

# Package ‘CircNNTSR’

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**Type** Package

**Title** Statistical Analysis of Circular Data using Nonnegative Trigonometric Sums (NNTS) Models

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**Description** Includes functions for the analysis of circular data using distributions based on Nonnegative Trigonometric Sums (NNTS). The package includes functions for calculation of densities and distributions, for the estimation of parameters, for plotting and more.

**Depends** stats

**License** GPL (>= 2)

**LazyLoad** yes

**NeedsCompilation** no

**Repository** CRAN

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CircNNTSR-package	<i>CircNNTSR: An R Package for the statistical analysis of circular data using nonnegative trigonometric sums (NNTS) models</i>
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## Description

A collection of utilities for the statistical analysis of circular and spherical data using nonnegative trigonometric sum (NNTS) models

## Details

Package:	CircNNTSR
Type:	Package
Version:	2.2-1
Date:	2020-02-16
License:	GLP (>=2)
LazyLoad:	yes

Fernandez-Duran, J.J. (2004) proposed a new family of distributions for circular random variables based on nonnegative trigonometric sums. This package provides functions for working with circular distributions based on nonnegative trigonometric sums, including functions for estimating the parameters and plotting the densities.

The distribution function in this package is a circular distribution based on nonnegative trigonometric sums (Fernandez-Duran, 2004). Fejer (1915) expressed a univariate nonnegative trigonometric (Fourier) sum (series), for a variable  $\theta$ , as the squared modulus of a sum of complex numbers, i.e.,

$$\left\| \sum_{k=0}^M c_k e^{ik\theta} \right\|^2 \quad (1)$$

where  $i = \sqrt{-1}$ . From this result, the parameters  $(a_k, b_k)$  for  $k = 1, \dots, M$  of the trigonometric sum of order  $M, T(\theta)$ ,

$$T(\theta) = a_0 + \sum_{k=1}^M (a_k \cos(k\theta) + b_k \sin(k\theta))$$

are expressed in terms of the complex parameters in Equation 1,  $c_k$ , for  $k = 0, \dots, M$ , as  $a_k - ib_k = 2 \sum_{\nu=0}^{n-k} c_{\nu+k} \bar{c}_\nu$ . The additional constraint,  $\sum_{k=0}^n \|c_k\|^2 = \frac{1}{2\pi} = a_0$ , is imposed to make the trigonometric sum to integrate one. Thus,  $c_0$  must be real and positive, and there are  $2*M$  free parameters. Then, the probability density function for a circular (angular) random variable is defined as (Fernandez-Duran, 2004)

$$f(\theta; \underline{a}, \underline{b}, M) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{k=1}^M (a_k \cos(k\theta) + b_k \sin(k\theta)).$$

Note that Equation 1 can also be expressed as a double sum as

$$\sum_{k=0}^M \sum_{m=0}^M c_k \bar{c}_m e^{i(k-m)\theta}$$

The  $\underline{c}$  parameters can also be expressed in polar coordinates as  $c_k = \rho_k e^{i\phi_k}$  for  $\rho_k \geq 0$  and  $\phi_k \in [0, 2\pi)$ ; where  $\rho_k$  is the modulus of  $c_k$  and  $\phi_k$  is the argument of  $c_k$  for  $k = 1, \dots, M$ . Many functions of the packages use as parameters the squared moduli and the arguments of  $c_k$ ,  $\rho_k^2$  and  $\phi_k$ , for  $k = 1, \dots, M$ . We refer to the parameter  $M$  as the number of components in the NNTS.

### Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Maintainer: Maria Mercedes Gregorio Dominguez <mercedes@itam.mx>

### References

- Fernandez-Duran, J.J. (2004). Circular Distributions Based on Nonnegative Trigonometric Sums, *Biometrics*, 60(2), 499-503.
- Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2010). A Likelihood-Ratio Test for Homogeneity in Circular Data. *Journal of Biometrics & Biostatistics*, 1(3), 107. doi:10.4172/2155-6180.1000107
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- Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Testing for Seasonality Using Circular Distributions Based on Nonnegative Trigonometric Sums as Alternative Hypotheses. *Statistical Methods in Medical Research*, 23(3), 279-292. doi:10.1177/0962280211411531.
- Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. *Journal of Statistical Software*, 70(6), 1-19. doi:10.18637/jss.v070.i06

### Examples

```
set.seed(200)
data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componentes for data
```

```
est<-nntsmanifoldnewtonestimation(Turtles_radians,3,iter=100)
est
#plot the estimated density
nntsplot(est$cestimates[,2],3)
#add the histogram to the estimated density plot
plot(Turtles_hist, freq=FALSE, add=TRUE)

b<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
estS<-nntsestimationSymmetric(2,b)
nntsplotSymmetric(estS$coef,2)

M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplot(cest, M)
```

---

Ants

*Movements of ants*

---

### **Description**

Directions chosen by 100 ants in response to an evenly illuminated black target.

### **Usage**

```
data(Ants)
```

### **Format**

Directions chosen by 100 ants in degrees

### **Source**

Randomly selected values by Fisher (1993) from Jander (1957)

### **References**

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

---

Ants_radians	<i>Movements of ants</i>
--------------	--------------------------

---

**Description**

Direction chosen by 100 ants in response to an evenly illuminated black target.

**Usage**

```
data(Ants_radians)
```

**Format**

Directions chosen by 100 ants in radians

**Source**

Randomly selected values by Fisher (1993) from Jander (1957)

**References**

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

---

Datab3fisher	<i>Database B3 from Fisher</i>
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---

**Description**

Database B3 from Fisher et al. (1987)

**Usage**

```
data(Datab3fisher)
```

**Format**

Datab3fisher

**Details**

The dataset Datab3fisher consists of 148 observations of the arrival directions of low-mu showers of cosmic rays (Toyoda *et al.*, 1965; see Fisher *et al.*, 1987, pp. 280-281). The observations are measured in declination and right ascension coordinates.

**Source**

Fisher, et al. (1987)

**References**

Toyoda, Y., Suga, K., Murakami, K., Hasegawa, H., Shibata, S., Domingo, V., Escobar, I., Kamata, K., Bradt, H., Clark, G. and La Pointe, M. (1965). Studies of Primary Cosmic Rays in the Energy Region  $10^{14}$  eV to  $10^{17}$  eV (Bolivian Air Shower Joint Experiment), Proceedings of the International Conference on Cosmic Rays, vol. 2, London, September, 1965, 708–711. London: The Institute of Physics and the Physical Society.

Fisher, N.I., Lewis, T. and Embleton, B.J.J. (1987). Statistical Analysis of Spherical Data, Cambridge U.K.: Cambridge University Press.

