intercepts through residual plots
f0 <- MultiChainLadder2(auto, type = "MCL")
```

```

parold <- par(mfrow = c(2, 3), mar = c(3, 3, 2, 1))
mt <- list(c("Personal Paid", "Personal Incurred", "Commercial Paid"))
plot(f0, which.plot = 3, main = mt)
plot(f1, which.plot = 3, main = mt)
par(parold)

## summary statistics
summary(f1, portfolio = "1+3")@report.summary[[4]]

# model for joint development of paid and incurred triangles
da <- auto[1:2]
# MCL with diagonal development
M0 <- MultiChainLadder(da)
# non-diagonal development matrix with no intercepts
M1 <- MultiChainLadder2(da, type = "GMCL-int")
# Munich chain-ladder
M2 <- MunichChainLadder(da[[1]], da[[2]])
# compile results and compare projected paid to incurred ratios
r1 <- lapply(list(M0, M1), function(x){
  ult <- summary(x)@Ultimate
  ult[, 1] / ult[, 2]
})
names(r1) <- c("MCL", "GMCL")
r2 <- summary(M2)[[1]][, 6]
r2 <- c(r2, summary(M2)[[2]][2, 3])
print(do.call(cbind, c(r1, list(MuCl = r2))) * 100, digits = 4)

## End(Not run)

# To reproduce results in Zhang (2010) and see more examples, use:
## Not run:
demo(MultiChainLadder)

## End(Not run)

```

---

MultiChainLadder-class

*Class "MultiChainLadder" of Multivariate Chain-Ladder Results*


---

## Description

This class includes the first and second moment estimation result using the multivariate reserving methods in chain-ladder. Several primitive methods and statistical methods are also created to facilitate further analysis.

### Objects from the Class

Objects can be created by calls of the form `new("MultiChainLadder", ...)`, or they could also be a result of calls from `MultiChainLadder` or `JoinFitMse`.

### Slots

`model`: Object of class "character". Either "MCL" or "GMCL".  
`Triangles`: Object of class "triangles". Input triangles.  
`models`: Object of class "list". Fitted regression models using `systemfit`.  
`coefficients`: Object of class "list". Estimated regression coefficients.  
`coefCov`: Object of class "list". Estimated variance-covariance matrix of coefficients.  
`residCov`: Object of class "list". Estimated residual covariance matrix.  
`fit.method`: Object of class "character". Could be values of "SUR" or "OLS".  
`delta`: Object of class "numeric". Parameter for weights.  
`int`: Object of class "NullNum". Indicator of which periods have intercepts.  
`mse.ay`: Object of class "matrix". Conditional mse for each accident year.  
`mse.ay.est`: Object of class "matrix". Conditional estimation mse for each accident year.  
`mse.ay.proc`: Object of class "matrix". Conditional process mse for each accident year.  
`mse.total`: Object of class "matrix". Conditional mse for aggregated accident years.  
`mse.total.est`: Object of class "matrix". Conditional estimation mse for aggregated accident years.  
`mse.total.proc`: Object of class "matrix". Conditional process mse for aggregated accident years.  
`FullTriangles`: Object of class "triangles". Completed triangles.  
`restrict.regMat`: Object of class "NullList"

### Extends

Class "`MultiChainLadderFit`", directly. Class "`MultiChainLadderMse`", directly.

### Methods

`$` signature(`x = "MultiChainLadder"`): Method for primitive function "`$`". It extracts a slot of `x` with a specified slot name, just as in list.  
`[[` signature(`x = "MultiChainLadder"`, `i = "numeric"`, `j = "missing"`): Method for primitive function "`[[`". It extracts the `i`-th slot of a "`MultiChainLadder`" object, just as in list. `i` could be a vector.  
`[[` signature(`x = "MultiChainLadder"`, `i = "character"`, `j = "missing"`): Method for primitive function "`[[`". It extracts the slots of a "`MultiChainLadder`" object with names in `i`, just as in list. `i` could be a vector.  
`coef` signature(`object = "MultiChainLadder"`): Method for function `coef`, to extract the estimated development matrix. The output is a list.

- fitted** signature(object = "MultiChainLadder"): Method for function fitted, to calculate the fitted values in the original triangles. Note that the return value is a list of fitted values based on the original scale, not the model scale which is first divided by  $Y_{i,k}^{\delta/2}$ .
- names** signature(x = "MultiChainLadder"): Method for function names, which returns the slot names of a "MultiChainLadder" object.
- plot** signature(x = "MultiChainLadder", y = "missing"): See [plot, MultiChainLadder, missing-method](#).
- residCov** signature(object = "MultiChainLadder"): S4 generic function and method to extract residual covariance from a "MultiChainLadder" object.
- residCor** signature(object = "MultiChainLadder"): S4 generic function and method to extract residual correlation from a "MultiChainLadder" object.
- residuals** signature(object = "MultiChainLadder"): Method for function residuals, to extract residuals from a system of regression equations. These residuals are based on model scale, and will not be equivalent to those on the original scale if  $\delta$  is not set to be 0. One should use rstandard instead, which is independent of the scale.
- resid** signature(object = "MultiChainLadder"): Same as residuals.
- rstandard** signature(model = "MultiChainLadder"): S4 generic function and method to extract standardized residuals from a "MultiChainLadder" object.
- show** signature(object = "MultiChainLadder"): Method for show.
- summary** signature(object = "MultiChainLadder"): See [summary, MultiChainLadder-method](#).
- vcov** signature(object = "MultiChainLadder"): Method for function vcov, to extract the variance-covariance matrix of a "MultiChainLadder" object. Note that the result is a list of Bcov, that is the variance-covariance matrix of the vectorized  $B$ .

### Author(s)

Wayne Zhang <actuary\_zhang@hotmail.com>

### See Also

See also [MultiChainLadder, summary, MultiChainLadder-method](#) and [plot, MultiChainLadder, missing-method](#).

### Examples

```
# example for class "MultiChainLadder"
data(liab)
fit.liab <- MultiChainLadder(Triangles = liab)
fit.liab

names(fit.liab)
fit.liab[[1]]
fit.liab$model
fit.liab@model

do.call("rbind", coef(fit.liab))
vcov(fit.liab)[[1]]
residCov(fit.liab)[[1]]
head(do.call("rbind", rstandard(fit.liab)))
```

---

 MultiChainLadderFit-class

 Class "MultiChainLadderFit", "MCLFit" and "GMCLFit"
 

---

### Description

"MultiChainLadderFit" is a virtual class for the fitted models in the multivariate chain ladder reserving framework, "MCLFit" is a result from the internal call `.FitMCL` to store results in model MCL and "GMCLFit" is a result from the internal call `.FitGMCL` to store results in model GMCL. The two classes "MCLFit" and "GMCLFit" differ only in the presentation of  $B_k$  and  $\Sigma_{B_k}$ , and different methods of `Mse` and `predict` will be dispatched according to these classes.

### Objects from the Class

"MultiChainLadderFit" is a virtual Class: No objects may be created from it. For "MCLFit" and "GMCLFit", objects can be created by calls of the form `new("MCLFit", ...)` and `new("GMCLFit", ...)` respectively.

### Slots

**Triangles:** Object of class "triangles"  
**models:** Object of class "list"  
**B:** Object of class "list"  
**Bcov:** Object of class "list"  
**ecov:** Object of class "list"  
**fit.method:** Object of class "character"  
**delta:** Object of class "numeric"  
**int:** Object of class "NullNum"  
**restrict.regMat:** Object of class "NullList"

### Extends

"MCLFit" and "GMCLFit" extends class "[MultiChainLadderFit](#)", directly.

### Methods

No methods defined with class "MultiChainLadderFit" in the signature.

For "MCLFit", the following methods are defined:

`Mse` signature(`ModelFit = "MCLFit"`, `FullTriangles = "triangles"`): Calculate Mse estimations.

`predict` signature(`object = "MCLFit"`): Predict ultimate losses and complete the triangles. The output is an object of class "triangles".

For "GMCLFit", the following methods are defined:

Mse signature(ModelFit = "GMCLFit", FullTriangles = "triangles"): Calculate Mse estimations.

predict signature(object = "GMCLFit"): Predict ultimate losses and complete the triangles. The output is an object of class "triangles".

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**See Also**

See also [Mse](#).

**Examples**

```
showClass("MultiChainLadderFit")
```

---

MultiChainLadderMse-class

*Class "MultiChainLadderMse"*

---

**Description**

This class is used to define the structure in storing the MSE results.

**Objects from the Class**

Objects can be created by calls of the form `new("MultiChainLadderMse", ...)`, or as a result of a call to `Mse`.

**Slots**

mse.ay: Object of class "matrix"  
mse.ay.est: Object of class "matrix"  
mse.ay.proc: Object of class "matrix"  
mse.total: Object of class "matrix"  
mse.total.est: Object of class "matrix"  
mse.total.proc: Object of class "matrix"  
FullTriangles: Object of class "triangles"

**Methods**

No methods defined with class "MultiChainLadderMse" in the signature.

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**See Also**

See Also [MultiChainLadder](#) and [Mse](#).

**Examples**

