

# Package ‘REndo’

May 8, 2026

**Title** Fitting Linear Models with Endogenous Regressors using Latent Instrumental Variables

**Type** Package

**Version** 2.5.0

**Date** 2026-05-06

## Description

Fits linear models with endogenous regressor using latent instrumental variable approaches. The methods included in the package are Lewbel's (1997) <doi:10.2307/2171884> higher moments approach as well as Lewbel's (2012) <doi:10.1080/07350015.2012.643126> heteroscedasticity approach, Park and Gupta's (2012) <doi:10.1287/mksc.1120.0718> joint estimation method that uses Gaussian copula and Kim and Frees's (2007) <doi:10.1007/s11336-007-9008-1> multilevel generalized method of moment approach that deals with endogeneity in a multilevel setting. These are statistical techniques to address the endogeneity problem where no external instrumental variables are needed. See the publication related to this package in the Journal of Statistical Software for more details: <doi:10.18637/jss.v107.i03>. Note that with version 2.0.0 sweeping changes were introduced which greatly improve functionality and usability but break backwards compatibility.

**License** GPL-3

**URL** <https://github.com/mmeierer/REndo>

**BugReports** <https://github.com/mmeierer/REndo/issues>

**NeedsCompilation** yes

**Depends** R (>= 3.5)

**Imports** methods (>= 3.4), stats (>= 3.4), utils (>= 3.4), Formula (>= 1.2), optimx (>= 2013.8.7), mvtnorm (>= 1.0-8), AER (>= 1.2-5), Matrix (>= 1.2-14), lme4 (>= 1.1-18-1), reformulas (>= 0.4.4), data.table (>= 1.11.8), corpcor (>= 1.6.9), Rcpp (>= 1.0.3), lmtest (>= 0.9-35), copula (>= 1.0-0), ks (>= 1.10.0)

**Suggests** testthat, covr, R.rsp

**LinkingTo** Rcpp, RcppEigen

**Encoding** UTF-8

**RoxygenNote** 7.3.3

**VignetteBuilder** R.rsp

**LazyData** true

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**Date/Publication** 2026-05-08 05:10:03 UTC

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confint.rendo.boots     *Confidence Intervals for Bootstrapped Model Parameters*

---

### Description

Confidence Intervals for Bootstrapped Model Parameters

### Usage

```
## S3 method for class 'rendo.boots'
confint(object, parm, level = 0.95, ...)
```

### Arguments

object	a fitted model object with bootstrapped parameters. Typically from <code>copulaCorrection</code>
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
level	the confidence level required.
...	ignored, for consistency with the generic function.

### Details

Computes the two-sided percentile confidence intervals from the bootstrapped parameter estimates. The intervals are obtained by selecting the quantile of the bootstrapped parameter estimates corresponding to the given alpha level.

A minimum of  $\frac{1}{\min(\text{level}, 1-\text{level})}$  parameters estimates are needed to derive the confidence interval. The reason for this is that there is otherwise no natural way to derive the percentiles (ie one cannot reasonably estimate the 95% quantile of only 7 values).

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copulaCorrection     *Fitting Linear Models Endogenous Regressors using Gaussian Copula*

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### Description

Fits linear models with continuous or discrete endogenous regressors (or a mixture of both) using Gaussian copulas, as presented in Park and Gupta (2012). This is a statistical technique to address the endogeneity problem where no external instrumental variables are needed. The important assumption of the model is that the endogenous variables should NOT be normally distributed, if continuous, preferably with a skewed distribution. The corrections proposed by Qian, Koschmann, and Xie (2024, p.19-22) are implemented. These mitigate the bias of the original paper for small and moderate sample sizes.

**Usage**

```
copulaCorrection(formula, data, num.boots = 1000, verbose = TRUE, ...)
```

**Arguments**

formula	A symbolic description of the model to be fitted. See the "Details" section for the exact notation.
data	A data.frame containing the data of all parts specified in the formula parameter.
num.boots	Positive integer giving the number of bootstrap replications used for standard error estimation. A minimum of 1000 is recommended.
verbose	Show details about the running of the function.
...	Arguments for the log-likelihood optimization function in the case of a single continuous endogenous regressor. Ignored with a warning otherwise.
	<b>start.params</b> A named vector containing a set of parameters to use in the first optimization iteration. The names have to correspond exactly to the names of the components specified in the formula parameter. If not provided, a linear model is fitted to derive them.
	<b>optimx.args</b> A named list of arguments which are passed to <code>optimx</code> . This allows users to tweak optimization settings to their liking.