---

Datab3fisher\_ready     *Data transformed from Datab3fisher*

---

**Description**

Data transformed from Datab3fisher

**Usage**

```
data(Datab3fisher_ready)
```

**Format**

Datab3fisher\_ready

**Details**

```
datab3fisher[,2] <- 90 + datab3fisher[,2]; datab3fisher_ready <- datab3fisher*(pi/180)
```

---

DataB5FisherSpherical     *Spherical Data on Magnetic Remanence*

---

**Description**

Measurements of magnetic remanence from 52 specimens of red beds from the Bowen Basin, Queensland.

**Usage**

```
data(DataB5FisherSpherical)
```

**Format**

Declination -inclination in degrees

**Source**

P.W. Schmidt

**References**

Fisher N.I., Lewis T. and Embleton B.J.J. (1987) *Statistical Analysis of Spherical Data*. Cambridge University Press, Cambridge. Data B.5.

**Examples**

```
data(DataB5FisherSpherical)
```

---

Datab6fisher

*Database B6 from Fisher et al. (1987)*

---

**Description**

datab6fisher

**Usage**

```
data(Datab6fisher)
```

**Format**

The coordinates are declination and inclination measured in degrees

**Details**

The data-set Datab6fisher contains 107 measurements of magnetic remanence in samples of Precambrian volcanics collected in Northwest Australia. (Schmidt and Embleton, 1985; see Fisher et al., 1987, pp. 285).

**Source**

Fisher, et al. (1987)

**References**

Schmidt, P.W. and Embleton, B.J.J. (1985). Pre-folding and overprint magnetic signatures in Precambrian (~2.9-2.7ga) igneous rocks from the Pilbara Craton and Hamersley Basin, N.W. Australia, *Journal of Geophysical Research*, 90 (B4), 2967–2984.

Fisher, N.I., Lewis, T. and Embleton, B.J.J. (1987). *Statistical Analysis of Spherical Data*, Cambridge U.K.: Cambridge University Press.

---

Datab6fisher\_ready      *Data transformed from datab6fisher*

---

**Description**

Data transformed from datab6fisher

**Usage**

```
data(Datab6fisher_ready)
```

**Format**

Datab6fisher\_ready

**Details**

```
dataaux <- datab6fisher; datab6fisher[,1] <- dataaux[,2]; datab6fisher[,2] <- dataaux[,1]; datab6fisher[,1]
<- 360 - datab6fisher[,1]; datab6fisher[,2] <- 90 + datab6fisher[,2]; datab6fisher_ready <- datab6fisher*(pi/180)
```

---

DataUniformBivariate200obs  
*Uniform Bivariate Circular Data*

---

**Description**

200 realizations of a uniform distribution on the torus

**Usage**

```
data(DataUniformBivariate200obs)
```

**Format**

Angles in radians

EarthquakesPacificMexicogt6

*Date of Occurrence of Earthquakes*

---

**Description**

The time of occurrence of earthquakes of intensity greater than 6.0° Richter with an epicenter occurring in the coast of the Pacific Ocean in Mexico from 1920 to 2002. There is a total of 241 observations.

**Usage**

```
data(EarthquakesPacificMexicogt6)
```

**Format**

Time in years. All observations in the interval (0,1]

**Source**

Mexican Database of Strong Earthquakes. CENAPRED.

---

EarthquakesPacificMexicogt7

*Date of Occurrence of Earthquakes 2*

---

**Description**

The time of occurrence of earthquakes of intensity greater than 7.0° Richter with an epicenter occurring in the coast of the Pacific Ocean in Mexico from 1920 to 2002. There are a total of 76 observations.

**Usage**

```
data(EarthquakesPacificMexicogt7)
```

**Format**

Time in years. All observations in the interval (0,1]

**Source**

Mexican Database of Strong Earthquakes. CENAPRED.

---

HomicidesMexico2005    *Homicides in Mexico during 2005*

---

**Description**

Monthly number of homicides in Mexico during 2005

**Usage**

```
data(HomicidesMexico2005)
```

**Format**

Integer values

**Source**

INEGI (Mexican National Statistical Agency) [www.inegi.gob.mx](http://www.inegi.gob.mx)

---

HurricanesGulfofMexico1951to1970  
*Hurricanes in Mexico from 1951 to 1970*

---

**Description**

The time of occurrence (starting times) of hurricanes in the Gulf of Mexico for the 1951-1970 period. There are a total of 196 observations.

**Usage**

```
data(HurricanesGulfofMexico1951to1970)
```

**Format**

Time in years. All observations in the interval (0,1]

**Source**

<http://weather.unisys.com/hurricane/atlantic/1978/index.html>

---

HurricanesGulfofMexico1971to2008

*Hurricanes in Mexico from 1971 to 2008*

---

### Description

The time of occurrence (starting times) of hurricanes in the Gulf of Mexico for the 1971-2008 period. There are a total of 417 observations

### Usage

```
data(HurricanesGulfofMexico1971to2008)
```

### Format

Time in years. All observations in the interval (0,1]

### Source

<http://weather.unisys.com/hurricane/atlantic/1978/index.html>

---

mnntsdensity

*MNNTS density function*

---

### Description

Density function for the MNNTS model

### Usage

```
mnntsdensity(data, cparams = 1/sqrt(2 * pi), M = 0, R=1)
```

### Arguments

data	Matrix of angles in radians, a column for each dimension, a row for each data point
cparams	Parameters of the model. A vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to dimension one, the next M[2]+1 elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to $\left(\frac{1}{2 * pi}\right)^R$ .
M	Vector of length R with number of components in the MNNTS for each dimension
R	Number of dimensions

**Value**

The function returns the density function evaluated at each row in data

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. *Journal of Statistical Software*, 70(6), 1-19. doi:10.18637/jss.v070.i06

**Examples**

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data<-c(0,pi,pi/2,pi,pi,3*pi/2,pi,2*pi,2*pi,pi)
data<-matrix(data,ncol=2,byrow=TRUE)
data
ccoef<-mnntsrandominitial(M,R)
mnntsdensity(data,ccoef,M,R)

M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
ccoef<-est$cestimates[,3]
mnntsdensity(data,ccoef,M,R)
```

---

mnntsloglik

*MNNTS log-likelihood function*


---

**Description**

Computes the log-likelihood function with MNNTS density for data

**Usage**

```
mnntsloglik(data, cpars = 1/sqrt(2 * pi), M = 0, R = 1)
```

**Arguments**

data	Matrix of angles in radians, a column for each dimension, a row for each data point.
cpars	Parameters of the model. A vector of complex numbers of dimension $\text{prod}(M+1)$ . The first element is a real and positive number. The first $M[1]+1$ elements correspond to dimension one, next $M[2]+1$ elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to $\left(\frac{1}{2*\pi i}\right)^R$ .
M	Vector of length R with number of components in the MNNTS for each dimension.
R	Number of dimensions.

**Value**

The function returns the value of the log-likelihood function for the data.

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

**Examples**