```
showClass("MultiChainLadderMse")
```

---

MultiChainLadderSummary-class

*Class "MultiChainLadderSummary"*

---

**Description**

This class stores the summary statistics from a "MultiChainLadder" object. These summary statistics include both model summary and report summary.

**Objects from the Class**

Objects can be created by calls of the form `new("MultiChainLadderSummary", ...)`, or a call from `summary`.

**Slots**

Triangles: Object of class "triangles"  
 FullTriangles: Object of class "triangles"  
 S.E.Full: Object of class "list"  
 S.E.Est.Full: Object of class "list"  
 S.E.Proc.Full: Object of class "list"  
 Ultimate: Object of class "matrix"  
 IBNR: Object of class "matrix"  
 S.E.Ult: Object of class "matrix"  
 S.E.Est.Ult: Object of class "matrix"  
 S.E.Proc.Ult: Object of class "matrix"  
 report.summary: Object of class "list"  
 coefficients: Object of class "list"  
 coefCov: Object of class "list"  
 residCov: Object of class "list"  
 rstandard: Object of class "matrix"  
 fitted.values: Object of class "matrix"  
 residCor: Object of class "matrix"  
 model.summary: Object of class "matrix"  
 portfolio: Object of class "NullChar"



**Arguments**

Paid	cumulative paid claims triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix $P_{ik}$ which is filled for $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$
Incurred	cumulative incurred claims triangle. Assume columns are the development period, use transpose otherwise. A (mxn)-matrix $I_{ik}$ which is filled for $k \leq n + 1 - i; i = 1, \dots, m; m \geq n$
est.sigmaP	defines how $\sigma_{n-1}$ for the Paid triangle is estimated, see est.sigma in <a href="#">MackChainLadder</a> for more details, as est.sigmaP gets passed on to MackChainLadder
est.sigmaI	defines how $\sigma_{n-1}$ for the Incurred triangle is estimated, see est.sigma in <a href="#">MackChainLadder</a> for more details, as est.sigmaI is passed on to MackChainLadder
tailP	defines how the tail of the Paid triangle is estimated and is passed on to <a href="#">MackChainLadder</a> , see tail just there.
tailI	defines how the tail of the Incurred triangle is estimated and is passed on to <a href="#">MackChainLadder</a> , see tail just there.
weights	weights. Default: 1, which sets the weights for all triangle entries to 1. Otherwise specify weights as a matrix of the same dimension as Triangle with all weight entries in [0; 1]. Hence, any entry set to 0 or NA eliminates that age-to-age factor from inclusion in the model. See also 'Details' in MackChainladder function. The weight matrix is the same for Paid and Incurred.

**Value**

MunichChainLadder returns a list with the following elements

call	matched call
Paid	input paid triangle
Incurred	input incurred triangle
MCLPaid	Munich-chain-ladder forecasted full triangle on paid data
MCLIncurred	Munich-chain-ladder forecasted full triangle on incurred data
MackPaid	Mack-chain-ladder output of the paid triangle
MackIncurred	Mack-chain-ladder output of the incurred triangle
PaidResiduals	paid residuals
IncurredResiduals	incurred residuals
QResiduals	paid/incurred residuals
QinverseResiduals	incurred/paid residuals
lambdaP	dependency coefficient between paid chain-ladder age-to-age factors and incurred/paid age-to-age factors
lambdaI	dependency coefficient between incurred chain-ladder ratios and paid/incurred ratios
qinverse.f	chain-ladder-link age-to-age factors of the incurred/paid triangle

rhoP.sigma      estimated conditional deviation around the paid/incurred age-to-age factors  
 q.f              chain-ladder age-to-age factors of the paid/incurred triangle  
 rhoI.sigma      estimated conditional deviation around the incurred/paid age-to-age factors

### Author(s)

Markus Gesmann <markus.gesmann@gmail.com>

### References

*Gerhard Quarg and Thomas Mack. Munich Chain Ladder. Blatter DGVM 26, Munich, 2004.*

### See Also

See also [summary.MunichChainLadder](#), [plot.MunichChainLadder](#), [MackChainLadder](#)

### Examples

```
MCLpaid
MCLincurred
op <- par(mfrow=c(1,2))
plot(MCLpaid)
plot(MCLincurred)
par(op)

# Following the example in Quarg's (2004) paper:
MCL <- MunichChainLadder(MCLpaid, MCLincurred, est.sigmaP=0.1, est.sigmaI=0.1)
MCL
plot(MCL)
# You can access the standard chain-ladder (Mack) output via
MCL$MackPaid
MCL$MackIncurred

# Input triangles section 3.3.1
MCL$Paid
MCL$Incurred
# Parameters from section 3.3.2
# Standard chain-ladder age-to-age factors
MCL$MackPaid$f
MCL$MackIncurred$f
MCL$MackPaid$sigma
MCL$MackIncurred$sigma
# Check Mack's assumptions graphically
plot(MCL$MackPaid)
plot(MCL$MackIncurred)

MCL$q.f
MCL$rhoP.sigma
MCL$rhoI.sigma

MCL$PaidResiduals
MCL$IncurredResiduals
```

```

MCL$QinverseResiduals
MCL$QResiduals

MCL$lambdaP
MCL$lambdaI
# Section 3.3.3 Results
MCL$MCLPaid
MCL$MCLIncurred

```

---

MW2008

*Run-off claims triangle*


---

### Description

Cumulative claims development triangle

### Format

A matrix with 9 accident years and 9 development years.

### Source

*Modelling the claims development result for solvency purposes. Michael Merz, Mario V. Wüthrich. Casualty Actuarial Society E-Forum, Fall 2008.*

### Examples

```

MW2008
plot(MW2008, lattice=TRUE)

```

---

MW2014

*Run-off claims triangle*


---

### Description

Cumulative claims development triangle

### Format

A matrix with 17 accident years and 17 development years.

### Source

*Claims Run-Off Uncertainty: The Full Picture. Michael Merz, Mario V. Wüthrich. Swiss Finance Institute Research Paper No. 14-69. 2014*

**Examples**

```
MW2014
plot(MW2014, lattice=TRUE)
```

---

NullNum-class	<i>Class "NullNum", "NullChar" and "NullList"</i>
---------------	---

---

**Description**

Virtual class for `c("null", "numeric")`, `c("null", "character")` and `c("null", "list")`

**Objects from the Class**

A virtual Class: No objects may be created from it.

**Methods**

No methods defined with class "NullNum" in the signature.

---

PaidIncurredChain	<i>PaidIncurredChain</i>
-------------------	--------------------------

---

**Description**

The Paid-incurred Chain model (Merz, Wuthrich (2010)) combines claims payments and incurred losses information to get a unified ultimate loss prediction.

**Usage**

```
PaidIncurredChain(triangleP, triangleI)
```

**Arguments**

triangleP	Cumulative claims payments triangle
triangleI	Incurred losses triangle.

**Details**

The method uses some basic properties of multivariate Gaussian distributions to obtain a mathematically rigorous and consistent model for the combination of the two information channels.

We assume as usual that  $I=J$ . The model assumptions for the Log-Normal PIC Model are the following:

- Conditionally, given  $\Theta = (\Phi_0, \dots, \Phi_I, \Psi_0, \dots, \Psi_{I-1}, \sigma_0, \dots, \sigma_{I-1}, \tau_0, \dots, \tau_{I-1})$  we have

- the random vector  $(\xi_{0,0}, \dots, \xi_{I,I}, \zeta_{0,0}, \dots, \zeta_{I,I-1})$  has multivariate Gaussian distribution with uncorrelated components given by

$$\xi_{i,j} \sim N(\Phi_j, \sigma_j^2),$$

$$\zeta_{k,l} \sim N(\Psi_l, \tau_l^2);$$

- cumulative payments are given by the recursion

$$P_{i,j} = P_{i,j-1} \exp(\xi_{i,j}),$$

with initial value  $P_{i,0} = \exp(\xi_{i,0})$ ;

- incurred losses  $I_{i,j}$  are given by the backwards recursion

$$I_{i,j-1} = I_{i,j} \exp(-\zeta_{i,j-1}),$$

with initial value  $I_{i,I} = P_{i,I}$ .

- The components of  $\Theta$  are independent and  $\sigma_j, \tau_j > 0$  for all j.

Parameters  $\Theta$  in the model are in general not known and need to be estimated from observations. They are estimated in a Bayesian framework. In the Bayesian PIC model they assume that the previous assumptions hold true with deterministic  $\sigma_0, \dots, \sigma_J$  and  $\tau_0, \dots, \tau_{J-1}$  and

$$\Phi_m \sim N(\phi_m, s_m^2),$$

$$\Psi_n \sim N(\psi_n, t_n^2).$$

This is not a full Bayesian approach but has the advantage to give analytical expressions for the posterior distributions and the prediction uncertainty.

## Value

The function returns:

- **Ult.Loss.Origin** Ultimate losses for different origin years.
- **Ult.Loss** Total ultimate loss.
- **Res.Origin** Claims reserves for different origin years.
- **Res.Tot** Total reserve.
- **s.e.** Square root of mean square error of prediction for the total ultimate loss.

## Note

The model is implemented in the special case of non-informative priors.

## Author(s)

Fabio Concina, <fabio.concina@gmail.com>

## References

Merz, M., Wuthrich, M. (2010). Paid-incurred chain claims reserving method. Insurance: Mathematics and Economics, 46(3), 568-579.

**See Also**

[MackChainLadder](#), [MunichChainLadder](#)

**Examples**

```
PaidIncurredChain(USAApaid, USAAincurred)
```

---

plot-MultiChainLadder *Methods for Function plot*

---

**Description**

Methods for function plot to produce different diagnostic plots for an object of class "MultiChainLadder".

**Usage**

```
## S4 method for signature 'MultiChainLadder,missing'
plot(x, y, which.plot=1:4,
     which.triangle=NULL,
     main=NULL,
     portfolio=NULL,
     lowess=TRUE,
     legend.cex=0.75,...)
```

**Arguments**

x	An object of class "MultiChainLadder".
y	"missing"
which.plot	This specifies which type of plot is desired. Its range is 1:5, but defaults to 1:4. "1" is the barplot of observed losses and predicted IBNR stacked and MSE predictions as error bars; "2" is a trajectory plot of the development pattern; "3" is the residual plot of standardized residuals against the fitted values; "4" is the Normal-QQ plot of the standardized residuals. "5" is the "xyplot" of development with confidence intervals for each accident year. Note that "3" and "4" are not available for portfolio.
which.triangle	This specifies which triangles are to be plotted. Default value is NULL, where all triangles plus the portfolio result will be plotted.
main	It should be a list of titles for each plot. If not supplied, use default titles.
portfolio	It specifies which triangles are to be summed as the portfolio, to be passed on to summary.
lowess	Logical. If TRUE, smoothing lines will be added on residual plots.
legend.cex	plotting parameter to be passes on to cex in legend if which.plot=1.
...	optional graphical arguments.

**See Also**

See also [MultiChainLadder](#)

**Examples**

```
## Not run:
data(liab)
fit.liab <- MultiChainLadder(liab)