**Details****Method:**

The underlying idea of the joint estimation method is that using information contained in the observed data, one selects marginal distributions for the endogenous regressor and the structural error term, respectively. Then, the copula model enables the construction of a flexible multivariate joint distribution allowing a wide range of correlations between the two marginals.

Consider the model:

$$Y_t = \beta_0 + \beta_1 P_t + \beta_2 X_t + \epsilon_t$$

where  $t = 1, \dots, T$  indexes either time or cross-sectional units,  $Y_t$  is a  $1 \times 1$  response variable,  $X_t$  is a  $k \times n$  exogenous regressor,  $P_t$  is a  $k \times 1$  continuous endogenous regressor,  $\epsilon_t$  is a normally distributed structural error term with mean zero and  $E(\epsilon^2) = \sigma_\epsilon^2$ ,  $\alpha$  and  $\beta$  are model parameters.

The marginal distribution of the endogenous regressor  $P_t$  is obtained using the Epanechnikov kernel density estimator (Epanechnikov, 1969), as below:

$$\hat{h}(p) = \frac{1}{T \cdot b} \sum_{t=1}^T K\left(\frac{p - P_t}{b}\right)$$

where  $P_t$  is the endogenous regressor,  $K(x) = 0.75 \cdot (1 - x^2) \cdot I(\|x\| \leq 1)$  and the bandwidth  $b$  is the one proposed by Silverman (1986), and is equal to  $b = 0.9 \cdot T^{-1/5} \cdot \min(s, IQR/1.34)$ .  $IQR$  is the interquartile range while  $s$  is the data sample standard deviation and  $T$  is the number of time periods observed in the data. After obtaining the joint distribution of the error term and the continuous endogenous regressor, the model parameters are estimated using maximum likelihood estimation.

The additional parameters used during model fitting and printed in summary hence are:

rho The correlation between the endogenous regressor and the error.

sigma The variance of the model's error.

With more than one continuous endogenous regressor or an endogenous discrete regressor, an alternative approach to the estimation using Gaussian copula should be applied. This approach is similar to the control function approach (Petrin and Train, 2010). The core idea is to apply OLS estimation on the original set of explanatory variables in the model equation above, plus an additional regressor  $P_t^* = \Phi^{-1}(H(P_t))$ . Here,  $H(P_t)$  is the marginal distribution of the endogenous regressor  $P$ . Including this regressor solves the correlation between the endogenous regressor and the structural error,  $\epsilon$ , OLS providing consistent parameter estimates. Due to identification problems, the discrete endogenous regressor cannot have a binomial distribution.

Hence, only in the case of a single continuous endogenous regressor maximum likelihood estimation is used. In all other cases, augmented OLS based on Gaussian copula is applied. This includes cases of multiple endogenous regressors of both discrete and continuous distributions.

In the case of discrete endogenous regressors, a random seed needs to be assigned because the marginal distribution function of the endogenous regressor is a step function in this case. This means that the value of  $P^*$  lies between 2 values,  $\Phi^{-1}(H(P_t - 1))$  and  $\Phi^{-1}(H(P_t))$ . However, the reported upper and lower bounds of the 95% bootstrapped confidence interval gives indication of the variance of the estimates.

Since the inference procedure in both cases, augmented OLS and maximum likelihood, occurs in two stages (first the empirical distribution of the endogenous regressor is computed and then used in constructing the likelihood function), the standard errors are not correct. Therefore, in both cases, the standard errors and the confidence intervals are obtained based on the sampling distributions resulted from bootstrapping. Since the distribution of the bootstrapped parameters is highly skewed, we report the percentile confidence intervals. Moreover, the variance-covariance matrix is also computed based on the bootstrapped parameters, and not based on the Hessian.

**Formula parameter:** The formula argument follows a two part notation:

A two-sided formula describing the model (e.g.  $y \sim X1 + X2 + P$ ) to be estimated and a second right-hand side part in which the endogenous regressors and their distributional assumptions are indicated (e.g. `continuous(P)`). These two parts are separated by a single vertical bar (`|`). In the second part, the special functions `continuous`, `discrete`, or a combination of both, are used to indicate the endogenous regressors and their respective distribution. Both functions use the `...` parameter in which the respective endogenous regressors is specified.

Note that no argument to `continuous` or `discrete` is to be supplied as character but as symbols without quotation marks.