```
M<-c(2,3)
R<-length(M)
data<-c(0,pi,pi/2,pi,pi,3*pi/2,pi,2*pi,2*pi,pi)
data<-matrix(data,ncol=2,byrow=TRUE)
data
ccoef<-mnntsrandominitial(M,R)
mnntsdensity(data,ccoef,M,R)
mnntsloglik(data,ccoef,M,R)
```

---

mnntsmanifoldnewtonestimation

*Parameter estimation for the MNNTS distributions*

---

**Description**

Computes the maximum likelihood estimates of the MNNTS parameters using a Newton algorithm on the hypersphere

**Usage**

```
mnntsmanifoldnewtonestimation(data,M=0,R=1,iter=1000,initialpoint=FALSE,cinitial)
```

**Arguments**

data	Matrix of angles in radians, a column for each dimension, a row for each data point
M	Vector of length R with number of components in the MNNTS for each dimension
R	Number of dimensions
iter	Number of iterations for the Newton algorithm
initialpoint	TRUE if an initial point for the optimization algorithm will be used
cinicial	Initial value for cpars (parameters of the model) for the optimization algorithm. Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The first M[1]+1 elements correspond to dimension one, the next M[2]+1 elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to $\left(\frac{1}{2\pi i}\right)^R$ .

**Value**

cestimates	Matrix of prod(M+1)*(R+1). The first R columns are the parameter number, and the last column is the c parameter's estimators
loglik	Optimum log-likelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after the last iteration

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

- Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1
- Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. *Statistical Applications in Genetics and Molecular Biology*, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

**Examples**

```

set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest*(pi/180)
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est

```

---

mnntsmarginal

*Marginal density function of the MNNTS model*


---

**Description**

Marginal density function for one dimension of the MNNTS model evaluated at a point

**Usage**

```
mnntsmarginal(cestimatesarray, M, component, theta)
```

**Arguments**

cestimatesarray	Matrix of $\text{prod}(M+1)*(R+1)$ . The first $R$ columns are the parameter number, and the last column is the $c$ parameter's estimators
M	Vector of length $R$ with number of components in the MNNTS for each dimension
component	Number of the dimension for computing the marginal
theta	An angle in radians (or a vector of angles)

**Value**

The function returns the density function evaluated at theta

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. *Statistical Applications in Genetics and Molecular Biology*, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

**Examples**

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsmarginal(cest,M,1,pi)
```

---

mnntsplot

*Plots an MNNTS bivariate density*

---

**Description**

Plots the MNNTS bivariate density function

**Usage**

```
mnntsplot(cestimates, M, ...)
```

**Arguments**

cestimates	Matrix of $\text{prod}(M+1)*(R+1)$ . The first $R$ columns are the parameter number, and the last column is the $c$ parameter's estimators. $R=2$ for a bivariate distribution
M	Vector with the number of components in the MNNTS for each dimension
...	Arguments passed to the function plot

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. *Statistical Applications in Genetics and Molecular Biology*, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

### Examples

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplot(cest, M)
```

---

mnntsplotmarginal      *Plots an MNNTS marginal density*

---

### Description

Plots the MNNTS marginal density function

### Usage

```
mnntsplotmarginal(cestimates, M, component, ...)
```

### Arguments

cestimates	Matrix of $\text{prod}(M+1)*(R+1)$ . The first R columns are the parameter number, and the last column the c parameter's estimators. The matrix could be the output of <code>mnntsmanifoldnewtonestimation \$cestimates</code>
M	Vector with number of components in the MNNTS for each dimension
component	Number of the dimension for computing the marginal density
...	Arguments passed to the function <code>plot</code>

### Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

### References

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. *Statistical Applications in Genetics and Molecular Biology*, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

**Examples**

```

set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplotmarginal(cest, M, 1)
mnntsplotmarginal(cest, M, 2)

```

---

mnntsplotwithmarginals

*Plots an MNNTS bivariate density together with the marginals*

---

**Description**

Plots the MNNTS bivariate density function together with the marginals

**Usage**

```
mnntsplotwithmarginals(cestimates, M, ...)
```

**Arguments**

cestimates	Matrix of $\text{prod}(M+1)*(R+1)$ . The first R columns are the parameter number, and the last column the c parameter's estimators. The matrix could be the output of <code>mnntsmanifoldnewtonestimation\$cestimates</code> .
M	Vector of length R with number of components in the MNNTS for each dimension
...	Arguments passed to the function plot

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2014). Modeling Angles in Proteins and Circular Genomes Using Multivariate Angular Distributions Based on Nonnegative Trigonometric Sums. *Statistical Applications in Genetics and Molecular Biology*, 13(1), 1-18. doi:10.1515/sagmb-2012-0012

**Examples**

```
set.seed(200)
M<-c(2,3)
R<-length(M)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,M,R,100)
est
cest<-est$cestimates
mnntsplotwithmarginals(cest, M)
```

---

mnntsrandominitial      *Initial random point*

---

**Description**

This function generates a random point on the surface of the  $\text{prod}(M+1)$ -dimensional unit hypersphere

**Usage**

```
mnntsrandominitial(M = 1, R = 1)
```

**Arguments**

M	Vector of length R with number of components in the MNNTS for each dimension
R	Number of dimensions

**Value**

Returns a valid initial point for estimation functions

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

**Examples**

```
M<-c(2,3)
R<-length(M)
mnntsrandominitial(M,R)
```

---

mnytssimulation	<i>MNNTS density simulation function</i>
-----------------	--

---

**Description**

Simulation for the density function for the MNNTS model

**Usage**

```
mnytssimulation(nsim=1, cpars = 1/(2 * pi), M = c(0,0), R=2)
```

**Arguments**

nsim	Number of simulations
cpars	Parameters of the model. A vector of complex numbers of dimension $\text{prod}(M+1)$ . The first element is a real and positive number. The first $M[1]+1$ elements correspond to dimension one, next $M[2]+1$ elements correspond to dimension two, and so on. The sum of the SQUARED moduli of the c parameters must be equal to $1/(2*\pi)$ .
M	Vector of length R with number of components in the MNNTS for each dimension
R	Number of dimensions

**Value**

simulations	The function generates nsim random values from the MNNTS density function
conteo	Number of uniform random numbers used for simulations

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2009) Multivariate Angular Distributions Based on Multiple Nonnegative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C09.1

**Examples**

```
M<-c(2,3)
R<-length(M)
ccoef<-mnytssrandominitial(M,R)
data<-mnytssimulation(10,ccoef,M,R)
data
```

---

Nest	<i>Nest orientations and creek directions</i>
------	---

---

**Description**

Orientation of nests of 50 noisy scrub birds ( $\theta$ ) along the bank of a creek bed, together with the corresponding directions ( $\phi$ ) of creek flow at the nearest point to the nest.

**Usage**

```
data(Nest)
```

**Format**

Orientation of 50 nests (vectors)

**Source**

Data supplied by Dr. Graham Smith

**References**

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

---

nntsABcoefficients	<i>AB coefficients</i>
--------------------	------------------------

---

**Description**

This function transforms the complex parameters  $c$  to the parameters  $ab$  for a reparameterization of the density function

**Usage**