# generate diagnostic plots
par(mfcol=(c(3,2)))
plot(fit.liab,which.plot=1:2)

par(mfrow=(c(2,2)))
plot(fit.liab,which.plot=3:4)

plot(fit.liab,which.triangle=1,which.plot=5)
graphics.off()

## End(Not run)
```

---

plot.BootChainLadder *Plot method for a BootChainLadder object*

---

**Description**

plot.BootChainLadder, a method to plot the output of [BootChainLadder](#). It is designed to give a quick overview of a BootChainLadder object and to check the model assumptions.

**Usage**

```
## S3 method for class 'BootChainLadder'
plot(x, mfrow=NULL, title=NULL, log=FALSE,
     which=1:4, ...)
```

**Arguments**

x	output from BootChainLadder
mfrow	see <a href="#">par</a>
title	see <a href="#">title</a>
log	logical. If TRUE the y-axes of the 'latest incremental actual vs. simulated' plot will be on a log-scale
which	if a subset of the plots is required, specify a subset of the numbers 1:4.
...	optional arguments. See <a href="#">plot.default</a> for more details.

## Details

plot.BootChainLadder shows four graphs, starting with a histogram of the total simulated IBNRs over all origin periods, including a rug plot; a plot of the empirical cumulative distribution of the total IBNRs over all origin periods; a box-whisker plot of simulated ultimate claims costs against origin periods; and a box-whisker plot of simulated incremental claims cost for the latest available calendar period against actual incremental claims of the same period. In the last plot the simulated data should follow the same trend as the actual data, otherwise the original data might have some intrinsic trends which are not reflected in the model.

## Note

The box-whisker plot of latest actual incremental claims against simulated claims follows is based on ideas from Barnett and Zehnwirth in: *Barnett and Zehnwirth. The need for diagnostic assessment of bootstrap predictive models*, Insureware technical report. 2007

## Author(s)

Markus Gesmann

## See Also

See also [BootChainLadder](#)

## Examples

```
B <- BootChainLadder(RAA)
plot(B)
plot(B, log=TRUE)
```

---

plot.checkTriangleInflation

*Plot method for a checkTriangleInflation object*

---

## Description

plot.checkTriangleInflation, a method to plot the output of [checkTriangleInflation](#). It is designed to give a quick overview of a checkTriangleInflation object and to check the inflation regression curves.

## Usage

```
## S3 method for class 'checkTriangleInflation'
plot(x, col.line = "black", type = "b", xlab = "dev. period", ylab = NULL, ...)
```

**Arguments**

x	output from checkTriangleInflation
col.line	Default: "black". Line color
type	Default: "b". What type of plot should be drawn
xlab	Default: "dev. period". X axis label
ylab	Default: NULL. Y axis label
...	optional arguments. See <a href="#">plot.default</a> for more details.

**Details**

plot.cyeff.test plots all the columns of the triangle (Development Periods) against the regression curves.

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See Also [checkTriangleInflation](#), [summary.checkTriangleInflation](#)

**Examples**

```
test <- checkTriangleInflation(MedMal$MedMalOutstanding / MedMal$MedMalOpen)
plot(test)
```

---

plot.clark

*Plot Clark method residuals*

---

**Description**

Function to plot the residuals of the Clark LDF and Cape Cod methods.

**Usage**

```
## S3 method for class 'clark'
plot(x, ...)
```

**Arguments**

x	object resulting from a run of the ClarkLDF or ClarkCapeCod functions.
...	not used.

## Details

If Clark's model is appropriate for the actual data, then the standardized residuals should appear as independent standard normal random variables. This function creates four plots of standardized residuals on a single page:

1. By origin
2. By age
3. By fitted value
4. Normal Q-Q plot with results of Shapiro-Wilk test

If the model is appropriate then there should not appear to be any trend in the standardized residuals or any systematic differences in the spread about the line  $y = 0$ . The Shapiro-Wilk p-value shown in the fourth plot gives an indication of how closely the standardized residuals can be considered "draws" from a standard normal random variable.

## Author(s)

Daniel Murphy

## References

Clark, David R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach", *Casualty Actuarial Society Forum*, Fall, 2003

## See Also

[ClarkLDF](#), [ClarkCapeCod](#)

## Examples

```
X <- GenIns
Y <- ClarkLDF(GenIns, maxage=Inf, G="weibull")
plot(Y) # One obvious outlier, shapiro test flunked
X[4,4] <- NA # remove the outlier
Z <- ClarkLDF(GenIns, maxage=Inf, G="weibull")
plot(Z) # Q-Q plot looks good
```

---

plot.cyEffTest

*Plot method for a cyEffTest object*

---

## Description

plot.cyEffTest, a method to plot the output of [cyEffTest](#). It is designed to give a quick overview of a cyEffTest object and to check the assumption of independence between calendar years.

**Usage**

```
## S3 method for class 'cyEffTest'
plot(x, type = "l", xlab = "Z", ylab = "Density",
      main = "Calendar Year Effect", col.area = "gray",
      border = NA, ...)
```

**Arguments**

x	output from cyEffTest
type	Default: "l". What type of plot should be drawn
xlab	Default: "Z". X axis label
ylab	Default: "Density". Y axis label
main	Default: "Calendar Year Effect". Plot title
col.area	Default: "gray". Color of the shaded area.
border	Default: NULL, the color to draw the border. Use border = NA to omit borders.
...	optional arguments. See <a href="#">plot.default</a> for more details.

**Details**

plot.cyEffTest shows the underlying distribution, the test statistic  $Z$  and the relative Confidence Interval. If the test statistic  $Z$  is within the highlighted region the hypothesis of dependence between calendar years could be rejected.

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See Also [cyEffTest](#), [summary.cyEffTest](#)

**Examples**

```
plot(cyEffTest(RAA))
```

---

plot.dfCorTest	<i>Plot method for a dfCorTest object</i>
----------------	---

---

**Description**

plot.dfCorTest, a method to plot the output of [dfCorTest](#). It is designed to give a quick overview of a dfCorTest object and to check the assumption of independence between development factors.

**Usage**

```
## S3 method for class 'dfCorTest'
plot(x, type = "l", xlab = "T", ylab = "Density",
      main = "Development Factor Correlation", col.area = "gray",
      border = NA, ...)
```

**Arguments**

x	output from dfCorTest
type	Default: "l". What type of plot should be drawn
xlab	Default: "Z". X axis label
ylab	Default: "Density". Y axis label
main	Default: "Development Factor Correlation". Plot title
col.area	Default: "gray". Color of the shaded area.
border	Default: NULL, the color to draw the border. Use border = NA to omit borders.
...	optional arguments. See <a href="#">plot.default</a> for more details.

**Details**

plot.dfCorTest shows the underlying distribution, the test statistic  $Z$  and the relative Confidence Interval. If the test statistic  $Z$  is within the highlighted region the hypothesis of correlation between development factors could be rejected.

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See Also [dfCorTest](#), [summary.dfCorTest](#)

**Examples**

```
plot(dfCorTest(RAA))
```

---

plot.MackChainLadder    *Plot method for a MackChainLadder object*

---

**Description**

plot.MackChainLadder, a method to plot the output of [MackChainLadder](#). It is designed to give a quick overview of a MackChainLadder object and to check Mack's model assumptions.

**Usage**

```
## S3 method for class 'MackChainLadder'  
plot(x, mfrow=NULL, title=NULL,  
     lattice=FALSE, which=1:6, ...)
```

**Arguments**

x	output from MackChainLadder
mfrow	see <a href="#">par</a>
title	see <a href="#">title</a>
lattice	logical. Default is set to FALSE and plots as described in the details section are produced. If lattice=TRUE, the function <a href="#">xyplot</a> of the lattice package is used to plot developments by origin period in different panels, plus Mack's S.E.
which	if a subset of the plots is required, specify a subset of the numbers 1:6.
...	optional arguments. See <a href="#">plot.default</a> for more details.

**Details**

plot.MackChainLadder shows six graphs, starting from the top left with a stacked bar-chart of the latest claims position plus IBNR and Mack's standard error by origin period; next right to it is a plot of the forecasted development patterns for all origin periods (numbered, starting with 1 for the oldest origin period), and 4 residual plots. The residual plots show the standardised residuals against fitted values, origin period, calendar period and development period. All residual plot should show no patterns or directions for Mack's method to be applicable. Pattern in any direction can be the result of trends and should be further investigated, see *Barnett and Zehnwirth. Best estimates for reserves. Proceedings of the CAS, LXXXVII(167), November 2000.* for more details on trends.

**Author(s)**

Markus Gesmann

**See Also**

See Also [MackChainLadder](#), [residuals.MackChainLadder](#)

**Examples**

```
plot(MackChainLadder(RAA))
```

---

`plot.MunichChainLadder`*Plot method for a MunichChainLadder object*

---

### Description

`plot.MunichChainLadder`, a method to plot the output of [MunichChainLadder](#) object. It is designed to give a quick overview of a `MunichChainLadder` object and to check the correlation between the paid and incurred residuals.

### Usage

```
## S3 method for class 'MunichChainLadder'  
plot(x, mfrow=c(2,2), title=NULL, ...)
```

### Arguments

<code>x</code>	output from <code>MunichChainLadder</code>
<code>mfrow</code>	see <a href="#">par</a>
<code>title</code>	see <a href="#">title</a>
<code>...</code>	optional arguments. See <a href="#">plot.default</a> for more details.

### Details

`plot.MunichChainLadder` shows four plots, starting from the top left with a barchart of forecasted ultimate claims costs by Munich-chain-ladder (MCL) on paid and incurred data by origin period; the barchart next to it compares the ratio of forecasted ultimate claims cost on paid and incurred data based on the Mack-chain-ladder and Munich-chain-ladder methods; the two residual plots at the bottom show the correlation of (incurred/paid)-chain-ladder factors against the paid-chain-ladder factors and the correlation of (paid/incurred)-chain-ladder factors against the incurred-chain-ladder factors.

### Note

The design of the plots follows those in Quarg's (2004) paper: *Gerhard Quarg and Thomas Mack. Munich Chain Ladder. Blatter DGVFM 26, Munich, 2004.*

### Author(s)

Markus Gesmann

### See Also

See also [MunichChainLadder](#)

### Examples

