

Package ‘Copula.surv’

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Type Package

Title Analysis of Bivariate Survival Data Based on Copulas

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Description Simulating bivariate survival data from various copula models.

Estimating bivariate copula models with semiparametric or Weibull margins under various copulas.

Two different ways to estimate the association parameter in copula models are implemented.

A goodness-of-

fit test for the Gumbel and Clayton copulas is also implemented for semiparametric models.

See Emura, Lin and Wang (2010) <[doi:10.1016/j.csda.2010.03.013](https://doi.org/10.1016/j.csda.2010.03.013)> for details.

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Copula.surv-package *Analysis of Bivariate Survival Data*

Description

Simulating bivariate survival data from various copula models. Estimating bivariate copula models with semiparametric or Weibull margins under various copulas. Two different ways to estimate the association parameter in copula models are implemented. A goodness-of-fit test for the Gumbel and Clayton copulas is also implemented for semiparametric models. See Emura, Lin and Wang (2010) <doi:10.1016/j.csda.2010.03.013> for details. Also, Weibull regression is implemented (Section 2.6.3 of Emura et al. (2019)).

Details

Details are seen from the references.

Author(s)

Takeshi Emura Maintainer: Takeshi Emura <takeshiemura@gmail.com>

References

- Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43
- Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

simu.BB1 *Simulating data from the BB1 copula*

Description

n pairs of (U,V) are generated from the BB1 copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($scale1=scale2=shape1=shape2=1$) give the unit exponential distributions.

Usage

```
simu.BB1(n,alpha,delta=0,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter
delta	BB1 copula's departure parameter from the Clayton (0 is the default)
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.BB1(n=n,alpha=1,delta=2,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[,"U"],Dat[,"V"])
cor(Dat[,"U"],Dat[,"V"],method="kendall")
plot(Dat[,"X"],Dat[,"Y"])
cor(Dat[,"X"],Dat[,"Y"],method="kendall")
```

simu.BB1reg

Simulating data from the BB1 copula regression model

Description

n pairs of (U,V) are generated from the BB1 copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($scale1=scale2=shape1=shape2=1$) give the unit exponential distributions.

Usage

```
simu.BB1reg(n,alpha,delta=0,scale1=1,scale2=1,shape1=1,shape2=1,
  beta1=0,beta2=0,beta12=0,Z.dist=runif,...)
```

Arguments

<code>n</code>	sample size
<code>alpha</code>	association (copula) parameter
<code>delta</code>	BB1 copula's departure parameter from the Clayton (0 is the default)
<code>scale1</code>	scale parameter for X
<code>scale2</code>	scale parameter for Y
<code>shape1</code>	shape parameter for X
<code>shape2</code>	shape parameter for Y
<code>beta1</code>	regression coefficient for X
<code>beta2</code>	regression coefficient for Y
<code>beta12</code>	regression coefficient for copula
<code>Z.dist</code>	distribution for covariates
<code>...</code>	parameters for Z.dist

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)
Z	Covariates

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=10
simu.BB1reg(n=n,alpha=1,delta=2,scale1=1,scale2=2,shape1=0.5,beta1=1,beta2=-1,beta12=2,shape2=2)
```

simu.CC

Simulating data from the Celebioglu-Cuadras (CC) copula

Description

n pairs of (U,V) are generated from the CC copula. n paris of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.CC(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter, $-1 \leq \alpha \leq 1$
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.CC(n=n,alpha=-1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.Clayton

Simulating data from the Clayton copula

Description

n pairs of (U,V) are generated from the Clayton copula. n paris of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Clayton(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Clayton(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[,"U"],Dat[,"V"])
cor(Dat[,"U"],Dat[,"V"],method="kendall")
plot(Dat[,"X"],Dat[,"Y"])
cor(Dat[,"X"],Dat[,"Y"],method="kendall")
```

simu.FGM

*Simulating data from the FGM copula***Description**

n pairs of (U, V) are generated from the FGM copula. n pairs of (X, Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($\text{scale1}=\text{scale2}=\text{shape1}=\text{shape2}=1$) give the unit exponential distributions.