```
nntsABcoefficients(cpars = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

cpars	Vector of complex numbers of dimension $M+1$ . The first element is a real and positive number. The sum of the SQUARED moduli of the $c$ parameters must be equal to $1/(2*\pi)$ .
M	Number of components in the NNTS

**Value**

The function returns the parameters `ab` associated with the parameters `cpars` and returns a vector of real numbers of size  $2*M$ , where the first  $M$  elements are the  $a_k$ ,  $k=1,\dots,M$ , and the next  $M$  elements are the  $b_k$ ,  $k=1,\dots,M$

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```
#random generation of c parameters
ccoef<-nntsrandominitial(3)
ccoef
ab<-nntsABcoefficients(ccoef,3)
ab
```

---

nntsABcoefficientsSymmetric  
*AB coefficients*

---

**Description**

This function transforms the complex parameters `c` to the parameters `ab` for a reparameterization of the density function

**Usage**

```
nntsABcoefficientsSymmetric(cpars = c(0,0), M = 0)
```

**Arguments**

<code>cpars</code>	Vector of complex numbers of dimension $2*M$
<code>M</code>	Number of components in the NNTS

**Value**

The function returns the parameters `ab` associated with the parameters `cpars`

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

---

nntsABDensity      *Density function with AB coefficients*

---

**Description**

Density function expressed in terms of the ab parameters at theta

**Usage**

```
nntsABDensity(theta, cpars = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

theta	Vector of angles in radians
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi)
M	Number of components in the NNTS

**Value**

Returns the density function in terms of the ab coefficients evaluated at theta

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```
ccoef<-nntsrandominitial(3)
nntsABDensity(1,ccoef,3)
nntsABDensity(1+2*pi,ccoef,3)
```

---

nntsABDensitySymmetric      *Density function with AB coefficients*

---

**Description**

Density function expressed in terms of the ab parameters at theta

**Usage**

```
nntsABDensitySymmetric(cpars = c(0, 0), M = 0, theta)
```

**Arguments**

theta	Vector of angles in radians
cpars	Vector of complex numbers of dimension $2 \cdot M$
M	Number of components in the NNTS

**Value**

Returns the density function in terms of the ab coefficients evaluated at theta

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

---

nntsdensity	<i>NNTS density function</i>
-------------	------------------------------

---

**Description**

Density function for the NNTS model

**Usage**

```
nntsdensity(data, cpars = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

data	Vector of angles in radians
cpars	Vector of complex numbers of dimension $M+1$ . The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to $1/(2 \cdot \pi)$ .
M	Number of components in the NNTS

**Value**

The function returns the density function evaluated at each point in data

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

- Fernandez-Duran, J.J. (2004). Circular Distributions Based on Nonnegative Trigonometric Sums, *Biometrics*, 60(2), 499-503.
- Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. *Journal of Statistical Software*, 70(6), 1-19. doi:10.18637/jss.v070.i06

**Examples**

```

ccoef<-nntsrandominitial(3)
nntsdensity(1,ccoef,3)
nntsdensity(1+pi,ccoef,3)
nntsdensity(c(1,1+pi),ccoef,3)

```

---

nntsDensityInterval0to1

*NNTS density function for a variable defined in the interval [0,1)*


---

**Description**

Computes the density function at theta for a variable defined in the interval [0,1)

**Usage**

```
nntsDensityInterval0to1(S, cpars = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

S	Vector of values defined in the interval [0,1) at which the density function is computed
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to $1/(2*\pi)$
M	Number of components in the NNTS

**Details**

This function computes the density function of a variable S (S in the interval [0,1)). If theta is defined in radians (theta in the interval  $[0,2*\pi)$ ), the relation between S and theta is  $\theta=2*\pi*S$ .

**Value**

Value of the density function at each component of S

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```

ccoef<-nntsrandominitial(3)
nntsDensityInterval0to1(c(.8,1.8),ccoef,3)

```

---

nntsDistribution      *NNTS Distribution function*

---

**Description**

Cumulative distribution function in terms of the c parameters at theta, measured in radians  $[0, 2\pi]$ .

**Usage**

```
nntsDistribution(theta, cparams = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

theta	Vector of angles in radians at which the distribution is computed
cparams	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to $1/(2\pi)$ .
M	Number of components in the NNTS

**Value**

The function returns the value of the distribution function evaluated at each component of theta

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```
ccoef<-nntsrandominitial(3)
nntsDistribution(c(0,pi/2,pi,2*pi-0.00000001,2*pi),ccoef,3)
```

---

```
nntsDistributioninterval0to1
```

*NNTS distribution function for the incidence data defined in the interval  $[0, 1]$*

---

**Description**

Computes the distribution function at theta for the incidence data (number of observed values in certain intervals defined in the interval  $[0, 1]$ )

**Usage**

```
nntsDistributioninterval0to1(theta, cparams = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

theta	Value at which the distribution function is computed
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to $1/(2*\pi)$ .
M	Number of components in the NNTS

**Value**

The function returns the value of the distribution function at theta

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```
cpars<-nntsrandominitial(2)
nntsDistributioninterval0to1(pi, cpars, 2)
```

---

`nntsDistributioninterval0to2pi`

*NNTS distribution function for data defined in the interval  $[0, 2*\pi)$*

---

**Description**

Computes the distribution function for the data at theta

**Usage**

```
nntsDistributioninterval0to2pi(theta, cpars = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

theta	Value at which the distribution function is computed
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to $1/(2*\pi)$ .
M	Number of components in the NNTS

**Value**

The function returns the value of the distribution function at theta

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```

cpars<-nntsrandominitial(3)
nntsDistributioninterval0to2pi(0, cpars, 3)
nntsDistributioninterval0to2pi(pi, cpars, 3)
nntsDistributioninterval0to2pi(2*pi-0.00000001, cpars, 3)
nntsDistributioninterval0to2pi(2*pi, cpars, 3)
nntsDistributioninterval0to2pi(3*pi, cpars, 3)

```

---

nntsestimationSymmetric

*NNTS Symmetric Coefficient estimation*


---

**Description**

Computes the maximum likelihood estimates of the symmetric NNTS parameters

**Usage**

```
nntsestimationSymmetric(M = 0, data, maxit = 500)
```

**Arguments**

M	Number of components in the NNTS
data	Vector of angles in radians
maxit	Maximum number of iterations in the optimization algorithm

**Value**

coef	Vector of length M+1. The first M components are the squared moduli of the c parameters, and the last number is the mean of symmetry
loglik	Optimum log-likelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
convergence	An integer code: zero indicates successful convergence; error codes are the following: one indicates that the iteration limit maxit has been reached, and 10 indicates degeneracy of the Nelder-Mead simplex

**Note**

For the maximization of the loglikelihood function the function constrOptim from the package stats is used

**Author(s)**

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

**Examples**

```
b<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
estS<-nntsestimationSymmetric(2,b)
nntsplotSymmetric(estS$coef,2)
```

---

nntsloglik	<i>NNTS log-likelihood function</i>
------------	-------------------------------------

---

**Description**

Computes the log-likelihood function with NNTS density for data

**Usage**

```
nntsloglik(data, cpars = 1/sqrt(2 * pi), M = 0)
```

**Arguments**

data	Vector with observed angles in radians.
cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to $1/(2*\pi)$ .
M	Number of components in the NNTS

**Value**

The function returns the value of the log-likelihood function for the data

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J. (2004). Circular Distributions Based on Nonnegative Trigonometric Sums, *Biometrics*, 60(2), 499-503.