```
M <- MunichChainLadder(MCLpaid, MCLincurred)
plot(M)
```

---

`predict.TriangleModel` *Prediction of a claims triangle*

---

### Description

The function is internally used by [MackChainLadder](#) to forecast future claims.

### Usage

```
## S3 method for class 'TriangleModel'
predict(object,...)
## S3 method for class 'ChainLadder'
predict(object,...)
```

### Arguments

<code>object</code>	a list with two items: <code>Models</code> , <code>Triangle</code> <code>Models</code> list of linear models for each development period <code>Triangle</code> input triangle to forecast
<code>...</code>	not in use

### Value

`FullTriangle` forecasted claims triangle

### Author(s)

Markus Gesmann

### See Also

See also [chainladder](#), [MackChainLadder](#)

### Examples

```
RAA
CL <- chainladder(RAA)
CL
predict(CL)
```

---

`print.ata`*Print Age-to-Age factors*

---

**Description**

Function to print the results of a call to the `ata` function.

**Usage**

```
## S3 method for class 'ata'  
print(x, ...)
```

**Arguments**

<code>x</code>	object resulting from a call to the <a href="#">ata</a> function
<code>...</code>	further arguments passed to <code>print</code>

**Details**

`print.ata` simply prints [summary.ata](#).

**Value**

A `summary.ata` matrix, invisibly.

**Author(s)**

Daniel Murphy

**See Also**

[ata](#) and [summary.ata](#)

**Examples**

```
x <- ata(GenIns)  
  
## Print ata factors rounded to 3 decimal places, the summary.ata default  
print(x)  
  
## Round to 4 decimal places and print cells corresponding  
## to future observations as blanks.  
print(summary(x, digits=4), na.print="")
```

---

```
print.checkTriangleInflation
```

*Print function for a checkTriangleInflation object*

---

## **Description**

print method for a checkTriangleInflation object

## **Usage**

```
## S3 method for class 'checkTriangleInflation'  
print(x, ...)
```

## **Arguments**

x	object of class checkTriangleInflation
...	optional arguments for a print method

## **Details**

print.checkTriangleInflation show the print of a checkTriangleInflation object.

## **Value**

print.checkTriangleInflation displays the default information resulting from a call of the checkTriangleInflation method

## **Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

## **See Also**

See also [checkTriangleInflation](#), [plot.checkTriangleInflation](#), [summary.checkTriangleInflation](#)

## **Examples**

```
test <- checkTriangleInflation(MedMal$MedMalOutstanding / MedMal$MedMalOpen)  
print(test)
```

---

```
print.clark          Print results of Clark methods
```

---

### Description

Functions to print the results of the ClarkLDF and ClarkCapeCod methods.

### Usage

```
## S3 method for class 'ClarkLDF'
print(x, Amountdigits=0, LDFdigits=3, CVdigits=3,
      row.names = FALSE, ...)

## S3 method for class 'ClarkCapeCod'
print(x, Amountdigits=0, ELRdigits=3, Gdigits=4, CVdigits=3,
      row.names = FALSE, ...)
```

### Arguments

x	object resulting from a run of the ClarkLDF or ClarkCapeCod function.
Amountdigits	number of digits to display to the right of the decimal point for "amount" columns
LDFdigits	number of digits to display to the right of the decimal point for the loss development factor (LDF) column
CVdigits	number of digits to display to the right of the decimal point for the coefficient of variation (CV) column
ELRdigits	number of digits to display to the right of the decimal point for the expected loss ratio (ELR) column
Gdigits	number of digits to display to the right of the decimal point for the "growth function factor" column; default of 4 conforms with the table on pp. 67, 68 of Clark's paper
row.names	logical (or character vector), indicating whether (or what) row names should be printed (same as for <a href="#">print.data.frame</a> )
...	further arguments passed to print

### Details

Display the default information in "pretty format" resulting from a run of the "LDF Method" or "Cape Cod Method" – a "Development-type" exhibit for Clark's "LDF Method," a "Bornhuetter-Ferguson-type" exhibit for Clark's "Cape Cod Method."

As usual, typing the name of such an object at the console invokes its print method.

### Value

data.frames whose columns are the character representation of their respective [summary.ClarkLDF](#) or [summary.ClarkCapeCod](#) data.frames.

**Author(s)**

Daniel Murphy

**References**

Clark, David R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach", *Casualty Actuarial Society Forum*, Fall, 2003

**See Also**

[summary.ClarkLDF](#) and [summary.ClarkCapeCod](#)

**Examples**

```
X <- GenIns
colnames(X) <- 12*as.numeric(colnames(X))
y <- ClarkCapeCod(X, Premium=10000000+400000*0:9, maxage=240)
summary(y)
print(y) # (or simply 'y') Same as summary(y) but with "pretty formats"

## Greater growth factors when projecting to infinite maximum age
ClarkCapeCod(X, Premium=10000000+400000*0:9, maxage=Inf)
```

---

print.cyEffTest	<i>Print function for a cyEffTest object</i>
-----------------	--

---

**Description**

print method for a cyEffTest object

**Usage**

```
## S3 method for class 'cyEffTest'
print(x, ...)
```

**Arguments**

x	object of class cyEffTest
...	optional arguments for a print method

**Details**

print.cyEffTest show the print of a cyEffTest object.

**Value**

print.cyEffTest displays the default information resulting from a call of the cyEffTest method

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See also [cyEffTest](#), [plot.cyEffTest](#), [summary.cyEffTest](#)

**Examples**

```
test <- cyEffTest(RAA)
print(test)
```

---

print.dfCorTest	<i>Print function for a dfCorTest object</i>
-----------------	--

---

**Description**

print method for a dfCorTest object

**Usage**

```
## S3 method for class 'dfCorTest'
print(x, ...)
```

**Arguments**

x	object of class dfCorTest
...	optional arguments for a print method

**Details**

print.dfCorTest show the print of a dfCorTest object.

**Value**

print.dfCorTest displays the default information resulting from a call of the dfCorTest method

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See also [dfCorTest](#), [plot.dfCorTest](#), [summary.dfCorTest](#)

**Examples**

```
test <- dfCorTest(RAA)
print(test)
```

---

qpaid

*Quarterly run off triangle of accumulated claims data*


---

**Description**

Sample data to demonstrate how to work with triangles with a higher development period frequency than origin period frequency

**Usage**

```
data(qpaid); data(qincurred)
```

**Format**

A matrix with 12 accident years and 45 development quarters of claims costs.

**Source**

Made up data for testing purpose

**Examples**

```
dim(qpaid)
dim(qincurred)
op=par(mfrow=c(1,2))
ymax <- max(c(qpaid,qincurred),na.rm=TRUE)*1.05
matplot(t(qpaid), type="l", main="Paid development",
        xlab="Dev. quarter", ylab="$", ylim=c(0,ymax))
matplot(t(qincurred), type="l", main="Incurred development",
        xlab="Dev. quarter", ylab="$", ylim=c(0,ymax))
par(op)
## MackChainLadder expects a quadratic matrix so let's expand
## the triangle to a quarterly origin period.
n <- ncol(qpaid)
Paid <- matrix(NA, n, n)
Paid[seq(1,n,4),] <- qpaid
M <- MackChainLadder(Paid)
plot(M)

# We expand the incurred triangle in the same way
Incurred <- matrix(NA, n, n)
Incurred[seq(1,n,4),] <- qincurred

# With the expanded triangles we can apply MunichChainLadder
MunichChainLadder(Paid, Incurred)

# In the same way we can apply BootChainLadder
# We reduce the size of bootstrap replicates R
# from the default of 999 to 99 purely to reduce run time.
BootChainLadder(Paid, R=99)
```

---

 quantile.MackChainLadder

*quantile function for Mack-chain-ladder*


---

## Description

quantile methods for a MackChainLadder object

## Usage

```
## S3 method for class 'MackChainLadder'
quantile(x, probs=c(0.75, 0.95), na.rm = FALSE,
         names = TRUE, type = 7,...)
```

## Arguments

x	object of class "MackChainLadder"
probs	numeric vector of probabilities with values in [0,1], see <a href="#">quantile</a> for more help
na.rm	not used
names	not used
type	not used
...	not used

## Details

Reserves at the desired quantile using the Cornish-Fisher expansion.

The Cornish-Fisher expansion relies on the first three moments of the reserve risk distribution: The Best estimate resulting from the Chain-Ladder projection, the Mack standard deviation and the skewness of the distribution (for skewness estimation, see references below).

The quantile estimation requires only that the standard Mack assumptions are met.

For details of the underlying calculations, see references below.

## Value

quantile.MackChainLadder gives a list with two elements back:

ByOrigin	data frame with skewness and quantile statistics by origin period
Totals	data frame with total skewness and quantile statistics across all origin periods

## Author(s)

Eric Dal Moro <eric\_dal\_moro@yahoo.com>

## References

Eric Dal Moro and Yuriy Krvavych. *Probability of sufficiency of Solvency II Reserve risk margins: Practical approximations*. ASTIN Bulletin, 47(3), 737-785

Dal Moro, Eric, *A Closed-Form Formula for the Skewness Estimation of Non-Life Reserve Risk Distribution* (September 15, 2013). Available at SSRN: <https://ssrn.com/abstract=2344297> or <https://dx.doi.org/10.2139/ssrn>.

## See Also

See also [MackChainLadder](#)

## Examples

```
M <- MackChainLadder(GenIns, est.sigma="Mack")
quantile(M, c(0.65, 0.75, 0.9))
```

---

QuantileIFRS17

*Quantile estimation for the IFRS 17 Risk Adjustment*

---

## Description

The Quantile IFRS 17 function provides an estimate of the quantile attained on the reserve risk distribution that corresponds to the booked Risk Adjustment plus the Best Estimate.

## Usage