Usage

```
simu.FGM(n, alpha, scale1=1, scale2=1, shape1=1, shape2=1, Print=FALSE)
```

Arguments

<code>n</code>	sample size
<code>alpha</code>	association (copula) parameter; $-1 \leq \alpha \leq 1$
<code>scale1</code>	scale parameter for X
<code>scale2</code>	scale parameter for Y
<code>shape1</code>	shape parameter for X
<code>shape2</code>	shape parameter for Y
<code>Print</code>	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

<code>U</code>	uniformly distributed on $(0,1)$
<code>V</code>	uniformly distributed on $(0,1)$
<code>X</code>	Weibull distributed (scale1 , shape1)
<code>Y</code>	Weibull distributed (scale2 , shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.FGM(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.Frank

*Simulating data from the Frank copula***Description**

n pairs of (U,V) are generated from the Frank copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Frank(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Frank(n=n,alpha=10,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.GB

Simulating data from the Gumbel-Barnett (GB) copula

Description

n pairs of (U,V) are generated from the GB copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.GB(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter, $0 \leq \alpha \leq 1$
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.GB(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[,"U"],Dat[,"V"])
cor(Dat[,"U"],Dat[,"V"],method="kendall")
plot(Dat[,"X"],Dat[,"Y"])
cor(Dat[,"X"],Dat[,"Y"],method="kendall")
```

simu.Gumbel

Simulating data from the Gumbel copula

Description

n pairs of (U,V) are generated from the Gumbel copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Gumbel(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Gumbel(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

simu.Joe

Simulating data from the Joe copula

Description

n pairs of (U,V) are generated from the Joe copula. n paris of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters (scale1=scale2=shape1=shape2=1) give the unit exponential distributions.

Usage

```
simu.Joe(n,alpha,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.Joe(n=n,alpha=1,scale1=1,scale2=2,shape1=0.5,shape2=2)
plot(Dat[,"U"],Dat[,"V"])
cor(Dat[,"U"],Dat[,"V"],method="kendall")
plot(Dat[,"X"],Dat[,"Y"])
cor(Dat[,"X"],Dat[,"Y"],method="kendall")
```

simu.t *Simulating data from the t-copula*

Description

n pairs of (U,V) are generated from the t -copula. n pairs of (X,Y) are generated from the corresponding bivariate survival model with the Weibull marginal distributions. The default parameters ($scale1=scale2=shape1=shape2=1$) give the unit exponential distributions.

Usage

```
simu.t(n,alpha,df=1,scale1=1,scale2=1,shape1=1,shape2=1,Print=FALSE)
```

Arguments

n	sample size
alpha	association (copula) parameter
df	degrees of freedom (d=1 is the default)
scale1	scale parameter for X
scale2	scale parameter for Y
shape1	shape parameter for X
shape2	shape parameter for Y
Print	print Kendall's tau and means of X and Y if "TRUE"

Details

See Section 2.6 of Emura et al.(2019) for copulas and bivariate survival times.

Value

U	uniformly distributed on (0,1)
V	uniformly distributed on (0,1)
X	Weibull distributed (scale1, shape1)
Y	Weibull distributed (scale2, shape2)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Emura T, Matsui S, Rondeau V (2019), *Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models*, JSS Research Series in Statistics, Springer

Examples

```
n=100
Dat=simu.t(n=n,alpha=0.8,df=1,scale1=1,scale2=2,shape1=0.5,shape2=2,Print=TRUE)
plot(Dat[, "U"],Dat[, "V"])
cor(Dat[, "U"],Dat[, "V"],method="kendall")
plot(Dat[, "X"],Dat[, "Y"])
cor(Dat[, "X"],Dat[, "Y"],method="kendall")
```

Test.Clayton

A goodness-of-fit test for the Clayton copula

Description

Perform a goodness-of-fit test for the Clayton copula based on Emura, Lin and Wang (2010). The test is asymptotically equivalent to the test of Shih (1998).

Usage

```
Test.Clayton(x.obs,y.obs,dx,dy,lower=0.001,upper=50,U.plot=TRUE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
lower	lower bound for the association parameter
upper	upper bound for the association parameter
U.plot	if TRUE, draw the plot of $U_1(\theta)$

Details

See the references.

