Package ‘EL’

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Title Two-Sample Empirical Likelihood

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Description Empirical likelihood (EL) inference for two-sample problems. The following statistics are included: the difference of two-sample means, smooth Huber estimators, quantile (qdiff) and cumulative distribution functions (fdiff), probability-probability (P-P) and quantile-quantile (Q-Q) plots as well as receiver operating characteristic (ROC) curves. Also includes two-sample block-wise empirical likelihood (BEL) and a frequency-domain empirical likelihood test for autocorrelation differences (FDEL). Methods for EL, P-P, Q-Q, ROC, qdiff and fdiff are based on Valeinis and Cers (2011) <http://home.lu.lv/~valeinis/lv/petnieciba/EL_TwoSample_2011.pdf>.

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Contents

`BEL.means` *The two-sample blockwise empirical likelihood statistic for differences in means*

Description

Calculates blockwise empirical likelihood test for the difference of two sample means.

Usage

```
BEL.means(X, Y, M_1, M_2, Delta = 0)
```

Arguments

`X, Y`	vectors of data values.
`M_1, M_2`	positive integers specifying block length for X and Y, respectively.
`Delta`	hypothesized difference of two populations.

Value

A list of class "htest" containing following components: method - the character string of the test. data.name - a character string with the names of the input data. Delta0 - the specified hypothesized value of mean differences under the null hypothesis statistic - the value of the test statistic. p.value - the p-value for the test.

Author(s)

R. Alksnis, J. Valeinis

Examples

```
# Basic example
Delta0 <- 1.5
X <- arima.sim(n = 400, model = list(ar = .3))
Y <- arima.sim(n = 400, model = list(ar = .5)) + Delta0
BEL.means(X, Y, 10, 20, Delta = Delta0)
```

EL.Huber *Empirical likelihood test for the difference of smoothed Huber estimators*

Description

Empirical likelihood inference for the difference of smoothed Huber estimators. This includes a test for the null hypothesis of a constant difference of smoothed Huber estimators, a confidence interval, and the EL estimator.

Usage

```
EL.Huber(
  X,
  Y,
  mu = 0,
  conf.level = 0.95,
  scaleX = 1,
  scaleY = 1,
  VX = 2.046,
  VY = 2.046,
  k = 1.35
)
```

Arguments

`X`	A numeric vector of data values.
`Y`	A numeric vector of data values.
`mu`	A number specifying the null hypothesis value for the difference. Default is `0`.
`conf.level`	Confidence level for the reported confidence interval (default `0.95`).
`scaleX`	The scale estimate of sample `X`. Default is `1`.
`scaleY`	The scale estimate of sample `Y`. Default is `1`.
`VX`	The asymptotic variance of the initial (nonsmooth) Huber estimator for sample `X`. Default is `2.046`.
`VY`	The asymptotic variance of the initial (nonsmooth) Huber estimator for sample `Y`. Default is `2.046`.
`k`	Tuning parameter for the Huber estimator. Default is `1.35`.

Details

A common choice for a robust scale estimate (parameters `scaleX` and `scaleY`) is the median absolute deviation (MAD).

Value

A list of class "htest" with components:

`estimate` The empirical likelihood estimate for the difference of two smoothed Huber estimators.

`conf.int` A confidence interval for the difference of two smoothed Huber estimators.

`p.value` The p-value for the test.

`statistic` The value of the test statistic.

`method` The character string "Empirical likelihood smoothed Huber estimator difference test".

`null.value` The hypothesised difference `mu` under H_0.

`data.name` A character string giving the names of the data.

Author(s)

E. Cers, J. Valeinis

References

Valeinis, J. and Cers, E. Extending the two-sample empirical likelihood. Preprint: http://home.lu.lv/~valeinis/lv/petnieciba/EL_TwoSample_2011.pdf

Hampel, F., Hennig, C. and Ronchetti, E. A. (2011). A smoothing principle for the Huber and other location M-estimators. *Computational Statistics & Data Analysis*, **55**(1), 324–337.

See Also

`EL.means`

Examples

```
X <- rnorm(100)
Y <- rnorm(100)
t.test(X, Y)
EL.means(X, Y)
EL.Huber(X, Y)
```

EL.means — *Empirical likelihood test for the difference of two sample means*

Description

Empirical likelihood inference for the difference of two sample means. This includes a test for the null hypothesis of a constant mean difference, a confidence interval, and the EL estimator.

Usage

```
EL.means(X, Y, mu = 0, conf.level = 0.95)
```

Arguments

`X`	A numeric vector of data values.
`Y`	A numeric vector of data values.
`mu`	A number specifying the null hypothesis value for the mean difference. Default is `0`.
`conf.level`	Confidence level for the reported confidence interval (default `0.95`).