```
QuantileIFRS17(MCL, Correlation, RiskMargin)
```

## Arguments

MCL	a list of MackChainLadder objects
Correlation	Correlation matrix depicting the correlations between each triangle imported. The correlation matrix is of dimension $n \times n$ , with $n$ the number of items in the list of MackChainLadder objects. For correlation estimations between P&C risks, please refer to the article of Arbenz et al. below.
RiskMargin	Input the risk margin as a single number. The risk margin corresponds to the IFRS 17 risk adjustment. It is estimated outside this function and can come from e.g. Solvency 2 standard formula. See International Actuarial Association reference below for details on risk adjustment calculations.

## Details

The IFRS 17 quantile is a mandatory disclosure when producing Financial Statements under the IFRS 17 framework: Such quantile reflects the Probability of Sufficiency of the reserves defined as Best Estimate plus Risk Adjustment i.e. the probability that the reserves will cover any negative deviations up to the disclosed quantile.

When a risk measure other than the quantile measure (Value At risk) is used for determining the Risk Adjustment, the quantile has to be estimated. The purpose of this function is to provide such an estimation on deriving the first three moments of the reserve risk distribution. These moments are estimated on the triangles input into the function. These triangles are projected using chain-ladder methods and the standard Best Estimate, Mack volatility and skewness are estimated. The resulting moments of the different triangles are then aggregated using Fleishman polynomials.

On using a Cornish-Fisher expansion based on the three aggregated moments, the Probability of Sufficiency of the reserves including the Risk Adjustment (given as an input to the function) can be easily derived.

### Value

QuantileIFRS17 returns a vector with the following elements

QuantileIFRS_17	Quantile attained on the reserve risk distribution with the booked Risk Adjustment
Skewness	Skewness of the overall aggregated risk distribution across all triangles
CoV	Coefficient of Variation of the overall aggregated risk distribution across all triangles
Reserve	Sum of reserves of the input MackChainLadder objects

### Note

The use of Fleishman polynomials and Cornish-Fisher expansion imply that the different risks involved in the triangles inputs should be "close to normality". If the risks involved in the input triangles are far from normal distributions (e.g. extreme events, nat cats ...), the proposed framework will not apply and the quantile derived from the function will not be relevant.

### Author(s)

Eric Dal Moro, Yuriy Krvavych

### References

- Thomas Mack. Distribution-free calculation of the standard error of chain ladder reserve estimates. Astin Bulletin. Vol. 23. No 2. 1993. pp.213:225*
- Thomas Mack. The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor. Astin Bulletin. Vol. 29. No 2. 1999. pp.361:366*
- Dal Moro, Krvavych. Probability of sufficiency of Solvency II Reserve risk margins: Practical approximations. ASTIN Bulletin, 47(3), 737-785*
- P. Arbenz, D. Canestraro (2012) Estimating Copulas for Insurance from Scarce Observations, Expert Opinion and Prior Information: A Bayesian Approach, Astin Bulletin, vol. 42(1), pages 271-290.*
- International Actuarial Association (2018) Risk Adjustments for Insurance Contracts under IFRS 17*

**See Also**

See also [MackChainLadder](#), [quantile.MackChainLadder](#)

**Examples**

```
QuantileIFRS17(MCL=list(M1=MackChainLadder(RAA, est.sigma = "Mack"),
  M2=MackChainLadder(GenIns/1000, est.sigma = "Mack")),
  Correlation=matrix(c(1,0.3, 0.3, 1), ncol=2),
  RiskMargin = 20000)
```

---

RAA

*Run off triangle of accumulated claims data*

---

**Description**

Run-off triangle of Automatic Factultative business in General Liability

**Usage**

```
data(RAA)
```

**Format**

A matrix with 10 accident years and 10 development years.

**Source**

*Historical Loss Development*, Reinsurance Association of Ammerica (RAA), **1991**, p.96

**References**

See Also: *Which Stochastic Model is Underlying the Chain Ladder Method?*, Thomas Mack, Insurance Mathematics and Economics, **15**, **2/3**, pp133-138, 1994

*P.D.England and R.J.Verrall, Stochastic Claims Reserving in General Insurance*, British Actuarial Journal, **Vol. 8**, pp443-544, 2002

**Examples**

```
RAA
plot(RAA)
plot(RAA, lattice=TRUE)
```

---

residCov	<i>Generic function for residCov and residCor</i>
----------	---

---

**Description**

residCov and residCor are a generic functions to extract residual covariance and residual correlation from a system of fitted regressions respectively.

**Usage**

```
residCov(object,...)
residCor(object,...)

## S4 method for signature 'MultiChainLadder'
residCov(object,...)
## S4 method for signature 'MultiChainLadder'
residCor(object,...)
```

**Arguments**

object	An object of class "MultiChainLadder".
...	Currently not used.

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**See Also**

See also [MultiChainLadder](#).

---

residuals.MackChainLadder	<i>Extract residuals of a MackChainLadder model</i>
---------------------------	---

---

**Description**

Extract residuals of a [MackChainLadder](#) model by origin-, calendar- and development period.

**Usage**

```
## S3 method for class 'MackChainLadder'
residuals(object, ...)
```

**Arguments**

object	output of <a href="#">MackChainLadder</a>
...	not in use

**Value**

The function returns a data.frame of residuals and standardised residuals by origin-, calendar- and development period.

**Author(s)**

Markus Gesmann

**See Also**

See Also [MackChainLadder](#)

**Examples**

```
RAA
MCL=MackChainLadder(RAA)
MCL

residuals(MCL)
```

---

summary-methods

*Methods for Function summary*

---

**Description**

Methods for function summary to calculate summary statistics from a "MultiChainLadder" object.

**Usage**

```
## S4 method for signature 'MultiChainLadder'
summary(object, portfolio=NULL,...)
```

**Arguments**

object	object of class "MultiChainLadder"
portfolio	character strings specifying which triangles to be summed up as portfolio.
...	optional arguments to summary methods

**Details**

summary calculations the summary statistics for each triangle and the whole portfolio from `portfolio`. `portfolio` defaults to the sum of all input triangles. It can also be specified as "i+j" format, which means the sum of the i-th and j-th triangle as portfolio. For example, "1+3" means the sum of the first and third triangle as portfolio.

**Value**

The summary function returns an object of class "MultiChainLadderSummary" that has the following slots:

<code>Triangles</code>	input triangles
<code>FullTriangles</code>	predicted triangles
<code>S.E.Full</code>	a list of prediction errors for each cell
<code>S.E.Est.Full</code>	a list of estimation errors for each cell
<code>S.E.Proc.Full</code>	a list of process errors for each cell
<code>Ultimate</code>	predicted ultimate losses for each triangle and portfolio
<code>Latest</code>	latest observed losses for each triangle and portfolio
<code>IBNR</code>	predicted IBNR for each triangle and portfolio
<code>S.E.Ult</code>	a matrix of prediction errors of ultimate losses for each triangle and portfolio
<code>S.E.Est.Ult</code>	a matrix of estimation errors of ultimate losses for each triangle and portfolio
<code>S.E.Proc.Ult</code>	a matrix of process errors of ultimate losses for each triangle and portfolio
<code>report.summary</code>	summary statistics for each triangle and portfolio
<code>coefficients</code>	estimated coefficients from <code>systemfit</code> . They are put into the matrix format for GMCL
<code>coefCov</code>	estimated variance-covariance matrix returned by <code>systemfit</code>
<code>residCov</code>	estimated residual covariance matrix returned by <code>systemfit</code>
<code>rstandard</code>	standardized residuals
<code>fitted.values</code>	fitted.values
<code>residCor</code>	residual correlation
<code>model.summary</code>	summary statistics for the coefficients including p-values
<code>portfolio</code>	how portfolio is calculated

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**See Also**

See Also [MultiChainLadder](#)

## Examples

```
data(GenIns)
fit.bbmw=MultiChainLadder(list(GenIns),fit.method="OLS", mse.method="Independence")
summary(fit.bbmw)
```

---

summary.ata

*Summary method for object of class 'ata'*

---

## Description

Summarize the age-to-age factors resulting from a call to the [ata](#) function.

## Usage

```
## S3 method for class 'ata'
summary(object, digits=3, ...)
```

## Arguments

object	object resulting from a call to <a href="#">ata</a>
digits	integer indicating the number of decimal places for rounding the factors. The default is 3. NULL indicates that rounding should take place.
...	not used

## Details

A call to [ata](#) produces a matrix of age-to-age factors with two attributes – the simple and volume weighted averages. `summary.ata` creates a new matrix with the averages appended as rows at the bottom.

## Value

A matrix.

## Author(s)

Dan Murphy

## See Also

See also [ata](#) and [print.ata](#)

## Examples

```
y <- ata(RAA)
summary(y, digits=4)
```

---

summary.BootChainLadder

*Methods for BootChainLadder objects*


---

### Description

summary, print, mean, and quantile methods for BootChainLadder objects

### Usage

```
## S3 method for class 'BootChainLadder'
summary(object, probs=c(0.75,0.95), ...)

## S3 method for class 'BootChainLadder'
print(x, probs=c(0.75,0.95), ...)

## S3 method for class 'BootChainLadder'
quantile(x, probs=c(0.75, 0.95), na.rm = FALSE,
         names = TRUE, type = 7,...)

