

# Package ‘Kurt’

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**Type** Package

**Title** Performs Kurtosis-Based Statistical Analyses

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**Description** Computes measures of multivariate kurtosis, matrices of fourth-order moments and cumulants, kurtosis-based projection pursuit.  
Franceschini, C. and Loperfido, N. (2018, ISBN:978-3-319-73905-2). ``An Algorithm for Finding Projections with Extreme Kurtosis".  
Loperfido, N. (2017,ISSN:0024-3795). ``A New Kurtosis Matrix, with Statistical Applications".

**License** GPL (>= 2)

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Nicola Loperfido [aut]

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 Kurt-package

*Kurt: Performs kurtosis-based statistical analyses*


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## Description

Computes measures of multivariate kurtosis, matrices of fourth-order moments and cumulants, kurtosis-based projection pursuit

## Details

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ExtKur	ExtKur: kurtosis based projection pursuit
ExtKurBiv	ExtKurBiv: kurtosis-based projection pursuit for bivariate random vectors
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Fourth4	Fourth4: fourth moment of a data matrix
Kurt-package	Kurt: Performs kurtosis-based statistical analyses
NoKurt	NoKurt: data projections whose excess kurtosis is as close to zero as possible
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optik	optik

ScalarKurt(), ExtKur(), ExtKurBiv(), optik(), NoKurt(), Cum4(), Fourth(), Fourth4()

## Author(s)

Cinzia Franceschini and Nicola Loperfido

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## References

- Franceschini, C. and Loperfido, N. (2012). On some Inequalities between Measures of Multivariate Kurtosis, with Application to Financial Returns. In "Mathematical and Statistical Methods for Actuarial Sciences and Finance", Perna, C. and Sibillo, M. (Eds.), Springer, 211-218.
- Franceschini, C. and Loperfido, N. (2018). An Algorithm for Finding Projections with Extreme Kurtosis. In "Studies in Theoretical and Applied Statistics: SIS2016-48th Meeting of the Italian Statistical Society, Salerno 8-10 June 2016", Perna C., Pratesi M. and Ruiz-Gazen A. (Eds.), Springer.
- Henze, N. (1994). On Mardia's kurtosis test for multivariate normality. *Communications in statistics-Theory and Methods* 23:4, 1031-1045.
- Kollo, T. (2008). Multivariate skewness and kurtosis measures with an application in ICA. *Journal of Multivariate Analysis* 99, 2328-2338.

- Kollo, T. and Srivastava, M.S. (2005). Estimation and testing of parameters in multivariate Laplace distribution. *Comm. Statist.* 33, 2363–2687.
- Koziol, J.A. (1987). An alternative formulation of Neyman's smooth goodness of fit tests under composite alternatives. *Metrika* 34, 17-24.
- Koziol, J.A. (1989). A note on measures of multivariate kurtosis. *Biometrical Journal* 31, 619-624.
- Loperfido, N. (2011). Spectral Analysis of the Fourth Moment Matrix. *Linear Algebra and its Applications* 435, 1837-1844.
- Loperfido, N. (2014). A Note on the Fourth Cumulant of a Finite Mixture Distribution. *Journal of Multivariate Analysis* 123, 386-394.
- Loperfido, N. (2017). A New Kurtosis Matrix, with Statistical Applications. *Linear Algebra and its Applications* 512, 1-17.
- Loperfido N. (2019). Kurtosis-Based Projection Pursuit for Outlier Detection in Financial Time Series. *The European Journal of Finance*, to appear.
- Loperfido, N. (2020). Some Remarks on Koziol's Kurtosis. *Journal of Multivariate Analysis* 175, to appear.
- Malkovich, J.F. and Afifi, A.A. (1973). On Tests for Multivariate Normality. *J. Amer. Statist. Ass.* 68, 176-179.
- Mardia, K.V. (1970). Measures of multivariate skewness and kurtosis with applications. *Biometrika* 57, 519-530.
- Mardia, K. V. and Kent, J. T. (1991). Rao Score Tests for Goodness of Fit and Independence. *Biometrika* 78, 355-36.
- Miettinen J., Taskinen S., Nordhausen K. and Oja H. (2015). Fourth Moments and Independent Component Analysis. *Statistical Science* 30, 372-390.
- Mori T.F., Rohatgi V.K. and Szekely G.J. (1993). On multivariate skewness and kurtosis. *Theory Probab. Appl.* 38, 547-551.

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 Cum4

*Cum4: fourth multivariate cumulant*


---

### Description

returns a matrix containing the fourth cumulants of the given data

### Usage

```
Cum4(data, type, shape)
```

### Arguments

data	data matrix
type	type=0 uses original data, type=1 uses centered data, type=2 uses standardized data
shape	if shape="square" the output is a $d^2 \times d^2$ matrix. If shape="rectangular", the output is a $d \times d^3$ matrix. Where d is the number of variables

**Value**

K4 is the matrix containing the fourth cumulants of the given data

**Author(s)**

Cinzia Franceschini and Nicola Loperfido

**Examples**

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
Cum4(iris[,1:4], 1, "square") # returns a matrix containing the fourth cumulants of the given data
```

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ExtKur

*ExtKur: kurtosis based projection pursuit*

---

**Description**

Returns a data projection with either maximal or minimal kurtosis.

**Usage**

```
ExtKur(data, iterations, maxmin)
```

**Arguments**

data	data matrix
iterations	number of required iterations
maxmin	is the choice to either maximise ("MAX") or minimise ("MIN") kurtosis

**Value**

linear	vector of coefficients
projection	vector of projected data
kurt	extreme kurtosis attainable by a data projection

**Author(s)**

Cinzia Franceschini and Nicola Loperfido

**Examples**

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
ExtKur(iris[,1:4],10,"MAX") #returns a data projection with maximal kurtosis
```

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ExtKurBiv	<i>ExtKurBiv: kurtosis-based projection pursuit for bivariate random vectors</i>
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### Description

Returns a projection of bivariate data with either maximal or minimal kurtosis.