See the example section for illustrations on how to specify the formula parameter.

## Value

For all cases, an object of classes `rendo.copula.correction`, `rendo.boots`, and `rendo.base` is returned which is a list and contains the following components:

<code>formula</code>	The formula given to specify the fitted model.
<code>terms</code>	The terms object used for model fitting.
<code>model</code>	The <code>model.frame</code> used for model fitting.
<code>coefficients</code>	A named vector of all coefficients resulting from model fitting.

`names.main.coefs` a vector specifying which coefficients are from the model. For internal usage.

`names.vars.continuous` The names of the continuous endogenous regressors.

`names.vars.discrete` The names of the discrete endogenous regressors.

`fitted.values` Fitted values at the found solution.

`residuals` The residuals at the found solution.

`boots.params` The bootstrapped coefficients.

For the case of a single continuous endogenous regressor, the returned object further contains the following components:

`start.params` A named vector with the initial set of parameters used to optimize the log-likelihood function.

`res.optimx` The result object returned by the function `optimx` after optimizing the log-likelihood function.

For all other cases, the returned object further contains the following component:

`res.lm.real.data` The linear model fitted on the original data together with generated p.star data.

The function summary can be used to obtain and print a summary of the results. Depending on the returned object, the generic accessor functions `coefficients`, `fitted.values`, `residuals`, `vcov`, `logLik`, `AIC`, `BIC`, and `nobs` are available.

## References

- Park, S. and Gupta, S., (2012), "Handling Endogenous Regressors by Joint Estimation Using Copulas", *Marketing Science*, 31(4), 567-86. doi:10.1287/mksc.1120.0718
- Qian, Y., Koschmann, A., and Xie, H. (2024). "A Practical Guide to Endogeneity Correction Using Copulas". National Bureau of Economic Research, w32231.
- Epanechnikov V (1969). "Nonparametric Estimation of a Multidimensional Probability Density." *Teoriya veroyatnostei i ee primeneniya*, 14(1), 156–161.
- Silverman B (1986). "Density Estimation for Statistics and Data Analysis". CRC Monographs on Statistics and Applied Probability. London: Chapman & Hall.
- Petrin A, Train K (2010). "A Control Function Approach to Endogeneity in Consumer Choice Models." *Journal of Marketing Research*, 47(1), 3–13.

## See Also

[summary](#) for how fitted models are summarized

[vcov](#) for how the variance-covariance matrix is derived

[confint](#) for how confidence intervals are derived

[optimx](#) for possible elements of parameter `optimx.arg`

Other copula-based methods: [copulaIMA\(\)](#)

**Examples**

```

data("dataCopCont")
data("dataCopCont2")
data("dataCopDis")
data("dataCopDis2")
data("dataCopDisCont")

## Not run:
# Single continuous: log-likelihood optimization
c1 <- copulaCorrection(y~X1+X2+P|continuous(P), num.boots=10, data=dataCopCont)
# same as above, with start.parameters and number of bootstrappings
c1 <- copulaCorrection(y~X1+X2+P|continuous(P), num.boots=10, data=dataCopCont,
  start.params = c("(Intercept)"=1, X1=1, X2=-2, P=-1))

# All following examples fit linear model with Gaussian copulas

# 2 continuous endogenous regressors
c2 <- copulaCorrection(y~X1+X2+P1+P2|continuous(P1, P2),
  num.boots=10, data=dataCopCont2)
# same as above
c2 <- copulaCorrection(y~X1+X2+P1+P2|continuous(P1)+continuous(P2),
  num.boots=10, data=dataCopCont2)

# single discrete endogenous regressor
d1 <- copulaCorrection(y~X1+X2+P|discrete(P), num.boots=10, data=dataCopDis)

# two discrete endogenous regressor
d2 <- copulaCorrection(y~X1+X2+P1+P2|discrete(P1)+discrete(P2),
  num.boots=10, data=dataCopDis2)
# same as above but less bootstrap runs
d2 <- copulaCorrection(y~X1+X2+P1+P2|discrete(P1, P2), num.boots = 10,
  data=dataCopDis2)

# single discrete, single continuous
cd <- copulaCorrection(y~X1+X2+P1+P2|discrete(P1)+continuous(P2),
  num.boots=10, data=dataCopDisCont)

# For single continuous only: use own optimization settings (see optimx())
# set maximum number of iterations to 50'000
res.c1 <- copulaCorrection(y~X1+X2+P|continuous(P),
  optimx.args = list(itnmax = 50000),
  num.boots=10, data=dataCopCont)

# print detailed tracing information on progress
res.c1 <- copulaCorrection(y~X1+X2+P|continuous(P),
  optimx.args = list(control = list(trace = 6)),
  num.boots=10, data=dataCopCont)

# use method L-BFGS-B instead of Nelder-Mead and print report every 50 iterations
res.c1 <- copulaCorrection(y~X1+X2+P|continuous(P),
  optimx.args = list(method = "L-BFGS-B",