## S3 method for class 'BootChainLadder'
mean(x, ...)

## S3 method for class 'BootChainLadder'
residuals(object, ...)
```

### Arguments

x, object	output from <a href="#">BootChainLadder</a>
probs	numeric vector of probabilities with values in [0,1], see <a href="#">quantile</a> for more help
na.rm	logical; if true, any NA and NaN's are removed from 'x' before the quantiles are computed, see <a href="#">quantile</a> for more help
names	logical; if true, the result has a names attribute. Set to FALSE for speedup with many 'probs', see <a href="#">quantile</a> for more help
type	an integer between 1 and 9 selecting one of the nine quantile algorithms detailed below to be used, see <a href="#">quantile</a>
...	further arguments passed to or from other methods

### Details

print.BootChainLadder calls summary.BootChainLadder and prints a formatted version of the summary. residuals.BootChainLadder gives the residual triangle of the expected chain-ladder minus the actual triangle back.

**Value**

summary.BootChainLadder, mean.BootChainLadder, and quantile.BootChainLadder, give a list with two elements back:

ByOrigin            data frame with summary/mean/quantile statistics by origin period  
Totals              data frame with total summary/mean/quantile statistics for all origin period

**Author(s)**

Markus Gesmann

**See Also**

See also [BootChainLadder](#)

**Examples**

```
B <- BootChainLadder(RAA, R=999, process.distr="gamma")
B
summary(B)
mean(B)
quantile(B, c(0.75,0.95,0.99, 0.995))
```

---

summary.checkTriangleInflation

*Summary function for a checkTriangleInflation object*

---

**Description**

summary method for a checkTriangleInflation object

**Usage**

```
## S3 method for class 'checkTriangleInflation'
summary(object, ...)
```

**Arguments**

object            object of class checkTriangleInflation  
...               optional arguments for a summary method

**Details**

summary.checkTriangleInflation shows the summary of a checkTriangleInflation object.

**Value**

summary.checkTriangleInflation gives a named numeric array of three rows.

rate	Inflation rate for the specific development period
R2	$R^2$ coefficient for each regression curve
points	Number of points used

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See also [checkTriangleInflation](#), [plot.checkTriangleInflation](#)

**Examples**

```
test <- checkTriangleInflation( MedMal$MedMalOutstanding / MedMal$MedMalOpen )
summary(test)
```

---

summary.clark

*Summary methods for Clark objects*

---

**Description**

summary methods for ClarkLDF and ClarkCapeCod objects

**Usage**

```
## S3 method for class 'ClarkLDF'
summary(object, ...)

## S3 method for class 'ClarkCapeCod'
summary(object, ...)
```

**Arguments**

object	object resulting from a run of the <a href="#">ClarkLDF</a> or <a href="#">ClarkCapeCod</a> functions.
...	not currently used

**Details**

summary.ClarkLDF returns a data.frame that holds the columns of a typical "Development-type" exhibit.

summary.ClarkCapeCod returns a data.frame that holds the columns of a typical "Bornhuetter-Ferguson-type" exhibit.

**Value**

summary.ClarkLDF and summary.ClarkCapeCod return data.frames whose columns are objects of the appropriate mode (i.e., character for "Origin", otherwise numeric)

**Author(s)**

Dan Murphy

**See Also**

See also [ClarkLDF](#)

**Examples**

```
y <- ClarkLDF(RAA)
summary(y)
```

---

summary.cyEffTest	<i>Summary function for a cyEffTest object</i>
-------------------	--

---

**Description**

summary method for a cyEffTest object

**Usage**

```
## S3 method for class 'cyEffTest'
summary(object, ...)
```

**Arguments**

object	object of class cyEffTest
...	optional arguments for a summary method

**Details**

summary.cyEffTest shows the summary of a cyEffTest object.

**Value**

summary.cyEffTest gives a list of three elements back

Table	data frame containing the statistics for each calendar year
Totals	data frame of totals of the main statistics from the dataframe Table
Range	data frame containing the upper and lower limits of the confidence interval range

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See also [cyEffTest](#), [plot.cyEffTest](#)

**Examples**

```
test <- cyEffTest(RAA)
summary(test)
```

---

summary.dfCorTest	<i>Summary function for a dfCorTest object</i>
-------------------	--

---

**Description**

summary method for a dfCorTest object

**Usage**

```
## S3 method for class 'dfCorTest'
summary(object, ...)
```

**Arguments**

object	object of class dfCorTest
...	optional arguments for a summary method

**Details**

summary.dfCorTest shows the summary of a dfCorTest object.

**Value**

summary.dfCorTest gives a list of two elements back

Results	data frame containing the summary statistics
---------	--

Range	data frame containing the upper and lower limits of the confidence interval range
-------	---

**Author(s)**

Marco De Virgilis <devirgilis.marco@gmail.com>

**See Also**

See also [dfCorTest](#), [plot.dfCorTest](#)

**Examples**

```
test <- dfCorTest(RAA)
summary(test)
```

---

```
summary.MackChainLadder
```

*Summary and print function for Mack-chain-ladder*

---

**Description**

summary and print methods for a MackChainLadder object

**Usage**

```
## S3 method for class 'MackChainLadder'
summary(object, ...)
```

```
## S3 method for class 'MackChainLadder'
print(x, ...)
```

**Arguments**

x, object	object of class "MackChainLadder"
...	optional arguments to print or summary methods

**Details**

print.MackChainLadder calls summary.MackChainLadder and prints a formatted version of the summary.

**Value**

summary.MackChainLadder gives a list of two elements back

ByOrigin	data frame with Latest (latest actual claims costs), Dev.To.Date (chain-ladder development to date), Ultimate (estimated ultimate claims cost), IBNR (estimated IBNR), Mack.S.E (Mack's estimation of the standard error of the IBNR), and CV(IBNR) (Coefficient of Variance=Mack.S.E/IBNR)
----------	---

Totals	data frame of totals over all origin periods. The items follow the same naming convention as in ByOrigin above
--------	--

**Author(s)**

Markus Gesmann

**See Also**

See also [MackChainLadder](#), [plot.MackChainLadder](#)

**Examples**

```
R <- MackChainLadder(RAA)
R
summary(R)
summary(R)$ByOrigin$Ultimate
```

---

```
summary.MunichChainLadder
```

*Summary and print function for Munich-chain-ladder*

---

**Description**

summary and print methods for a MunichChainLadder object

**Usage**

```
## S3 method for class 'MunichChainLadder'
summary(object, ...)

## S3 method for class 'MunichChainLadder'
print(x, ...)
```

**Arguments**

x, object      object of class "MunichChainLadder"  
 ...            optional arguments to print or summary methods

**Details**

print.MunichChainLadder calls summary.MunichChainLadder and prints a formatted version of the summary.

**Value**

summary.MunichChainLadder gives a list of two elements back

ByOrigin	data frame with <i>Latest Paid</i> (latest actual paid claims costs), <i>Latest Incurred</i> (latest actual incurred claims position), <i>Latest P/I Ratio</i> (ratio of latest paid/incurred claims), <i>Ult. Paid</i> (estimate ultimate claims cost based on the paid triangle), <i>Ult. Incurred</i> (estimate ultimate claims cost based on the incurred triangle), <i>Ult. P/I Ratio</i> (ratio of ultimate paid forecast / ultimate incurred forecast)
Totals	data frame of totals over all origin periods. The items follow the same naming convention as in ByOrigin above

**Author(s)**

Markus Gesmann

**See Also**

See also [MunichChainLadder](#), [plot.MunichChainLadder](#)

**Examples**

```
M <- MunichChainLadder(MCLpaid, MCLincurred)
M
summary(M)
summary(M)$ByOrigin
```

---

Table65

*Functions to Reproduce Clark's Tables*

---

**Description**

Print the tables on pages 64, 65, and 68 of Clark's paper.

**Usage**

```
Table64(x)
Table65(x)
Table68(x)
```

**Arguments**

x an object resulting from `ClarkLDF` or `ClarkCapeCod`

**Details**

These exhibits give some of the details behind the calculations producing the estimates of future values (a.k.a. "Reserves" in Clark's paper). Table65 works for both the "LDF" and the "CapeCod" methods. Table64 is specific to "LDF", Table68 to "CapeCod".

**Value**

A data.frame.

**Author(s)**

Daniel Murphy

## References

Clark, David R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach", *Casualty Actuarial Society Forum*, Fall, 2003 [https://www.casact.org/sites/default/files/database/forum\\_03fforum\\_03ff041.pdf](https://www.casact.org/sites/default/files/database/forum_03fforum_03ff041.pdf)

## Examples

```
Table65(ClarkLDF(GenIns, maxage=20))
Table64(ClarkLDF(GenIns, maxage=20))

X <- GenIns
colnames(X) <- 12*as.numeric(colnames(X))
Table65(ClarkCapeCod(X, Premium=10000000+400000*0:9, maxage=Inf))
Table68(ClarkCapeCod(X, Premium=10000000+400000*0:9, maxage=Inf))
```

---

triangle S3 Methods     *Generic functions for triangles*

---

## Description

Functions to ease the work with triangle shaped matrix data. A 'triangle' is a matrix with some generic functions.

triangle creates a triangle from the given set of vectors of observed data.

as.triangle attempts to turn its argument into a triangle. Triangles are usually stored in a "long" format in data bases. The function can transform a data.frame into a triangle shape.

as.data.frame turns a triangle into a data frame.

## Usage

```
triangle(..., bycol=FALSE, origin="origin", dev="dev", value="value")

## S3 method for class 'matrix'
as.triangle(Triangle, origin="origin", dev="dev", value="value", ...)
## S3 method for class 'data.frame'
as.triangle(Triangle, origin="origin", dev="dev", value="value", ...)
## S3 method for class 'triangle'
as.data.frame(x, row.names=NULL, optional, lob=NULL, na.rm=FALSE, ...)
as.triangle(Triangle, origin="origin", dev="dev", value="value", ...)
## S3 method for class 'triangle'
plot(x, type = "b", xlab = "dev. period", ylab = NULL, lattice=FALSE, ...)
```

**Arguments**

Triangle	a triangle
bycol	logical. If FALSE (the default) the triangle is filled by rows, otherwise the triangle is filled by columns.
origin	name of the origin period, default is "origin".
dev	name of the development period, default is "dev".
value	name of the value, default is "value".
row.names	default is set to NULL and will merge origin and dev. period to create row names.
lob	default is NULL. The idea is to use lob (line of business) as an additional column to label a triangle in a long format, see the examples for more details.
optional	not used
na.rm	logical. Remove missing values?
x	a matrix of class 'triangle'
xlab	a label for the x axis, defaults to 'dev. period'
ylab	a label for the y axis, defaults to NULL
lattice	logical. If FALSE the function <code>matplot</code> is used to plot the developments of the triangle in one graph, otherwise the <code>xyplot</code> function of the lattice package is used, to plot developments of each origin period in a different panel.
type	type, see <code>plot.default</code>
...	vectors of data in triangle, see details; arguments to be passed to other methods everywhere else.

**Details**

Function `triangle` builds a triangle matrix from the vectors of *known* data provided in `...`. Normally, each of these vectors should be one shorter than the preceeding one. The length of the first vector dictates the number of development periods or origin periods (respectively when `bycol` is FALSE or TRUE). As a special case, the function will build an  $n \times n$  triangle from a single vector of  $n(n+1)/2$  data points.

The names of the arguments in `...` for function `triangle` (when there are more than one) are retained for row/column names. Similarly, the names of the elements of the *first* argument are used as column/row names.

**Warning**

Please note that for the function `as.triangle` the origin and dev. period columns have to be of type numeric or a character which can be converted into numeric.

Also note that when converting from a data.frame to a matrix with `as.triangle`, multiple records with the same origin and dev will be aggregated.

**Author(s)**

Markus Gesmann, Dan Murphy, Vincent Goulet

**Examples**