**Examples**

```
a<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
est<-nntsmanifoldnewtonestimation(a,2)
ccoef<-est$cestimates[,2]
nntsloglik(a,ccoef,2)
```

---

`nntslolikInterval0to1`

*NNTS log-likelihood function for the incidence data defined in the interval [0,1)*

---

### Description

Computes the log-likelihood function for incidence data (number of observed values in certain intervals defined in the interval [0,1))

### Usage

```
nntslolikInterval0to1(data, cutpoints, cpars = 1/sqrt(2 * pi), M = 0)
```

### Arguments

<code>data</code>	Number of observations in each interval
<code>cutpoints</code>	Vector of size <code>length(data)+1</code> with the limits of the intervals
<code>cpars</code>	Vector of complex numbers of dimension <code>M+1</code> . The first element is a real and positive number. The sum of the SQUARED moduli of the <code>c</code> parameters must be equal to $1/(2*\pi)$ .
<code>M</code>	Number of components in the NNTS

### Value

The function returns the value of the log-likelihood function for data

### Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

### Examples

```
data<-c(1,2,6,4,1)
cutpoints<-c(0,0.2,0.4,0.6,0.8,0.9999999)
cpars<-nntsrandominitial(1)
nntslolikInterval0to1(data, cutpoints, cpars, 1)
```

---

nntsloglikInterval0to2pi

*NNTS log-likelihood function for the incidence data defined in the interval  $[0, 2\pi)$*

---

### Description

Computes the log-likelihood function for incidence data (number of observed values in certain intervals defined in the interval  $[0, 2\pi)$ )

### Usage

```
nntsloglikInterval0to2pi(data, cutpoints, cpars = 1/sqrt(2 * pi), M = 0)
```

### Arguments

data	Number of observations in each interval
cutpoints	Vector of size $\text{length}(\text{data})+1$ with the limits of the exhaustive and mutually exclusive intervals in which the interval $[0, 2\pi)$ is divided.
cpars	Vector of complex numbers of dimension $M+1$ . The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to $1/(2\pi)$ .
M	Number of components in the NNTS density

### Value

The function returns the value of the log-likelihood function for the data

### Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

### Examples

```
data<-c(2,3,6,4)
cutpoints<-c(0,pi/2,pi,3*pi/2,2*pi-0.00000001)
est<-nntsmanifoldnewtonestimationinterval0to2pi(data,cutpoints,M=1)
cpars<-est$cestimates[,2]
nntsloglikInterval0to2pi(data,cutpoints,cpars,M=1)
```

---

nntslolikSymmetric *NNTS symmetric log-likelihood function*

---

**Description**

Computes the log-likelihood function with NNTS symmetric density for the data

**Usage**

```
nntslolikSymmetric(cpars = c(0, 0), M = 0, data)
```

**Arguments**

cpars	Vector of real numbers of dimension M+1. The first M numbers are the squared moduli of the c parameters. The sum must be less than $1/(2*\pi)$ . The last number is the mean of symmetry
M	Number of components in the NNTS
data	Vector with angles in radians. The first column is used if data are a matrix

**Value**

The function returns the value of the log-likelihood function for the data

**Note**

The default values provide the Uniform circular log-likelihood for the data

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

**Examples**

```
nntslolikSymmetric(c(.01, .02, 2), 2, t(c(pi, pi/2, 2*pi, pi)))
```

---

 nntsmanifoldnewtonestimation

*Parameter estimation for NNTS distributions*


---

### Description

Computes the maximum likelihood estimates of the NNTS parameters, using a Newton algorithm on the hypersphere

### Usage

```
nntsmanifoldnewtonestimation(data, M=0, iter=1000, initialpoint = FALSE, cinitial)
```

### Arguments

data	Vector of angles in radians
M	Number of components in the NNTS
iter	Number of iterations
initialpoint	TRUE if an initial point for the optimization algorithm will be used
cinital	Vector of size M+1. The first element is real and the next M elements are complex (values for $c_0$ and $c_1, \dots, c_M$ ). The sum of the squared moduli of the parameters must be equal to $1/(2*\pi)$

### Value

cestimates	Matrix of $(M+1) \times 2$ . The first column is the parameter numbers, and the second column is the c parameter's estimators
loglik	Optimum log-likelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after the last iteration

### Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

### References

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2010). Maximum Likelihood Estimation of Nonnegative Trigonometric Sums Models by Using a Newton-like Algorithm on Manifolds, Working Paper, Department of Statistics, ITAM, DE-C10.8

**Examples**

```

set.seed(200)
a<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
#Estimation of the NNTSdensity with 2 components for data and 200 iterations
nntsmanifoldnewtonestimation(a,2,iter=200)

data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componetes for data
nntsmanifoldnewtonestimation(Turtles_radians,3,iter=200)

```

---

```
nntsmanifoldnewtonestimationgradientstop
```

*Maximum likelihood estimates of the NNTS parameters*

---

**Description**

Computes the maximum likelihood estimates of the NNTS parameters, using a Newton algorithm on the hypersphere with the option to specify a minimum value of the norm of the gradient error to stop the algorithm

**Usage**

```
nntsmanifoldnewtonestimationgradientstop(data, M = 0, iter = 1000, initialpoint = FALSE,
cinitial,gradientstop=1e-10)
```

**Arguments**

data	Vector of angles in radians
M	Number of components in the NNTS
iter	Number of iterations
initialpoint	TRUE if an initial point for the optimization algorithm will be used
cinitial	Vector of size M+1. The first element is real and the next M elements are complex (values for $c_0$ and $c_1, \dots, c_M$ ). The sum of the squared moduli of the parameters must be equal to $\frac{1}{7}2\pi$
gradientstop	The value of the norm of the gradient error of the Newton algorithm at which the algorithms stops

**Value**

cestimates	Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter???'s estimators
loglik	Optimum log-likelihood value
AIC	Value of Akaike???'s Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after the last iteration

**Author(s)**

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2010). Maximum Likelihood Estimation of Nonnegative Trigonometric Sums Models by Using a Newton-like Algorithm on Manifolds, Working Paper, Department of Statistics, ITAM, DE-C10.8