```

```

                                control=list(trace = 2, REPORT=50)),
                                num.boots=10, data=dataCopCont)

# For coef(), the parameter "complete" determines if only the
# main model parameters or also the auxiliary coefficients are returned

c1.all.coefs <- coef(res.c1) # also returns rho and sigma
# same as above
c1.all.coefs <- coef(res.c1, complete = TRUE)

# only main model coefs
c1.main.coefs <- coef(res.c1, complete = FALSE)

## End(Not run)

```

---

copulaIMA

*Copula-based Instrumental Variable Model (IMA)*


---

## Description

Fitting Haschka's copula-based Instrumental Variable Model (IMA) using Gaussian copulas to address endogeneity without external instruments, while allowing dependence between endogenous and exogenous regressors.

The copula-based IMA approach corrects endogeneity by constructing control functions from the marginal distributions of the endogenous regressors and incorporating them into the estimation equation. Inference is based on bootstrapping.

The key contribution of IMA over the original Park and Gupta (2012) copula correction ([copulaCorrection](#)) is that it does not require the endogenous and exogenous regressors to be independent. When this independence assumption is violated, which is common in practice, the original copula correction yields severely biased estimates of all model coefficients. IMA corrects for this by exploiting the content of the exogenous variables through a first-stage auxiliary regression, in a similar way to instrumental variable identification.

The IMA method requires a normally distributed structural error. At least one endogenous regressor  $P_{i,k}$  is continuously and nonnormally distributed. It does not require that the exogenous regressors  $X_i$  be independent of the endogenous regressors  $P_{i,k}$  or to satisfy any distributional requirement, i.e.,  $X_i$  may be binary or non-continuously distributed. This method supports only continuous endogenous regressors.

## Usage

```

copulaIMA(
  formula,
  data,
  cdf = c("adj.ecdf", "resc.ecdf", "ecdf", "kde"),
  num.boots = 1000,
  verbose = TRUE
)

```

**Arguments**

formula	A symbolic description of the model to be fitted. See the "Details" section for the exact notation.
data	A data.frame containing the data of all parts specified in the formula parameter.
cdf	Character string specifying the method used to estimate the marginal distribution functions of the endogenous regressors. One of "adj.ecdf", "resc.ecdf", "ecdf", or "kde". "adj.ecdf" Adjusted empirical CDF with midrank correction (Liengaard et al., 2024). Keeps values strictly inside (0,1). "resc.ecdf" Rescaled empirical CDF via copula::pobs (Qian et al., 2024). "ecdf" Empirical CDF with boundary replacement (Becker et al., 2022). "kde" Integral of a density estimator via ks::kcde used in (Park and Gupta, 2012).
num.boots	Positive integer giving the number of bootstrap replications used for standard error estimation. A minimum of 1000 is recommended.
verbose	Show details about the running of the function.

**Details****Model:**

Consider the structural regression model with  $K$  endogenous regressors:

$$Y_i = \mu + \sum_{k=1}^K P_{i,k} \alpha_k + X_i' \beta + \varepsilon_i$$

where  $i = 1, \dots, n$  is the number of observations,  $Y_i$  is the dependent variable,  $P_{i,k}$  are continuous endogenous regressors that may be correlated with both the structural error  $\varepsilon_i$  and the exogenous regressors  $X_i$ ,  $X_i$  is a vector of exogenous regressors uncorrelated with  $\varepsilon_i$ , and  $\mu, \alpha_k, \beta$  are the structural model parameters.

IMA is an augmented OLS estimator. The estimation proceeds in three steps:

1. For every explanatory variable  $W \in \{X_1, \dots, X_J, P_1, \dots, P_K\}$ , compute the normal score  $W^* = \Phi^{-1}(\hat{F}_W(W))$ , where  $\hat{F}_W$  is the estimated marginal CDF of  $W$  and  $\Phi^{-1}$  is the standard normal quantile function.
2. For each endogenous regressor  $P_{i,k}^*$ , regress it on the normal scores of the exogenous regressors  $X_1^*, \dots, X_J^*$  without intercept and retain the residuals  $\hat{\varepsilon}_k$  as the copula correction terms. This first-stage regression exploits the dependence between endogenous and exogenous regressors in a way analogous to instrumental variable identification.
3. Augment the structural model with  $\hat{\varepsilon}_1, \dots, \hat{\varepsilon}_K$  as additional regressors and estimate by OLS.