Value

A list of class `"htest"` with components:

`estimate` The empirical likelihood estimate of the mean difference.

`conf.int` A confidence interval for the mean difference.

`p.value` The p-value for the test.

`statistic` The value of the test statistic.

`method` The character string `"Empirical likelihood mean difference test"`.

`null.value` The hypothesised mean difference `mu` under H_0.

`data.name` A character string giving the names of the data.

Author(s)

E. Cers, J. Valeinis

References

Valeinis, J., Cers, E. and Cielens, J. (2010). Two-sample problems in statistical data modelling. *Mathematical Modelling and Analysis*, **15**(1), 137–151.

Valeinis, J. and Cers, E. Extending the two-sample empirical likelihood. Preprint: http://home.lu.lv/~valeinis/lv/petnieciba/EL_TwoSample_2011.pdf

See Also

`EL.Huber`

Examples

```
X <- rnorm(100)
Y <- rnorm(100)
t.test(X, Y)
EL.means(X, Y)
EL.Huber(X, Y)
```

EL.plot *Draws plots using the smoothed two-sample empirical likelihood method*

Description

Draws P-P and Q-Q plots, ROC curves, quantile differences (qdiff) and CDF differences (ddiff) and their respective confidence bands (pointwise or simultaneous) using the empirical likelihood method.

Usage

```
EL.plot(
  method,
  X,
  Y,
  bw = bw.nrd0,
  conf.level = NULL,
  simultaneous = FALSE,
  bootstrap.samples = 300,
  more.warnings = FALSE,
  ...
)
```

Arguments

method	One of "pp", "qq", "roc", "qdiff", or "fdiff".
X	A numeric vector of data values.
Y	A numeric vector of data values.
bw	A function taking a numeric vector and returning a bandwidth, or a numeric vector of length two giving the bandwidths for X and Y respectively. Default is bw.nrd0.
conf.level	Confidence level for the intervals, a number in $(0, 1)$, or NULL to skip confidence band calculation.
simultaneous	Logical. If TRUE, simultaneous confidence bands are constructed via a nonparametric bootstrap. Default is FALSE (pointwise intervals).
bootstrap.samples	Integer. Number of bootstrap samples used when simultaneous = TRUE. Default is 300.
more.warnings	Logical. If FALSE (default), a single warning is issued if any problem occurs. If TRUE, a warning is produced for every point with a problem.
...	Further arguments passed to the plot function.

Details

The plotting interval for P-P plots, ROC curves and differences of quantile functions is $[0, 1]$. The Q-Q plot is drawn from the minimum to the maximum of `Y`. For the plot of distribution function differences the interval from $\max(\min(X), \min(Y))$ to $\min(\max(X), \max(Y))$ is used.

Confidence bands are drawn only if `conf.level` is not `NULL`.

When constructing simultaneous confidence bands, the plot is drawn on an interval narrowed by 5% on both sides, since the procedure is usually sensitive at the endpoints. The confidence level is bootstrapped using 50 evenly spaced points in this interval. If the default interval produces excessively wide bands, use `EL.smooth` where intervals are specified manually. Note that calculation of simultaneous confidence bands can be slow.

Value

A `ggplot` object. The plot can be further customized using **ggplot2** functions, as shown in the examples.

Author(s)

E. Cers, J. Valeinis

References

Valeinis, J. and Cers, E. Extending the two-sample empirical likelihood. Preprint: `http://home.lu.lv/~valeinis/lv/petnieciba/EL_TwoSample_2011.pdf`

Hall, P. and Owen, A. (1993). Empirical likelihood bands in density estimation. *Journal of Computational and Graphical Statistics*, **2**(3), 273–289.