**Examples**

```
set.seed(200)
a<-c(runif(10,3*pi/2,2*pi-0.00000001),runif(10,pi/2,pi-0.00000001))
#Estimation of the NNTSdensity with 2 components for data and gradientstop at 1e-12
nntsmanifoldnewtonestimationgradientstop(a,2,gradientstop=1e-12)
data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componetes for data and gradientstop at 1e-12
nntsmanifoldnewtonestimationgradientstop(Turtles_radians,3,gradientstop=1e-12)
```

---

nntsmanifoldnewtonestimationinterval0to1

*Parameter estimation for grouped data defined in [0,1)*

---

**Description**

Parameter estimation for incidence data (number of observed values in certain intervals defined over [0,1))

**Usage**

```
nntsmanifoldnewtonestimationinterval0to1(data, cutpoints, subintervals, M = 0, iter=1000,
initialpoint = FALSE, cinitial)
```

**Arguments**

data	Frequency of data on each interval
cutpoints	Vector with the limits of intervals. The length of cutpoints must be one plus the length of the data
subintervals	Number of intervals
M	Number of components in the NNTS
iter	Number of iterations
initialpoint	TRUE if an initial point for the optimization algorithm will be used
cinicial	Vector of size M+1. The first element is real and the next M elements are complex (values for $c_0$ and $c_1, \dots, c_M$ ). The sum of the squared moduli of the parameters must be equal to $1/(2*\pi)$

**Value**

cestimates	Matrix of $M+1 * 2$ . The first column is the parameter numbers and the second column is the c parameter's estimators
loglik	Optimum loglikelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after the last iteration

**Author(s)**

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

**Examples**

```
data<-c(1,2,4,6,1)
cutpoints<-c(0,0.2,0.4,0.6,0.8,0.99999999)
nntsmanifoldnewtonestimationinterval0to1(data, cutpoints, length(data), 1)
```

---

nntsmanifoldnewtonestimationinterval0to2pi

*Parameter estimation for grouped data defined in  $[0,2*\pi)$*

---

**Description**

Parameter estimation for incidence data (number of observed values in certain intervals defined over  $[0,2*\pi)$ )

**Usage**

```
nntsmanifoldnewtonestimationinterval0to2pi(data, cutpoints,
subintervals,M = 0, iter=1000, initialpoint = FALSE, cinitial)
```

**Arguments**

data	Frequency of data on each interval
cutpoints	Vector with the limits of intervals. The length of cutpoints has to be one plus the length of the data
subintervals	Number of intervals
M	Number of components in the NNTS
iter	Number of iterations
initialpoint	TRUE if an initial point for the optimization algorithm will be used
cinicial	A vector of size $M+1$ . The first element is real, and the next $M$ elements are complex (values for $c_0$ and $c_1, \dots, c_M$ ). The sum of the squared moduli of the parameters must be equal to $1/(2*\pi)$

**Value**

cestimates	Matrix of $M+1 * 2$ . The first column is the parameter numbers, and the second column is the $c$ parameter's estimators
loglik	Optimum log-likelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after last iteration

**Author(s)**

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

**Examples**

```
data<-c(1,2,6,4)
cutpoints<-c(0,pi/2,pi,3*pi/2,2*pi-0.00000001)
nntsmanifoldnewtonestimationinterval0to2pi(data, cutpoints, length(data),1)
```

---

nntsplot

*Plots the NNTS density*

---

**Description**

Plots the NNTS density

**Usage**

```
nntsplot(cpars = 1/sqrt(2 * pi), M = 0, ...)
```

**Arguments**

cpars	Vector of complex numbers of dimension $M+1$ . The first element is a real and positive number. The sum of the SQUARED moduli of the $c$ parameters must be equal to $1/(2*\pi)$ .
M	Number of components in the NNTS
...	Arguments passed to the function curve

**Examples**

```
data(Turtles_radians)
#Empirical analysis of data
Turtles_hist<-hist(Turtles_radians,breaks=10,freq=FALSE)
#Estimation of the NNTS density with 3 componetes for data
est<-nntsmanifoldnewtonestimation(Turtles_radians,3,iter=200)
est
#plot the histogram
```

```
plot(Turtles_hist, freq=FALSE)
#add the estimated density to the histogram
nntsplo(est$cestimates[,2],3,add= TRUE)
```

---

nntsploInterval0to1 *Plots an NNTS density for a variable defined in the interval [0,1)*

---

### Description

Plots the NNTS density for a variable defined in the interval [0,1)

### Usage

```
nntsploInterval0to1(cpars = 1/sqrt(2 * pi), M = 0, ...)
```

### Arguments

cpars	Vector of complex numbers of dimension M+1. The first element is a real and positive number. The sum of the SQUARED moduli of the c parameters must be equal to 1/(2*pi).
M	Number of components in the NNTS
...	Arguments passed to the function curve

### Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

### Examples

```
data<-c(1,2,4,6,2)
cutpoints<-c(0,0.2,0.4,0.6,0.8,0.9999999)
est<-nntsmifoldnewtonestimationinterval0to1(data,cutpoints,5,1)
cpars<-est$cestimates[,2]
nntsploInterval0to1(cpars, 1)
```

---

nntsploSymmetric *Plots a symmetric NNTS density function*

---

### Description

Plots the Symmetric NNTS density function

### Usage

```
nntsploSymmetric(cpars = c(0, 0), M = 0, ...)
```

**Arguments**

<code>cpars</code>	Vector of real numbers of dimension $2M$ . The first $2M-1$ numbers are the squared moduli of the $c$ parameters. The sum must be less than $1/(2*\pi)$ . The last number is the mean of symmetry
<code>M</code>	Number of components in the NNTS
<code>...</code>	Arguments passed to the function curve

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

---

`nntsrandominitial`      *Initial random point*

---

**Description**

This function generates a random point on the surface of the  $(M+1)$ -dimensional unit hypersphere

**Usage**

```
nntsrandominitial(M=1)
```

**Arguments**

<code>M</code>	Number of components in the NNTS
----------------	----------------------------------

**Value**

Returns a valid initial point for the estimation functions

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```
nntsrandominitial(3)  
nntsrandominitial(7)
```



**Value**

simulations      The function generates nsim random values from the MNNTS density function  
 conteo            Number of uniform random numbers used for simulations

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```
M<-3
ccoef<-nntsrandominitial(M)
data<-nntssimulation(10,ccoef,M)
data
```

---

nntsSymmetricDensity    *Symmetric NNTS density function*

---

**Description**

Density function for the Symmetric NNTS

**Usage**

```
nntsSymmetricDensity(cpars = c(0, 0), M = 0, theta)
```

**Arguments**

cpars            Vector of real numbers of dimension  $2 \cdot M$ . The first  $M$  numbers are the squared moduli of the  $c$  parameters. The sum must be less than  $1/(2 \cdot \pi)$ . The last number is the mean of symmetry

M                Number of components in the NNTS

theta            Angle in radians

**Value**

The function returns the density function evaluated at theta

**Note**

The default values provide the uniform circular density

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran, J.J., Gregorio-Dominguez, M.M. (2009) Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums. Working Paper, DE-C09.12, Department of Statistics, ITAM, Mexico

---

nntsuniformitytestlikelihoodratio

*Computes the statistic and critical values of the circular uniformity test*

---

**Description**

Computes the statistic and critical values at 10%, 5% and 1% of the circular uniformity test based on the NNTS likelihood ratio for M values from 1 to 7 and any sample size.