Value

theta1	association parameter by the pseudo-likelihood estimator
theta2	association parameter by the unweighted estimator
Stat	$\log(\theta_1+1)-\log(\theta_2+1)$
Z	Z-value of the goodness-of-fit for the Clayton copula
P	P-value of the goodness-of-fit for the Clayton copula

Author(s)

Takeshi Emura

References

- Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43
- Shih JH (1998) A goodness-of-fit test for association in a bivariate survival model. *Biometrika* 85: 189-200

Examples

```
n=20
theta_true=2 ## association parameter ##
r1_true=2 ## hazard for X
r2_true=2 ## hazard for Y

set.seed(1)
V1=runif(n)
V2=runif(n)
X=-1/r1_true*log(1-V1)
W=(1-V1)^(-theta_true)
Y=1/theta_true/r2_true*log( 1-W+W*(1-V2)^(-theta_true/(theta_true+1)) )
C=runif(n,min=0,max=5)

x.obs=pmin(X,C)
y.obs=pmin(Y,C)
dx=X<=C
dy=Y<=C

Test.Clayton(x.obs,y.obs,dx,dy)
```

Test.Gumbel

A goodness-of-fit test for the Gumbel copula

Description

Perform a goodness-of-fit test for the Gumbel copula based on Emura, Lin and Wang (2010).

Usage

```
Test.Gumbel(x.obs,y.obs,dx,dy,lower=0.01,upper=50,U.plot=TRUE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
lower	lower bound for the association parameter
upper	upper bound for the association parameter
U.plot	if TRUE, draw the plot of $U_1(\theta)$ and $U_2(\theta)$

Details

See the references.

Value

theta1	association parameter by the pseudo-likelihood estimator
theta2	association parameter by the unweighted estimator
Stat	$\log(\text{theta1}) - \log(\text{theta2})$
Z	Z-value of the goodness-of-fit for the Clayton copula
P	P-value of the goodness-of-fit for the Clayton copula

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
x.obs=c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)
y.obs=c(2,1,4,5,6,8,3,7,10,9,11,12,13,14,15)
dx=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
dy=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
Test.Gumbel(x.obs,y.obs,dx,dy)
```

U1.Clayton

Estimation of an association parameter via the pseudo-likelihood

Description

Estimate the association parameter of the Clayton copula using bivariate survival data. The estimator was derived by Clayton (1978) and reformulated by Emura, Lin and Wang (2010).

Usage

```
U1.Clayton(x.obs,y.obs,dx,dy,lower=0.001,upper=50,U.plot=TRUE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
lower	lower bound for the association parameter
upper	upper bound for the association parameter
U.plot	if TRUE, draw the plot of $U_1(\theta)$

Details

Details are seen from the references.

Value

theta	association parameter
tau	Kendall's tau ($=\theta/(\theta+2)$)

Author(s)

Takeshi Emura

References

- Clayton DG (1978). A model for association in bivariate life tables and its application to epidemiological studies of familial tendency in chronic disease incidence. *Biometrika* 65: 141-51.
- Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
n=200
theta_true=2 ## association parameter ##
r1_true=1 ## hazard for X
r2_true=1 ## hazard for Y

set.seed(1)
V1=runif(n)
V2=runif(n)
X=-1/r1_true*log(1-V1)
W=(1-V1)^(-theta_true)
Y=1/theta_true/r2_true*log( 1-W+W*(1-V2)^(-theta_true/(theta_true+1)) )
C=runif(n,min=0,max=5)

x.obs=pmin(X,C)
y.obs=pmin(Y,C)
dx=X<=C
dy=Y<=C
```

```
U1.Clayton(x.obs,y.obs,dx,dy)
```

U1.Gumbel

Estimation of an association parameter via the unweighted estimator

Description

Estimate the association parameter of the Gumbel copula using bivariate survival data. The estimator was derived by Emura, Lin and Wang (2010).