### Usage

```
ExtKurBiv(data, maxmin)
```

### Arguments

data	data matrix
maxmin	choice between maximal ("MAX") and minimal ("MIN") kurtosis

### Value

linearMAX	coefficients of the projections maximising kurtosis
projectionMAX	projection with maximal kurtosis
kurtMAX	maximal kurtosis
linearMIN	coefficients of the projections minimising kurtosis
projectionMIN	projection with minimal kurtosis
kurtMIN	minimal kurtosis

### Author(s)

Cinzia Franceschini and Nicola Loperfido

### Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode

ExtKurBiv(iris[,1:2],"MAX")# returns a projection of bivariate data with maximal kurtosis
```

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Fourth

*Fourth: matrices of fourth moments or fourth cumulants*

---

### Description

Returns a matrix containing either the fourth moments or the fourth cumulants. It recalls the functions Fourth4 and Cum4.

### Usage

```
Fourth(data, type, shape, feature)
```

### Arguments

data	data matrix
type	type =0 is the ordinary fourth moment / cumulant; type =1 is the centered fourth moment / cumulant; type =2 is the standardized fourth moment / cumulant
shape	"square" or "rectangular"
feature	"moment" or "cumulant". If feature is "moment", the function computes the fourth moment of a data matrix. The function recalls the function Fourth4. If feature is "cumulant", the function computes the fourth multivariate cumulant. The function recalls the function Cum4.

### Value

M	Fourth square moment matrix
MM	Fourth rectangular moment matrix
K4	Fourth cumulants of the given data

### Author(s)

Cinzia Franceschini and Nicola Loperfido

### Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
Fourth(iris[,1:4], 1,"square", "moment")#returns a matrix containing the fourth moments
```

---

 Fourth4

*Fourth4: fourth moment of a data matrix*


---

**Description**

Returns a matrix containing the fourth moments.

**Usage**

```
Fourth4(data, type, shape)
```

**Arguments**

data	data matrix
type	type=0 is the ordinary fourth moment, type=1 is the centered fourth moment, type=2 is the standardized fourth moment
shape	"square" or "rectangular"

**Value**

M	Fourth square moment matrix
MM	Fourth rectangular moment matrix

**Author(s)**

Cinzia Franceschini and Nicola Loperfido

**Examples**

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
Fourth4(iris[,1:4], 1,"square") #returns a matrix containing the fourth moments
```

---

 NoKurt

*NoKurt: data projections whose excess kurtosis is as close to zero as possible*


---

**Description**

Data projections whose excess kurtosis is as close to zero as possible. Excess kurtosis is the fourth standardized cumulant, that is the fourth standardized moment minus three.

**Usage**

```
NoKurt(data, number)
```

**Arguments**

data	data matrix
number	number of required projections. It must be greater than one and less than the number of variables

**Value**

Nkurtoses	kurtoses of Nprojections
Nprojections	data projections ordered according to the absolute values of their excess kurtoses
MATRIX	matrix characterizing the projection

**Author(s)**

Cinzia Franceschini and Nicola Loperfido

**Examples**

```
data(iris)
iris<-data.matrix(iris[,1:4])
NoKurt(iris[,1:4],3)#returns data projections whose excess kurtosis is as close to zero as possible
```

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optik

*optik*

---

**Description**

It computes the matrix containing the smallest and largest kurtoses of data projections as well as the corresponding directions.

**Usage**

```
optik(data)
```

**Arguments**

data	data matrix
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**Value**

kurMAX	kurtosis of the projection maximizing kurtosis
pMAX	projection maximizing kurtosis
dMAX	direction maximizing kurtosis
kurMINbis	kurtosis of the projection minimizing kurtosis
pMINbis	projection minimizing kurtosis
dMINbis	direction minimizing kurtosis

**Author(s)**

Cinzia Franceschini and Nicola Loperfido

**Examples**

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
optik(iris[,1:4])#starting values of the projections with extreme kurtosis
```

---

ScalarKurt

*ScalarKurt*

---

**Description**

Returns the statistic and the p-value of either Mardia's kurtosis or Koziol's kurtosis tests for normality .

**Usage**

```
ScalarKurt(data, feature, type, prob)
```

**Arguments**

data	data matrix
feature	"moment" or "cumulant"
type	"Mardia" or "Koziol"
prob	"lower" if probability is $P[X \leq x]$ , "upper" if probability is $P[X > x]$ , "twoside" if probability is computed on both tails

**Details**

For Koziol kurtosis only the upper tail is meaningful

**Value**

```
statistic
pvalue
```

**Author(s)**

Cinzia Franceschini and Nicola Loperfido

**Examples**

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
ScalarKurt(iris[,1:4],"moment","Mardia","upper")#returns the statistic and the p-value
#of Mardia's kurtosis test for normality
```

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