Haschka (2025) reported that simulation results showed that copula-based IMA estimator may exhibit a slightly larger bias than alternative two-stage approaches when the sample size is very small (e.g.,  $n = 100$ ) and with intercept. However, the bias decreases rapidly as sample size increases and become negligible for moderate sample sizes (around  $n \geq 600$ ). The estimator appears asymptotically unbiased.

Bootstrap inference is performed by resampling the data with replacement. Degenerate bootstrap samples (e.g. singular design matrices or failed model estimation) are discarded and resampled

until the requested number of valid bootstrap replications is obtained. The percentage of discarded samples is reported as a warning. Confidence intervals are computed using only successful bootstrap replications.

#### Formula interface:

The formula argument follows a two-part notation separated by |. The first part specifies the structural model. The second part identifies the continuous endogenous regressors using continuous():

```
y ~ X + P | continuous(P) # typical use: with intercept
y ~ X + P1 + P2 | continuous(P1) + continuous(P2) # two endogenous regressors
y ~ X + P - 1 | continuous(P) # no intercept (in simulation settings)
```

In typical applied settings the model includes an intercept. The no-intercept specification (-1) is primarily used in simulation studies following the design of (Haschka (2025) Section 4.1) and is unlikely to be appropriate for real datasets.

#### Value

An object of class `rendo.copula.ima` which is a list that contains:

<code>formula</code>	The formula given to specify the fitted model.
<code>model</code>	The <code>model.frame</code> used for model fitting.
<code>terms</code>	The terms object used for model fitting.
<code>coefficients</code>	A named vector of all coefficients resulting from model fitting.
<code>names.main.coefs</code>	A vector specifying which coefficients are from the model. For internal usage.
<code>fitted.values</code>	Fitted values of the structural model.
<code>residuals</code>	The structural residuals.
<code>boots.params</code>	The bootstrapped coefficients.
<code>n.boots.attempted</code>	The number of bootstrap iterations that were attempted.
<code>n.boots.failed</code>	The number of bootstrap iterations resulting in a failed fit.
<code>cdf</code>	The used cdf function.
<code>names.endo.regs</code>	The names of the continuous endogenous regressors.
<code>res.lm.augmented</code>	The fitted augmented regression model, including the control function terms.

#### References

- Haschka, R. E. (2025). Robustness of copula-correction models in causal analysis: Exploiting between-regressor correlation. *IMA Journal of Management Mathematics*, 36, 161-180. doi:10.1093/imaman/dpae018
- Park, S. and Gupta, S., (2012), "Handling Endogenous Regressors by Joint Estimation Using Copulas", *Marketing Science*, 31(4), 567-86. doi:10.1287/mksc.1120.0718

Becker, J.M., Proksch, D., & Ringle, C.M. Revisiting Gaussian copulas to handle endogenous regressors. *Journal of the Academy of Marketing Science*, 50, 46–66 (2022). doi:10.1007/s11747-02100805y

Liengaard, B. D., Becker, J.-M., Bennedsen, M., Heiler, P., Taylor, L. N., & Ringle, C. M. (2025). Dealing with regression models' endogeneity by means of an adjusted estimator for the Gaussian copula approach. *Journal of the Academy of Marketing Science*, 53(1), 279-299 doi:10.1007/s11747024010554

Qian, Y., Koschmann, A., and Xie, H. (2024). A practical guide to endogeneity correction using copulas. NBER Working Paper No. w32231. doi:10.2139/ssrn.4754776

### See Also

Other copula-based methods: [copulaCorrection\(\)](#)