```

GenIns
plot(GenIns)
plot(GenIns, lattice=TRUE)

## Convert long format into triangle
## Triangles are usually stored as 'long' tables in data bases
head(GenInsLong)
as.triangle(GenInsLong, origin="accyear", dev="devyear", "incurred claims")

X <- as.data.frame(RAA)
head(X)

Y <- as.data.frame(RAA, lob="General Liability")
head(Y)

## Basic creation of a triangle from loss development data
triangle(c(100, 150, 175, 180, 200),
         c(110, 168, 192, 205),
         c(115, 169, 202),
         c(125, 185),
         150)

## Same, with named origin periods
triangle("2012" = c(100, 150, 175, 180, 200),
        "2013" = c(110, 168, 192, 205),
        "2014" = c(115, 169, 202),
        "2015" = c(125, 185),
        "2016" = 150)

## Again, with also named development periods
triangle("2012" = c("12 months" = 100,
                  "24 months" = 150,
                  "36 months" = 175,
                  "48 months" = 180,
                  "60 months" = 200),
        "2013" = c(110, 168, 192, 205),
        "2014" = c(115, 169, 202),
        "2015" = c(125, 185),
        "2016" = 150)

## Quick, simplified usage
triangle(c(100, 150, 175, 110, 168, 115))

```

**Description**

This is a S4 class that has "list" in the data part. This class is created to facilitate validation and extraction of data.

**Objects from the Class**

Objects can be created by calls of the form `new("triangles", ...)`, or use `as(..., "triangles")`, where ... is a "list".

**Slots**

.Data: Object of class "list"

**Extends**

Class "list", from data part. Class "vector", by class "list", distance 2.

**Methods**

**Mse** signature(ModelFit = "GMCLFit", FullTriangles = "triangles"): See [Mse](#)

**Mse** signature(ModelFit = "MCLFit", FullTriangles = "triangles"): See [Mse](#)

[ signature(x = "triangles", i = "missing", j = "numeric", drop = "logical"): Method for primitive function "[" to subset certain columns. If drop=TRUE, rows composed of all "NA"s are removed. Dimensions are not dropped.

[ signature(x = "triangles", i = "missing", j = "numeric", drop = "missing"): Method for primitive function "[" to subset certain columns, where rows composed of all "NA"s are removed. Dimensions are not dropped.

[ signature(x = "triangles", i = "numeric", j = "missing", drop = "logical"): Method for primitive function "[" to subset certain rows. If drop=TRUE, columns composed of all "NA"s are removed. Dimensions are not dropped.

[ signature(x = "triangles", i = "numeric", j = "missing", drop = "missing"): Method for primitive function "[" to subset certain rows, where columns composed of all "NA"s are removed. Dimensions are not dropped.

[ signature(x = "triangles", i = "numeric", j = "numeric", drop = "missing"): Method for primitive function "[" to subset certain rows and columns. Dimensions are not dropped.

[<- signature(x = "triangles", i = "numeric", j = "numeric", value = "list"): Method for primitive function "[<-" to replace one cell in each triangle with values specified in value.

**coerce** signature(from = "list", to = "triangles"): Method to construct a "triangles" object from "list".

**dim** signature(x = "triangles"): Method to get the dimensions. The return value is a vector of length 3, where the first element is the number of triangles, the second is the number of accident years, and the third is the number of development years.

**cbind2** signature(x = "triangles", y="missing"): Method to column bind all triangles using cbind internally.

**rbind2** signature(x = "triangles", y="missing"): Method to row bind all triangles using rbind internally.

**Author(s)**

Wayne Zhang <actuary\_zhang@hotmail.com>

**See Also**

See also [MultiChainLadder](#)

**Examples**

```
data(auto)

# "coerce"
auto <- as(auto,"triangles") # transform "list" to be "triangles"

# method for "["
auto[,4:6,drop=FALSE] # rows of all NA's not dropped
auto[,4:6] # drop rows of all NA's

auto[8:10, ,drop=FALSE] #columns of all NA's not dropped
auto[8:10, ] #columns of all NA's dropped

auto[1:2,1]

# replacement method
auto[1:2,1] <- list(1,2,3)
auto[1,2]

dim(auto)

cbind2(auto[1:2,1])
rbind2(auto[1:2,1])
```

---

tweedieReserve

*Tweedie Stochastic Reserving Model*


---

**Description**

This function implements loss reserving models within the generalized linear model framework in order to generate the full predictive distribution for loss reserves. Besides, it generates also the one year risk view useful to derive the reserve risk capital in a Solvency II framework. Finally, it allows the user to validate the model error while changing different model parameters, as the regression structure and diagnostics on the Tweedie  $p$  parameter.

**Usage**

```
tweedieReserve(triangle, var.power = 1,
               link.power = 0, design.type = c(1, 1, 0),
               rereserving = FALSE, cum = TRUE, exposure = FALSE,
               bootstrap = 1, boot.adj = 0, nsim = 1000,
```

```
proc.err = TRUE, p.optim = FALSE,
p.check = c(0, seq(1.1, 2.1, by = 0.1), 3),
progressBar = TRUE, ...)
```

## Arguments

triangle	An object of class <a href="#">triangle</a> .
var.power	The index ( $p$ ) of the power variance function $V(\mu) = \mu^p$ . Default to $p = 1$ , which is the over-dispersed Poisson model. If NULL, it will be assumed to be in (1, 2) and estimated using the <code>cplm</code> package. See <a href="#">tweedie</a> .
link.power	The index of power link function. The default <code>link.power = 0</code> produces a log link. See <a href="#">tweedie</a> .
design.type	It's a 3 dimension array that specifies the design matrix underlying the GLM. The dimensions represent respectively: origin period, development and calendar period. Accepted values are: 0 (not modelled), 1 (modelled as factor) and 2 (modelled as variable). Default to <code>c(1, 1, 0)</code> , which is the common specification in actuarial literature (origin and development period as factors, calendar period not modelled). If a parameter for the calendar period is specified, a linear regression on the log CY parameter is fitted to estimate future values, thus is recommended to validate them running a plot of the gamma values (see output <code>gamma_y</code> ).
reserving	Boolean, if TRUE the one year risk view loss reserve distribution is derived. Default to FALSE. Note, the runtime can materially increase if set to TRUE.
cum	Boolean, indicating whether the input triangle is cumulative or incremental along the development period. If TRUE, then <code>triangle</code> is assumed to be on the cumulative scale, and it will be converted to incremental losses internally before a GLM is fitted.
exposure	Boolean, if TRUE the exposure defined in the <a href="#">triangle</a> object is specified as offset in the GLM model specification. Default to FALSE.
bootstrap	Integer, it specifies the type of bootstrap for parameter error. Accepted values are: 0 (disabled), 1 (parametric), 2 (semi-parametric). Default to 1.
boot.adj	Integer, it specified the methodology when using semi-parametric bootstrapping. Accepted values are: 0 (cycles until all the values of the pseudo-triangle are $\geq 0$ ), 1 (overwrite negative values to 0.01). Default to 0. Note, runtime can materially increase when set to 0, as it could struggle to find pseudo-triangles $\geq 0$
nsim	Integer, number of simulations to derive the loss reserve distribution. Default to 1000. Note, high num of simulations could materially increase runtime, in particular if a re-reserving algorithm is used as well.
proc.err	Boolean, if TRUE a process error (coherent with the specified model) is added to the forecasted distribution. Default to TRUE.
p.optim	Boolean, if TRUE the model estimates the MLE for the Tweedie's $p$ parameter. Default to FALSE. Recommended to use to validate the Tweedie's $p$ parameter.
p.check	If <code>p.optim=TRUE</code> , a vector of $p$ values for consideration. The values must all be larger than one (if the response variable has exact zeros, the values must all

	be between one and two). Default to <code>c(0, seq(1.1, 2.1, by=0.1), 3)</code> . As fitting the Tweedie p-value isn't a straightforward process, please refer to <a href="#">tweedie.profile</a> , <code>p.vec</code> argument.
<code>progressBar</code>	Boolean, if TRUE a progress bar will be shown in the console to give an indication of bootstrap progress.
<code>...</code>	Arguments to be passed onto the function <code>glm</code> or <code>cpglm</code> such as <code>contrasts</code> or <code>control</code> . It is important that <code>offset</code> and <code>weight</code> should not be specified. Otherwise, an error will be reported and the program will quit.

## Value

The output is an object of class "glm" that has the following components:

<code>call</code>	the matched call.
<code>summary</code>	A data frame containing the predicted loss reserve statistics. The following items are displayed: <ul style="list-style-type: none"> <li>• Latest: Latest paid</li> <li>• Det.Reserve: Deterministic reserve, i.e. the MLE GLM estimate of the Reserve</li> <li>• Ultimate: Ultimate cost, defined as Latest+Det.Reserve</li> <li>• Dev.To.Date: Development to date, defined as Latest/Ultimate</li> </ul>

The following items are available if `bootstrap>0`

- Expected.Reserve: The expected reserve, defined as the average of the reserve simulations. Should be roughly as Det.Reserve.
- Prediction.Error: The prediction error of the reserve, defined as sqrt of the simulations. Please note that if `proc.err=FALSE`, this field contains only the parameter error given by the bootstrap.
- CoV: Coefficient of Variation, defined as Prediction.Error/Expected.Reserve.
- Expected Ultimate: The expected ultimate, defined as Expected.Reserve+Latest.

The following items are available if `bootstrap>0 & reserving=TRUE`

- Expected.Reserve\_1yr: The reserve derived as sum of next year payment and the expected value of the re-reserve at the end of the year. It should be similar to both Expected.Reserve and Det.Reserve. If it isn't, it's recommended to change regression structure and parameters.
- Prediction.Error\_1yr: The prediction error of the prospective Claims Development Result (CDR), as defined by Wüthrich ( $CDR=R(0)-X-R(1)$ ).
- Emergence.Pattern: It's the emergence pattern defined as  $Prediction.Error\_1yr/Prediction.E$

<code>Triangle</code>	The input triangle.
<code>FullTriangle</code>	The completed triangle, where empty cells in the original triangle are filled with model predictions.
<code>model</code>	The fitted GLM, a class of <code>glm</code> or <code>cpglm</code> . It is most convenient to work with this component when model fit information is wanted.
<code>scale</code>	The dispersion parameter <code>phi</code>
<code>bias</code>	The model bias, defined as <code>bias&lt;-sqrt(n/d.f)</code>

GLMReserve	Deterministic reserve, i.e. the MLE GLM estimate of the Reserve
gamma_y	When the calendar year is used, it displays the observed and fitted calendar year (usually called "gamma") factors.
res.diag	It's a data frame for residual diagnostics. It contains: <ul style="list-style-type: none"> <li>• unscaled: The GLM Pearson residuals.</li> <li>• unscaled.biasadj: The GLM Person residuals adjusted by the bias, i.e. <math>\text{unscaled.biasadj} = \text{unscaled} * \text{bias}</math>.</li> <li>• scaled: The GLM Person scaled residuals, i.e. <math>\text{scaled} = \text{unscaled} / \sqrt{\text{phi}}</math>.</li> <li>• scaled, biasadj: The GLM Person scaled residuals adjusted by the bias, i.e. <math>\text{scaled.biasadj} = \text{scaled} * \text{bias}</math>.</li> <li>• dev: Development year.</li> <li>• origin: Origin year.</li> <li>• cy: Calendar year.</li> </ul>
[If bootstrap>1]	
distr.res_ult	The full distribution "Ultimate View"
[If rereserve=TRUE]	
distr.res_1yr	The full distribution "1yr View"

**Warning**

Note that the runtime can materially increase for certain parameter setting. See above for more details.

**Note**

This function was born initially as a fork of the glmReserve by Wayne Zhang. I would like to thank him for his work that permitted me to speed up my coding.

**Author(s)**

Alessandro Carrato MSc FIA OA <alessandro.carrato@gmail.com>

**References**

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- Peters, Shevchenko, Wüthrich, *Model uncertainty in claims reserving within Tweedie's compound poisson models*. Astin Bulletin 39(1). 1-33. 2009
- Renshaw, Verrall. *A stochastic model underlying the chain-ladder technique*. B.A.J. 4, IV. 903-923. 1998

**See Also**

See also [summary.tweedieReserve](#).

**Examples**