**Usage**

```
nntsuniformitytestlikelihoodratio(data,M=1, iter=1000, initialpoint = FALSE,
cinitial,gradientstop=1e-10)
```

**Arguments**

data	Vector of angles in radians
M	Number of components in the NNTS
iter	Number of iterations
initialpoint	TRUE if an initial point for the optimization algorithm will be used
cinitial	Vector of size M+1. The first element is real and the next M elements are complex (values for $c_0$ and $c_1, \dots, c_M$ ). The sum of the squared moduli of the parameters must be equal to $\frac{1}{2\pi}$
.	.
gradientstop	The value of the gradient of the Newton algorithm at which the algorithms stops

**Value**

gradient	Gradient error after the last iteration
likratiounifstat	Value of the likelihood ratio NNTS circular uniformity test statistic
criticalvalue10percent	Critical value at a 10% significance level of the likelihood ratio NNTS circular uniformity test
criticalvalue05percent	Critical value at a 5% significance level of the likelihood ratio NNTS circular uniformity test
criticalvalue01percent	Critical value at a 1% significance level of the likelihood ratio NNTS circular uniformity test

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2022). Sums of Independent Circular Random Variables and Maximum Likelihood Circular Uniformity Tests Based on Nonnegative Trigonometric Sums Distributions, arXiv preprint arXiv:2212.01416

**Examples**

```
set.seed(200)
a<-2*pi*runif(50)
#NNTS likelihood ratio circular uniformity test for M=2 and gradientstop at 1e-09
nntsuniformitytestlikelihoodratio(data=a,M=2,gradientstop=1e-09)
data(Turtles_radians)
#NNTS likelihood ratio circular uniformity test for M=5 and gradientstop at 1e-12
nntsuniformitytestlikelihoodratio(data=Turtles_radians,M=5,gradientstop=1e-09)
```

---

ProteinsAAA

*Dihedral angles in protein*

---

**Description**

Dataset of the dihedral angles in a protein between three consecutive Alanine (Ala) amino acids. This dataset was constructed from the recommended July 2003 list of proteins via the algorithm in Hobohm et al. (1992). This algorithm selects a representative sample of proteins from the vast Protein Data Bank (PDB, Berman et al., 2000). The dataset contains 233 pairs of dihedral angles.

**Usage**

```
data(ProteinsAAA)
```

**Format**

Two columns of angles in radians

**Source**

Protein Data Bank (PDB)

**References**

Hobohm, U. and Scharf, M. and Schneider, R. and Sander, C. (1992) Selection of a Representative Set of Structures from the Brookhaven Protein Data Bank, Protein Science, 1, 409-417. Berman, H. M. and Westbrook, J. and Feng, Z. and Gilliland, G. and Bhat, T. N. and Weissing, H. and Shyndialov, I. N. and Bourne, P. E. (2000) The Protein Data Bank, Nucleic Acids Research, 28, 235-242.

---

snntsdensity                      *SNNTS density function for spherical data*

---

**Description**

Density function for the SNNTS model for spherical data

**Usage**

```
snntsdensity(data, cpars = 1, M = c(0,0))
```

**Arguments**

data	Matrix of angles in radians. The first column contains longitude data (between zero and $2\pi$ ), and second column contains latitude data (between zero and $\pi$ ), with one row for each data point
cpars	Vector of complex numbers of dimension $\text{prod}(M+1)$ . The first element is a real and positive number. The sum of the squared moduli of the $c$ parameters must be equal to one.
M	Vector with the number of components in the SNNTS for each dimension

**Value**

The function returns the density function evaluated for each row in the data

**Note**

The parameters `cinitial` and `cestimates` used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Juan Jose Fernandez-Duran, Maria Mercedes Gregorio-Dominguez (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. *Journal of Statistical Software*, 70(6), 1-19. doi:10.18637/jss.v070.i06

**Examples**

```
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(2,3)
cpars<-rnorm(prod(M+1))+rnorm(prod(M+1))*complex(real=0,imaginary=1)
cpars[1]<-Re(cpars[1])
cpars<- cpars/sqrt(sum(Mod(cpars)^2))
```

```
snntsdensity(data, cpars, M)
```

---

```
snntsdensityplot      Plots a SNNTS density for spherical data
```

---

**Description**

Computes the points needed to plot the SNNTS density function for spherical data

**Usage**

```
snntsdensityplot(long, lat, cpars = 1, M = c(0,0))
```

**Arguments**

long	Grid for longitude. Vector with values between zero and $2\pi$
lat	Grid for latitude. Vector with values between zero and $\pi$
cpars	Vector of complex numbers of dimension $\text{prod}(M+1)$ . The sum of the squared moduli of the c parameters must be equal to one
M	Vector with the number of components in the SNNTS for each dimension

**Value**

The points needed to plot the SNNTS density function

**Note**

The parameters cpars used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

**Examples**

```

set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(4,4)
cest<-snntsmanifoldnewtonestimation(data, M, iter=150)
cpars<-cest$cestimates[,3]
longitud<-seq(0,360,10)*(pi/180)
latitud<-seq(0,180,5)*(pi/180)
z<-outer(longitud,latitud,FUN="snntsdensityplot",cpars,M)
persp(longitud,latitud,z,theta=45,phi=30)
contour(longitud,latitud,z)
points(data[,1],data[,2])

```

---

snntsloglik

*SNNTS log-likelihood function for spherical data*


---

**Description**

Computes the log-likelihood function with SNNTS density for spherical data

**Usage**

```
snntsloglik(data, cpars = 1, M = c(0,0))
```

**Arguments**

data	Matrix of angles in radians. The first column contains longitude data (between zero and $2\pi$ ), and the second column contains latitude data (between zero and $\pi$ ), with one row for each data point
cpars	Vector of complex numbers of dimension $\text{prod}(M+1)$ . The first element is a real and positive number. The first $M[1]+1$ elements correspond to longitude, the next $M[2]+1$ elements correspond to latitude. The sum of the squared moduli of the $c$ parameters must be equal to 1
M	Vector with number of components in the SNNTS for each dimension

**Value**

The function returns the value of the log-likelihood function for the data

**Note**

The parameters  $cpars$  used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

## References

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

## Examples

```
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(4,4)
cpars<-rnorm(prod(M+1))+rnorm(prod(M+1))*complex(real=0,imaginary=1)
cpars[1]<-Re(cpars[1])
cpars<- cpars/sqrt(sum(Mod(cpars)^2))
snntsdensity(data, cpars, M)
snntsloglik(data, cpars, M)
```

---

snntsmanifoldnewtonestimation

*Parameter estimation for SNNTS distributions for spherical data*

---

## Description

Computes the maximum likelihood estimates of the SNNTS model parameters using a Newton algorithm on the hypersphere

## Usage

```
snntsmanifoldnewtonestimation(data, M = c(0,0), iter = 1000,
initialpoint = FALSE, cinitial)
```

## Arguments

data	Matrix of angles in radians, with one row for each data point. The first column contains longitude data (between zero and $2\pi$ ), and second column contains latitude data (between zero and $\pi$ ), with one row for each data point
M	Vector with number of components in the SNNTS for each dimension
iter	Number of iterations
initialpoint	TRUE if an initial point for the optimization algorithm will be used
cinitial	Initial value for cpars for the optimization algorithm, a vector of complex numbers of dimension $\text{prod}(M+1)$ . The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to one.