See Also

`EL.smooth`, `EL.statistic`

Examples

```
## The examples showcase all available graphs
set.seed(42)
X1 <- rnorm(100, 0.5, 1.5)
X2 <- rnorm(100, 0, 1)


xlim <- c(min(X1, X2) - 0.5, max(X1, X2) + 0.5)
D1 <- density(X1)
D2 <- density(X2)
df <- data.frame(x1 = D1$x, y1 = D1$y, x2 = D2$x, y2 = D2$y)
p1 <- ggplot2::ggplot(data = df) +
    ggplot2::geom_line(ggplot2::aes(x = x2, y = y2,
        color = paste0('X2 (bw=', round(D2$bw, 2), ')'))) +
    ggplot2::geom_line(ggplot2::aes(x = x1, y = y1,
        color = paste0('X1 (bw=', round(D1$bw, 2), ')'))) +
    ggplot2::guides(color = ggplot2::guide_legend(title = NULL)) +
```

```
    ggplot2::theme_minimal() +
    ggplot2::theme(legend.position = "top") +
    ggplot2::labs(x = "X", y = "Density")
p1

# CDF differences
p2 <- EL.plot("fdiff", X1, X2, main = "F difference", conf.level = 0.95)
tt <- seq(max(c(min(X1), min(X2))), min(c(max(X1), max(X2))), length = 30)
ee <- ecdf(X2)(tt) - ecdf(X1)(tt)
p2 <- p2 + ggplot2::geom_point(data = data.frame(tt = tt, ee = ee),
                               ggplot2::aes(x = tt, y = ee))
p2

# Quantile differences
p3 <- EL.plot("qdiff", X1, X2, main = "Quantile difference", conf.level = 0.95)
tt <- seq(0.01, 0.99, length = 30)
ee <- quantile(X2, tt) - quantile(X1, tt)
p3 <- p3 + ggplot2::geom_point(data = data.frame(tt = tt, ee = ee),
                               ggplot2::aes(x = tt, y = ee))
p3

# Q-Q plot
p4 <- EL.plot("qq", X1, X2, main = "Q-Q plot", conf.level = 0.95)
tt <- seq(min(X2), max(X2), length = 30)
ee <- quantile(X1, ecdf(X2)(tt))
p4 <- p4 + ggplot2::geom_point(data = data.frame(tt = tt, ee = ee),
                               ggplot2::aes(x = tt, y = ee))
p4

# P-P plot
p5 <- EL.plot("pp", X1, X2, main = "P-P plot", conf.level = 0.95, ylim = c(0, 1))
tt <- seq(0.01, 0.99, length = 30)
ee <- ecdf(X1)(quantile(X2, tt))
p5 <- p5 + ggplot2::geom_point(data = data.frame(tt = tt, ee = ee),
                               ggplot2::aes(x = tt, y = ee))
p5

# ROC curve
p6 <- EL.plot("roc", X1, X2, main = "ROC curve", conf.level = 0.95, ylim = c(0, 1))
tt <- seq(0.01, 0.99, length = 30)
ee <- 1 - ecdf(X1)(quantile(X2, 1 - tt))
p6 <- p6 + ggplot2::geom_point(data = data.frame(tt = tt, ee = ee),
                               ggplot2::aes(x = tt, y = ee))
p6

# To show all plots at once:
# require(cowplot)
# cowplot::plot_grid(p1, p2, p3, p4, p5, p6, ncol = 2)
```

EL.smooth — *Smooth estimates and confidence intervals using the smoothed two-sample EL method*

Description

Calculates estimates and pointwise confidence intervals (or simultaneous bands) for P-P and Q-Q plots, ROC curves, quantile differences (`qdiff`) and CDF differences (`ddiff`) using the smoothed empirical likelihood method.

Usage

```
EL.smooth(
  method,
  X,
  Y,
  t,
  bw = bw.nrd0,
  conf.level = NULL,
  simultaneous = FALSE,
  bootstrap.samples = 300,
  more.warnings = FALSE
)
```

Arguments

`method`	One of `"pp"`, `"qq"`, `"roc"`, `"qdiff"`, or `"fdiff"`.
`X`	A numeric vector of data values.
`Y`	A numeric vector of data values.
`t`	A numeric vector of points at which to calculate estimates and confidence intervals.
`bw`	A function taking a numeric vector and returning a bandwidth, or a numeric vector of length two giving the bandwidths for `X` and `Y` respectively. Default is `bw.nrd0`.
`conf.level`	Confidence level for the intervals, a number in $(0, 1)$, or `NULL` to skip confidence band calculation.
`simultaneous`	Logical. If `TRUE`, simultaneous confidence bands are constructed via a nonparametric bootstrap. Default is `FALSE` (pointwise intervals).
`bootstrap.samples`	Integer. Number of bootstrap samples used when `simultaneous = TRUE`. Default is `300`.
`more.warnings`	Logical. If `FALSE` (default), a single warning is issued if any problem occurs. If `TRUE`, a warning is produced for every point with a problem.

Details

Confidence bands are drawn only if `conf.level` is not `NULL`.

When constructing simultaneous confidence bands, check that the chosen range of `t` values does not produce excessively wide bands (for example, for a P-P plot the interval $[0.05, 0.95]$ is typically a sensible choice). This should be verified for each dataset. Note that simultaneous band calculation can be slow.

Value

A list with components:

`estimate` Estimated values at points `t`.

`conf.int` A two-row matrix where each column gives the lower and upper confidence bounds at the corresponding point in `t`.

`simultaneous.conf.int` Logical; `TRUE` if simultaneous bands were constructed.

`bootstrap.crit` The bootstrap critical value of the $-2\log$-likelihood statistic for simultaneous bands at level `conf.level`. Only present when `conf.level` is not `NULL` and `simultaneous = TRUE`.