```
## Not run:
## Verrall's ODP Model is a Tweedie with p=1, log link and
## origin/development periods as factors, thus c(1,1,0)
res1 <- tweedieReserve(MW2008, var.power=1, link.power=0,
                      design.type=c(1,1,0), rereserving=TRUE,
                      progressBar=TRUE)

## To get directly ultimate view and respective one year view
## at selected percentiles
summary(res1)

#To get other interesting statistics
res1$summary

## In order to validate the Tweedie parameter 'p', it is interesting to
## review its loglikelihood profile. Please note that, given the nature
## of our data, it is expected that we may have some fitting issues for
## given 'p' parameters, thus any results/errors should be considered
## only indicatively. Considering different regression structures is anyway
## recommended. Different 'p' values can be defined via the p.check array
## as input of the function.
## See help(tweedie.profile), p.vec parameter, for further information.
## Note: The parameters rereserving and bootstrap can be set to 0 to speed up
## the process, as they aren't needed.

## Runs a 'p' loglikelihood profile on the parameters
## p=c(0,1.1,1.2,1.3,1.4,1.5,2,3)
res2 <- tweedieReserve(MW2008, p.optim=TRUE,
                      p.check=c(0,1.1,1.2,1.3,1.4,1.5,2,3),
                      design.type=c(1,1,0),
                      rereserving=FALSE, bootstrap=0,
                      progressBar=FALSE)

## As it is possible to see in this example, the MLE of p (or xi) results
## between 0 and 1, which is not possible as Tweedie models aren't
## defined for  $0 < p < 1$ , thus the Error message.
## But, despite this, we can conclude that overall the value p=1 could be
## reasonable for this dataset, as anyway it seems to be near the MLE.

## In order to consider an inflation parameter across the origin period,
## it may be interesting to change the regression structure to c(0,1,1)
## to get the same estimates of the Arithmetic Separation Method, as
## referred in Gigante/Sigalotti.
res3 <- tweedieReserve(MW2008, var.power=1, link.power=0,
                      design.type=c(0,1,1), rereserving=TRUE,
                      progressBar=TRUE)

res3
```

```
## An assessment on future fitted calendar year values (usually defined
## as "gamma") is recommended
plot(res3$gamma_y)

## Model residuals can be plotted using the res.diag output
plot(scaled.biasadj ~ dev, data=res3$res.diag) # Development year
plot(scaled.biasadj ~ cy, data=res3$res.diag) # Calendar year
plot(scaled.biasadj ~ origin, data=res3$res.diag) # Origin year

## End(Not run)
```

---

tweedieReserve methods

*Reserve Risk Capital Report*

---

### Description

Main purpose of this function is to create a report to assess the reserve risk capital given an object of the `tweedieReserve` class. It displays both the ultimate and one year risk views at given percentiles.

### Usage

```
## S3 method for class 'tweedieReserve'
print(x, ...)
## S3 method for class 'tweedieReserve'
summary(object, q = c(0.5, 0.75, 0.9, 0.95, 0.995),...)
```

### Arguments

x	An object of class <code>tweedieReserve</code> .
object	An object of class <code>tweedieReserve</code> .
q	Array of percentiles to be displayed.
...	Not used

### Value

A list with two items

Predicton	a data.frame with ultimate view reserve risk and the one year view reserve risk at the given percentiles.
Diagnostic	Quick diagnostic to show the deterministic reserve vs ultimate view and one year view best estimate. If the model is working properly, then these three value shouldn't be much different.

### Author(s)

Alessandro Carrato MSc FIA OA <alessandro.carrato@gmail.com>

**See Also**

See also [tweedieReserve](#).

**Examples**

```
## Not run:
tw <- tweedieReserve(MW2008, rereserving = TRUE)
summary(tw)
# For comparison
CDR.BootChainLadder(BootChainLadder(MW2008))

## End(Not run)
```

---

UKMotor

*UK motor claims triangle*

---

**Description**

Triangle of cumulative claims payments for four origin (accident) years over time (development years).

**Usage**

```
data("UKMotor")
```

**Format**

A matrix with 7 accident years and 7 development years.

**Source**

<https://www.actuaries.org.uk/system/files/documents/pdf/crm2-D5.pdf>

**References**

Stavros Christofides. Regression models based on log-incremental payments. Claims Reserving Manual. Volume 2 D5. September 1997

**Examples**

```
data(UKMotor)
plot(UKMotor)
MackChainLadder(UKMotor, est.sigma="Mack")
```

---

USAA triangle

*Example paid and incurred triangle data from CAS web site.*

---

**Description**

Paid and incurred triangle data from the United Services Automobile Association company for the private passenger auto liability/medical line of business.

**Usage**

```
data("USAApaid")
```

**Format**

A triangle with 10 accident years and 10 development years.

**Details**

The claims data comes from Schedule P - Analysis of Losses and Loss Expenses in the National Association of Insurance Commissioners (NAIC) database. CAS obtained permission from the NAIC to make this data available to all interested researchers on the CAS website. NAIC Schedule P contains information on claims for major personal and commercial lines for all property-casualty insurers that write business in US.

**Source**

Casualty Actuarial Society

**References**

CAS website: <https://www.casact.org>

**Examples**

```
data(USAApaid)
```

---

vcov.clark

*Covariance Matrix of Parameter Estimates – Clark's methods*

---

**Description**

Function to compute the covariance matrix of the parameter estimates for the ClarkLDF and Clark-CapeCod methods.

**Usage**

```
## S3 method for class 'clark'
vcov(object, ...)
```

**Arguments**

```
object      object resulting from a run of the ClarkLDF or ClarkCapeCod functions.
...         not used.
```

**Details**

The covariance matrix of the estimated parameters is estimated by the inverse of the Information matrix (see Clark, p. 53). This function uses the "FI" and "sigma2" values returned by ClarkLDF and by ClarkCapeCod and calculates the matrix  $-\text{sigma2} \cdot \text{FI}^{-1}$ .

**Author(s)**

Daniel Murphy

**References**

Clark, David R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach", *Casualty Actuarial Society Forum*, Fall, 2003

**See Also**

[ClarkLDF](#), [ClarkCapeCod](#)

**Examples**

```
x <- GenIns
colnames(x) <- 12*as.numeric(colnames(x))
Y <- ClarkCapeCod(x, Premium=10000000+400000*0:9, maxage=240)
round(vcov(Y),6) ## Compare to matrix on p. 69 of Clark's paper

# The estimates of the loglogistic parameters
Y$THETAG
# The standard errors of the estimated parameters
sqrt(tail(diag(vcov(Y)), 2))

# The parameter risks of the estimated reserves are calculated
# according to the formula on p. 54 of Clark's paper. For example, for
# the 5th accident year, pre- and post-multiply the covariance matrix
# by a matrix consisting of the gradient entries for just that accident year
FVgrad5 <- matrix(Y$FutureValueGradient[, 5], ncol=1)
sqrt(t(FVgrad5) %*% vcov(Y) %*% FVgrad5) ## compares to 314,829 in Clark's paper

# The estimated reserves for accident year 5:
Y$FutureValue[5] ## compares to 2,046,646 in the paper
```

```
# Recalculate the parameter risk CV for all accident years in total (10.6% in paper):  
sqrt(sum(t(Y$FutureValueGradient) %*% vcov(Y) %*% Y$FutureValueGradient)) /  
  Y$Total$FutureValue
```

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