**Value**

cestimates	Matrix of $\text{prod}(M+1)^*(3)$ . The first two columns are the parameter numbers, and the last column is the c parameter's estimators
loglik	Optimum log-likelihood value
AIC	Value of Akaike's Information Criterion
BIC	Value of Bayesian Information Criterion
gradnormerror	Gradient error after the last iteration

**Note**

The parameters cinitial and cestimates used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

**Examples**

```
set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready

M<-c(4,4)
cpar<-rnorm(prod(M+1))+rnorm(prod(M+1))*complex(real=0,imaginary=1)
cpar[1]<-Re(cpar[1])
cpar<- cpar/sqrt(sum(Mod(cpar)^2))

cest<-snntsmanifoldnewtonestimation(data,c(4,4),100,TRUE,cpar)
cest
cest<-snntsmanifoldnewtonestimation(data,c(1,2),100)
cest
```

---

snntsmarginallatitude *Marginal density function for latitude of the SNNTS model for spherical data*

---

**Description**

Marginal density function for latitude of the SNNTS model for spherical data

**Usage**

```
snntsmarginallatitude(data, cpars = 1, M = c(0,0))
```

**Arguments**

data	Vector of angles in radians, with one row for each data point. The data must be between zero and pi.
cpars	Vector of complex numbers of dimension prod(M+1). The first element is a real and positive number. The sum of the squared moduli of the c parameters must be equal to one
M	Vector with the number of components in the SNNTS for each dimension

**Value**

The function returns the SNNTS marginal density function for latitude evaluated at data

**Note**

The parameters cpars used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**References**

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

**Examples**

```
set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(1,2)
cest<-snntsmanifoldnewtonestimation(data, M, iter=150)
lat<-snntsmarginallatitude(seq(0,pi,.1),cest$cestimates[,3],M)
plot(seq(0,pi,.1),lat,type="l")
```

---

snntsmarginallongitude

*Marginal density function for the longitude of the SNNTS model for spherical data*

---

### Description

Marginal density function for the longitude of the SNNTS model for spherical data

### Usage

```
snntsmarginallongitude(data, cpars = 1, M = c(0,0))
```

### Arguments

data	Vector of angles in radians, with one row for each data point. The data must be between zero and $2\pi$
cpars	Vector of complex numbers of dimension $\text{prod}(M+1)$ . The first element is a real and positive number. The first $M[1]+1$ elements correspond to longitude, and the next $M[2]+1$ elements correspond to latitude. The sum of the squared moduli of the $c$ parameters must be equal to one.
M	Vector with number of components in the SNNTS for each dimension

### Value

The function returns the density function evaluated for the data

### Note

The parameters `cpars` used by this function are the transformed parameters of the SNNTS density function, which lie on the surface of the unit hypersphere

### Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

### References

Fernandez-Duran J. J. y Gregorio Dominguez, M. M. (2008) Spherical Distributions Based on Non-negative Trigonometric Sums, Working Paper, Statistics Department, ITAM, DE-C08.6

**Examples**

```

set.seed(200)
data(Datab6fisher_ready)
data<-Datab6fisher_ready
M<-c(1,2)
cest<-snntsmanifoldnewtonestimation(data, M, iter=150)
long<-snntsmarginallongitude(seq(0, 2*pi, .1), cest$cestimates[, 3], M)
plot(seq(0, 2*pi, .1), long, type="l")

```

---

sntssimulation      *SNNTS density simulation function*

---

**Description**

Simulation for the density function for the SNNTS model

**Usage**

```
sntssimulation(nsim=1, cpars =(1/(2*pi))^2, M = c(0,0))
```

**Arguments**

nsim	Number of simulations
cpars	Vector of complex numbers of dimension $\text{prod}(M+1)$ . The first element is a real and positive number. The first $M[1]+1$ elements correspond to longitude, the next $M[2]+1$ elements correspond to latitude. The sum of the squared moduli of the c parameters must be equal to one
M	Vector with the number of components in the SNNTS for each dimension

**Value**

simulations	The function generates nsim random values from the SNNTS density function
conteo	Number of uniform random numbers used for simulations

**Author(s)**

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

**Examples**

```

M<-c(2,3)
R<-length(M)
ccoef<-mnntsrandominitial(M,R)
data<-mnntssimulation(10,ccoef,M,R)
data

```

---

SuicidesMexico2005	<i>Suicides in Mexico during 2005</i>
--------------------	---------------------------------------

---

**Description**

Monthly number of suicides in Mexico during 2005

**Usage**

```
data(SuicidesMexico2005)
```

**Format**

Integer values

**Source**

INEGI (Mexican National Statistical Agency) [www.inegi.gob.mx](http://www.inegi.gob.mx)

---

Turtles	<i>Movements of turtles</i>
---------	-----------------------------

---

**Description**

Data measurement of the directions taken by 76 turtles after treatment

**Usage**

```
data(Turtles)
```

**Format**

Directions of turtles in degrees

**Source**

Stephens (1969) Techniques for directional data. Technical Report 150. Dept. of Statistics, Stanford University, Stanford, CA.

**References**

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

---

Turtles_radians	<i>Movements of turtles</i>
-----------------	-----------------------------

---

**Description**

Data measurement of the directions taken by 76 turtles after treatment

**Usage**

```
data(Turtles_radians)
```

**Format**

Directions of turtles in radians

**Source**

Stephens (1969) Techniques for directional data. Technical Report 150. Dept. of Statistics, Stanford University. Stanford, CA.

**References**

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

---

WindDirectionsTrivariate	<i>Wind directions</i>
--------------------------	------------------------

---

**Description**

Wind directions registered at the monitoring stations of San Agustin located in the north, Pedregal in the southwest, and Hangares in the southeast of the Mexico Central Valley's at 14:00 on days between January 1, 1993 and February 29, 2000. There are a total of 1,682 observations

**Usage**

```
data(WindDirectionsTrivariate)
```

**Format**

Three columns of angles in radians

**Source**

Mexico Central Valleys pollution monitoring network. RAMA SIMAT (Red Automatica de Monitoreo Ambiental)

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