### Examples

```
#-----
# Example 1: Single endogenous regressor with continuous
# and normal exogenous regressor (Haschka 2025, Section 4.1 Scenario 1)
# True values: alpha = 1 (P), beta = 1 (X), no intercept
#-----
data(dataCopIMAContExo)
res <- copulaIMA(
  y ~ X + P - 1 | continuous(P),
  data = dataCopIMAContExo,
  cdf = "adj.ecdf",
  num.boots = 1000
)
summary(res)

#-----
# Example 2: Two endogenous regressors with intercept and no exogenous regressor
# True values: mu=10, alpha1 = 1 (P1), alpha2 = 1 (P2)
# Extension of the first example
#-----
data("dataCopIMAMultiEndo")
res2 <- copulaIMA(
  # Alternative: y ~ P1 + P2 | continuous(P1, P2)
  y ~ P1 + P2 | continuous(P1) + continuous(P2),
  data = dataCopIMAMultiEndo,
  cdf = "adj.ecdf",
  num.boots = 1000
)
summary(res2)

#-----
# Example 3: Single endogenous regressor with binary exogenous regressor
# (Haschka 2025, section 4.1 scenario 2)

# This example shows one of the key example of IMA method, i.e., the exogenous
```

```
# regressor does not need to be continuous or normally distributed.
# X the exogenous regressor is binary (0 or 1).
# True values: alpha = 1 (P), beta = 1 (X), no intercept.
#-----
data("dataCopIMABinExo")
res3 <- copulaIMA(
  y ~ X + P - 1 | continuous(P),
  data = dataCopIMABinExo,
  cdf = "adj.ecdf",
  num.boots = 1000
)
summary(res3)
```

---

dataCopCont

*Simulated Dataset with One Endogenous Continuous Regressor*

---

### **Description**

A dataset with two exogenous regressors,  $X_1, X_2$ , and one endogenous, continuous regressor,  $P$ , having a T-distribution with 3 degrees of freedom. An intercept and a dependent variable,  $y$ , are also included. The true parameter values for the coefficients are:  $b_0 = 2$ ,  $b_1 = 1.5$ ,  $b_2 = -3$  and the coefficient of the endogenous regressor,  $P$ , is equal to  $a_1 = -1$ .

### **Usage**

```
data("dataCopCont")
```

### **Format**

A data frame with 2500 observations on 4 variables:

$y$  a numeric vector representing the dependent variable.

$X_1$  a numeric vector, normally distributed and exogenous.

$X_2$  a numeric vector, normally distributed and exogenous.

$P$  a numeric vector, continuous and endogenous having T-distribution with 3 degrees of freedom.

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---

`dataCopCont2`*Simulated Dataset with Two Endogenous Continuous Regressor*

---

**Description**

A dataset with two exogenous regressors,  $X_1, X_2$ , and two endogenous, continuous regressors,  $P_1$  and  $P_2$ , having a T-distribution with 3 degrees of freedom. An intercept and a dependent variable,  $y$ , are also included. The true parameter values for the intercept and the exogenous regressors' coefficients are:  $b_0 = 2$ ,  $b_1 = 1.5$ ,  $b_2 = -3$ . The coefficient of the endogenous regressor  $P_1$  is equal to  $a_1 = -1$  and of  $P_2$  is equal to  $a_2 = 0.8$ .

**Usage**

```
data("dataCopCont2")
```

**Format**

A data frame with 2500 observations on 5 variables:

$y$  a numeric vector representing the dependent variable.

$X_1$  a numeric vector, normally distributed and exogenous.

$X_2$  a numeric vector, normally distributed and exogenous.

$P_1$  a numeric vector, continuous and endogenous having T-distribution with 3 degrees of freedom.

$P_2$  a numeric vector, continuous and endogenous having T-distribution with 3 degrees of freedom.

**Author(s)**

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---

`dataCopDis`*Simulated Dataset with One Endogenous Discrete Regressor*

---

**Description**

A dataset with two exogenous regressors,  $X_1, X_2$ , and one endogenous, discrete (Poisson distributed) regressor,  $P$ . An intercept and a dependent variable,  $y$ , are also included. The true parameter values for the coefficients are:  $b_0 = 2$ ,  $b_1 = 1.5$ ,  $b_2 = -3$  and the coefficient of the endogenous regressor,  $P$ , is equal to  $a_1 = -1$ .

**Usage**

```
data("dataCopDis")
```

**Format**

A data frame with 2500 observations on 4 variables:

y a numeric vector representing the dependent variable.

X1 a numeric vector, normally distributed and exogenous.

X2 a numeric vector, normally distributed and exogenous.

P a numeric vector, continuous and endogenous having T-distribution with 3 degrees of freedom.