Usage

```
U1.Gumbel(x.obs,y.obs,dx,dy,lower=0.01,upper=50,U.plot=TRUE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
lower	lower bound for the association parameter
upper	upper bound for the association parameter
U.plot	if TRUE, draw the plot of $U_1(\theta)$

Details

Details are seen from the references.

Value

theta	association parameter
tau	Kendall's tau ($=\theta/(\theta+2)$)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
x.obs=c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)
y.obs=c(2,1,4,5,6,8,3,7,10,9,11,12,13,14,15)
dx=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
dy=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
U1.Gumbel(x.obs,y.obs,dx,dy)
```

U2.Clayton

*Estimation of an association parameter via the unweighted estimator***Description**

Estimate the association parameter of the Clayton copula using bivariate survival data. The estimator was defined as the unweighted estimator in Emura, Lin and Wang (2010).

Usage

```
U2.Clayton(x.obs,y.obs,dx,dy,lower=0.001,upper=50,U.plot=TRUE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
lower	lower bound for the association parameter
upper	upper bound for the association parameter
U.plot	if TRUE, draw the plot of U ₂ (theta)

Details

Details are seen from the references.

Value

theta	association parameter
tau	Kendall's tau (=theta/(theta+2))

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring. *Compt Stat Data Anal* 54: 3033-43

Examples

```

n=200
theta_true=2 ## association parameter ##
r1_true=1 ## hazard for X
r2_true=1 ## hazard for Y

set.seed(1)
V1=runif(n)
V2=runif(n)
X=-1/r1_true*log(1-V1)
W=(1-V1)^(-theta_true)
Y=1/theta_true/r2_true*log( 1-W+W*(1-V2)^(-theta_true/(theta_true+1)) )
C=runif(n,min=0,max=5)

x.obs=pmin(X,C)
y.obs=pmin(Y,C)
dx=X<=C
dy=Y<=C

U2.Clayton(x.obs,y.obs,dx,dy)

```

U2.Gumbel

Estimation of an association parameter via the pseudo-likelihood

Description

Estimate the association parameter of the Gumbel copula using bivariate survival data. The estimator was derived by Emura, Lin and Wang (2010).

Usage

```
U2.Gumbel(x.obs,y.obs,dx,dy,lower=0.01,upper=50,U.plot=TRUE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
lower	lower bound for the association parameter
upper	upper bound for the association parameter
U.plot	if TRUE, draw the plot of $U_2(\theta)$

Details

Details are seen from the references.

Value

theta	association parameter
tau	Kendall's tau ($=\text{theta}/(\text{theta}+1)$)

Author(s)

Takeshi Emura

References

Emura T, Lin CW, Wang W (2010) A goodness-of-fit test for Archimedean copula models in the presence of right censoring, *Compt Stat Data Anal* 54: 3033-43

Examples

```
x.obs=c(1,2,3,4,5)
y.obs=c(2,1,4,5,6)
dx=c(1,1,1,1,1)
dy=c(1,1,1,1,1)
U2.Gumbel(x.obs,y.obs,dx,dy)
```

Weib.reg.BB1

Weibull regression under the BB1 copula

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.BB1(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
zx	matrix of covariates for X
zy	matrix of covariates for Y
convergence.par	if TRUE, show the details

Details

Details are seen from the references.

Value

beta_x	regression coefficients for X
beta_y	regression coefficients for Y
alpha	copula parameter
delta	copula parameter
tau	Kendall's tau

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Emura T, Ha IL-Do, Pathak AK (2026), Flexible dependence modeling for bivariate survival and semi-competing risks data using the two-parameter BB1 copula, in revision

Examples

```
#TBA
```

```
Weib.reg.BB1a
```

Weibull regression under the BB1 copula with known "delta"

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.BB1a(x.obs,y.obs,dx,dy,zx,zy,delta=0,convergence.par=FALSE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
zx	matrix of covariates for X
zy	matrix of covariates for Y
delta	known copula parameter ($d \geq 0$)
convergence.par	if TRUE, show the details

Details

Details are seen from the references.