Author(s)

E. Cers, J. Valeinis

References

Valeinis, J. and Cers, E. Extending the two-sample empirical likelihood. Preprint: http://home.lu.lv/~valeinis/lv/petnieciba/EL_TwoSample_2011.pdf

Hall, P. and Owen, A. (1993). Empirical likelihood bands in density estimation. *Journal of Computational and Graphical Statistics*, **2**(3), 273–289.

See Also

`EL.plot`, `EL.statistic`

Examples

```
#### Simultaneous confidence bands for a P-P plot
X1 <- rnorm(200)
X2 <- rnorm(200, 1)
x <- seq(0.05, 0.95, length = 19)
y <- EL.smooth("pp", X1, X2, x, conf.level = 0.95,
               simultaneous = TRUE, bw = c(0.3, 0.3))
conf.int <- data.frame(x = x, ci.l = y$conf.int[1,], ci.u = y$conf.int[2,])

## Plot with both pointwise and simultaneous confidence bands
EL.plot("pp", X1, X2, conf.level = 0.95, bw = c(0.3, 0.3)) +
    ggplot2::geom_line(data = conf.int, ggplot2::aes(x = x, y = ci.u), lty = "dotted") +
    ggplot2::geom_line(data = conf.int, ggplot2::aes(x = x, y = ci.l), lty = "dotted")
```

EL.statistic *The two-sample empirical likelihood statistic*

Description

Calculates -2 times the log-likelihood ratio statistic when the function of interest (either of P-P or Q-Q plot, ROC curve, difference of quantile or distribution functions) at some point `t` is equal to `d`.

Usage

```
EL.statistic(method, X, Y, Delta, d, t, bw = bw.nrd0, conf.level = 0.95)
```

Arguments

`method`	One of "pp", "qq", "roc", "qdiff", or "fdiff".
`X`	A numeric vector of data values.
`Y`	A numeric vector of data values.
`Delta`	A number. The hypothesised value of the function at point `t`.
`d`	Deprecated. Use `Delta` instead.
`t`	A number. The point at which to evaluate the statistic.
`bw`	A function taking a numeric vector and returning a bandwidth, or a numeric vector of length two giving the bandwidths for `X` and `Y` respectively. Default is `bw.nrd0`.
`conf.level`	Confidence level for the reported confidence interval (default `0.95`).

Value

An object of class "htest".

Author(s)

E. Cers, J. Valeinis

References

Valeinis, J. and Cers, E. Extending the two-sample empirical likelihood. Preprint: http://home.lu.lv/~valeinis/lv/petnieciba/EL_TwoSample_2011.pdf

See Also

`EL.smooth`

Examples

```
EL.statistic(method = "pp", X = rnorm(100), Y = rnorm(100), Delta = 0.5, t = 0.5)
```

`FDEL.acf` *Two-sample frequency-domain empirical likelihood test for autocorrelation difference*

Description

Tests whether the lag-h autocorrelation of two independent stationary time series differ by a specified amount Δ.

Usage

```
FDEL.acf(
  X,
  Y,
  Delta = 0,
  lag = 1,
  bartlett = FALSE,
  bootstrap.samples = 500,
  center = TRUE,
  seed = NULL,
  span = 0.15,
  rho.lower = -0.99,
  rho.upper = 0.99
)
```

Arguments

`X, Y`	Numeric time-series vectors (length >= 10).
`Delta`	Hypothesised difference $\rho_x(h) - \rho_y(h) = \Delta$ (default 0).
`lag`	Positive integer lag h (default 1).
`bartlett`	Logical; apply a bootstrap Bartlett correction? (default `FALSE`).
`bootstrap.samples`	Number of bootstrap replicates for the Bartlett correction (used only when `bartlett = TRUE`).
`center`	Logical; subtract sample means before computing periodograms? (default `TRUE`).
`seed`	Optional integer seed for reproducibility.
`span`	Loess span for periodogram smoothing used in the Bartlett correction (default 0.15).
`rho.lower, rho.upper`	Search bounds for the profile autocorrelation optimization (defaults `-0.99`, `0.99`).

Value

An object of class `c("FDELacf", "htest")` with components `statistic`, `parameter`, `p.value`, `estimate`, `null.value`, `alternative`, `method`, `data.name`, `lag`, `bartlett`, `bartlett.factor`, `statistic.uncorrected`, `p.value.uncorrected`, `B`, `call`.

Author(s)

R. Alksnis, J. Valeinis

Examples

```
set.seed(1)
X <- arima.sim(n = 200, model = list(ar = 0.3))
Y <- arima.sim(n = 350, model = list(ar = 0.3))
FDEL.acf(X, Y)
## With Bartlett correction (slower)
## FDEL.acf(X, Y, bartlett = TRUE, B = 199)
```

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