**Author(s)**

Raluca Gui <raluca.gui@business.uzh.ch>

---

dataCopDis2

*Simulated Dataset with Two Endogenous Discrete Regressors*

---

**Description**

A dataset with two exogenous regressors, X1,X2, and two endogenous, discrete (Poisson distributed) regressors, P1 and P2. An intercept and a dependent variable, y, are also included. The true parameter values for the coefficients of the intercept and the exogenous variables are:  $b_0 = 2$ ,  $b_1 = 1.5$ ,  $b_2 = -3$ . The true parameter values for the coefficients of the endogenous regressors are  $a_1 = -1$  for P1 and  $a_2 = 0.8$  for P2.

**Usage**

```
data("dataCopDis2")
```

**Format**

A data frame with 2500 observations on 5 variables:

y a numeric vector representing the dependent variable.

X1 a numeric vector, normally distributed and exogenous.

X2 a numeric vector, normally distributed and exogenous.

P1 a numeric vector, having a Poisson distribution with parameter lambda equal to 3, and endogenous.

P2 a numeric vector, having a Poisson distribution with parameter lambda equal to 3, and endogenous.

**Author(s)**

Raluca Gui <raluca.gui@business.uzh.ch>

---

 dataCopDisCont

*Simulated Dataset with Two Endogenous Regressors*


---

**Description**

A dataset with two exogenous regressors,  $X_1, X_2$ , and two endogenous regressors,  $P_1$ , having a Poisson distribution with lambda parameter equal to 3, and  $P_2$ , having a T-distribution with 3 degrees of freedom. An intercept and a dependent variable,  $y$ , are also included. The true parameter values for the coefficients are:  $b_0 = 2$ ,  $b_1 = 1.5$ ,  $b_2 = -3$  and the coefficient of the endogenous regressor  $P_1$  is set to  $a_1 = -1$  and of  $P_2$  is set to  $a_2 = 0.8$ .

**Usage**

```
data("dataCopDisCont")
```

**Format**

A data frame with 2500 observations on 5 variables:

$y$  a numeric vector representing the dependent variable.

$X_1$  a numeric vector, normally distributed and exogenous.

$X_2$  a numeric vector, normally distributed and exogenous.

$P_1$  a numeric vector, continuous and endogenous having Poisson distribution with parameter lambda equal to 3.

$P_2$  a numeric vector, continuous and endogenous having T-distribution with 3 degrees of freedom.

**Author(s)**

Raluca Gui <raluca.gui@business.uzh.ch>

---

 dataCopIMABinExo

*Simulated Dataset for Copula IMA - Binary Exogenous Regressor*


---

**Description**

A dataset simulated with one binary exogenous regressor  $X$  and one endogenous continuous regressor  $P$ , and a dependent variable  $y$ . The exogenous regressor  $X$  is a binary variable (0 or 1) derived from a latent standard normal variable and is correlated with  $P$  ( $r = 0.5$ ). The endogenous regressor  $P$  follows a bounded continuous distribution ( $\Phi(P^*) + 0.5$ ). The endogeneity strength is given by  $\rho = 0.5$ . This dataset illustrates that the IMA estimator does not require the exogenous regressor to be continuously or normally distributed. There is no intercept in the data generating process. The true parameter values are  $\alpha = 1$  for  $P$  and  $\beta = 1$  for  $X$ .

**Usage**

```
data("dataCopIMABinExo")
```

**Format**

A data frame with 1000 observations on 3 variables:

*y* a numeric vector representing the dependent variable.

*X* a numeric vector, binary (0 or 1) and exogenous, correlated with *P*.

*P* a numeric vector, continuous and endogenous, following a bounded distribution  $\text{Phi}(P^*) + 0.5$  with values in (0.5, 1.5).

**Author(s)**

Kimberly Lew <kimberlylew12@gmail.com>

**References**

Haschka, R. E. (2025). Robustness of copula-correction models in causal analysis: Exploiting between-regressor correlation. *IMA Journal of Management Mathematics*, 36, 161-180. doi:[10.1093/imaman/dpae018](https://doi.org/10.1093/imaman/dpae018)

---

dataCopIMAContExo

*Simulated Dataset for Copula IMA - Continuous Exogenous Regressor*

---

**Description**

A dataset simulated with one exogenous regressor *X* and one endogenous continuous regressor *P*, and a dependent variable *y*. The exogenous regressor *X* follows a normal distribution with mean 1 and is correlated with *P* ( $r = 0.5$ ). The endogenous regressor *P* follows a bounded continuous distribution ( $\text{Phi}(P^*) + 0.5$ ). The endogeneity strength is  $\rho = 0.5$ . There is no intercept in the data generating process. The true parameter values are given as  $\alpha = 1$  for *P* and  $\beta = 1$  for *X*.