Value

beta_x	regression coefficients for X
beta_y	regression coefficients for Y
alpha	copula parameter
tau	Kendall's tau

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Emura T, Ha IL-Do, Pathak AK (2026), Flexible dependence modeling for bivariate survival and semi-competing risks data using the two-parameter BB1 copula, in revision

Examples

#TBA

Weib.reg.cBB1

Weibull regression under the conditional BB1 copula

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

Weib.reg.cBB1(x.obs,y.obs,dx,dy,zx,zy,zxy,alpha=NULL,convergence.par=FALSE)

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
zx	matrix of covariates for X
zy	matrix of covariates for Y

zxy	matrix of covariates for copula
alpha	copula parameter: the default is NULL (unknown)
convergence.par	if TRUE, show the details

Details

Details are seen from the references.

Value

beta_x	regression coefficients for X
beta_y	regression coefficients for Y
beta_xy	regression coefficients for copula
alpha	copula parameter
delta	copula parameter
tau	Kendall's tau given beta_xy=0

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Emura T, Ha IL-Do, Pathak AK (2026), Flexible dependence modeling for bivariate survival and semi-competing risks data using the two-parameter BB1 copula, in revision

Examples

#TBA

Weib.reg.cBB1a	<i>Weibull regression under the conditional BB1 copula with known "delta"</i>
----------------	---

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

Weib.reg.cBB1a(x.obs,y.obs,dx,dy,zx,zy,zxy,delta=0,convergence.par=FALSE)

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
zx	matrix of covariates for X
zy	matrix of covariates for Y
zxy	matrix of covariates for copula
delta	known copula parameter ($d \geq 0$)
convergence.par	if TRUE, show the details

Details

Details are seen from the references.

Value

beta_x	regression coefficients for X
beta_y	regression coefficients for Y
beta_xy	regression coefficients for copula
alpha	copula parameter
tau	Kendall's tau given $\beta_{xy}=0$

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Emura T, Ha IL-Do, Pathak AK (2026), Flexible dependence modeling for bivariate survival and semi-competing risks data using the two-parameter BB1 copula, in revision

Examples

#TBA

Weib.reg.Clayton *Weibull regression under the Clayton copula*

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.Clayton(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
zx	matrix of covariates for X
zy	matrix of covariates for Y
convergence.par	if TRUE, show the details

Details

Details are seen from the references.

Value

beta_x	regression coefficients for X
beta_y	regression coefficients for Y

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

```
#TBA
```

`Weib.reg.Frank`*Weibull regression under the Frank copula*

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.Frank(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

<code>x.obs</code>	censored times for X
<code>y.obs</code>	censored times for Y
<code>dx</code>	censoring indicators for X
<code>dy</code>	censoring indicators for Y
<code>zx</code>	matrix of covariates for X
<code>zy</code>	matrix of covariates for Y
<code>convergence.par</code>	if TRUE, show the details

Details

Details are seen from the references.

Value

<code>beta_x</code>	regression coefficients for X
<code>beta_y</code>	regression coefficients for Y

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

```
#TBA
```

Weib.reg.Gumbel	<i>Weibull regression under the Gumbel copula</i>
-----------------	---

Description

See Section 2.6.3 of Emura et al. (2019).

Usage

```
Weib.reg.Gumbel(x.obs,y.obs,dx,dy,zx,zy,convergence.par=FALSE)
```

Arguments

x.obs	censored times for X
y.obs	censored times for Y
dx	censoring indicators for X
dy	censoring indicators for Y
zx	matrix of covariates for X
zy	matrix of covariates for Y
convergence.par	if TRUE, show the details

Details

Details are seen from the references.

Value

beta_x	regression coefficients for X
beta_y	regression coefficients for Y

Author(s)

Takeshi Emura

References

Emura T, Matsui S, Rondeau V (2019), Survival Analysis with Correlated Endpoints, Joint Frailty-Copula Models, JSS Research Series in Statistics, Springer

Examples

```
#TBA
```

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