**Usage**

```
data("dataCopIMAContExo")
```

**Format**

A data frame with 1000 observations on 3 variables:

*y* a numeric vector representing the dependent variable.

*X* a numeric vector, normally distributed with mean 1, exogenous and correlated with *P*.

*P* a numeric vector, continuous and endogenous, following a bounded distribution  $\text{Phi}(P^*) + 0.5$  with values in (0.5, 1.5).

**Author(s)**

Kimberly Lew <kimberlylew12@gmail.com>

**References**

Haschka, R. E. (2025). Robustness of copula-correction models in causal analysis: Exploiting between-regressor correlation. *IMA Journal of Management Mathematics*, 36, 161-180. doi:[10.1093/imaman/dpae018](https://doi.org/10.1093/imaman/dpae018)

---

dataCopIMAMultiEndo     *Simulated Dataset for Copula IMA - Two Continuous Endogenous Regressors, Intercept and No Exogenous Regressor*

---

**Description**

A dataset simulated with two endogenous continuous regressors P1 and P2, an intercept, and a dependent variable y. No exogenous regressor. Both endogenous regressors follow a bounded continuous distribution ( $\Phi(P^*) + 0.5$ ) and are correlated with each other ( $r = 0.3$ ), and with the structural error (endogeneity strength  $\rho = 0.5$ ). The true parameter values are  $\alpha_1 = 1$  for P1,  $\alpha_2 = 1$  for P2, and  $\mu = 10$  for the intercept. This was simulated as an extension to the dataset 'dataCopIMAContExo'. This dataset was created to test how well `copulaIMA()` works with an intercept and no exogenous regressor.

**Usage**

```
data("dataCopIMAMultiEndo")
```

**Format**

A data frame with 1000 observations on 4 variables:

y a numeric vector representing the dependent variable.

P1 a numeric vector, continuous and endogenous, following a bounded distribution  $\Phi(P1^*) + 0.5$  with values in (0.5, 1.5), correlated with P2 ( $r = 0.3$ ).

P2 a numeric vector, continuous and endogenous, following a bounded distribution  $\Phi(P2^*) + 0.5$  with values in (0.5, 1.5), correlated with P1 ( $r = 0.3$ ).

**Author(s)**

Kimberly Lew <kimberlylew12@gmail.com>

**References**

Haschka, R. E. (2025). Robustness of copula-correction models in causal analysis: Exploiting between-regressor correlation. *IMA Journal of Management Mathematics*, 36, 161-180. doi:[10.1093/imaman/dpae018](https://doi.org/10.1093/imaman/dpae018)

---

`dataHetIV`*Simulated Dataset with One Endogenous Continuous Regressor*

---

**Description**

A dataset with two exogenous regressors,  $X_1, X_2$ , one endogenous, continuous regressor  $P$ , and the dependent variable  $y$ . The true parameter values for the coefficients are:  $b_0 = 2$ ,  $b_1 = 1.5$ ,  $b_2 = 3$  and the coefficient of the endogenous regressor,  $P$ , is equal to  $a_1 = -1$ .

**Usage**

```
data("dataHetIV")
```

**Format**

A data frame with 2500 observations on 4 variables:

$y$  a numeric vector representing the dependent variable.

$X_1$  a numeric vector, normally distributed and exogenous.

$X_2$  a numeric vector, normally distributed and exogenous.

$P$  a numeric vector, continuous and endogenous regressor, normally distributed.

**Author(s)**

Raluca Gui <raluca.gui@business.uzh.ch>

---

`dataHigherMoments`*Simulated Dataset with One Endogenous Regressor*

---

**Description**

A dataset with two exogenous regressors,  $X_1, X_2$ , and one endogenous, continuous regressor  $P$ . An intercept and a dependent variable,  $y$ , are also included. The true parameter values for the coefficients are:  $b_0 = 2$ ,  $b_1 = 1.5$ ,  $b_2 = 3$  and the coefficient of the endogenous regressor,  $P$ , is equal to  $a_1 = -1$ .

**Usage**

```
data("dataHigherMoments")
```

**Format**

A data frame with 2500 observations on 4 variables:

y a numeric vector representing the dependent variable.

X1 a numeric vector, normally distributed and exogenous.

X2 a numeric vector, normally distributed and exogenous.

P a numeric vector, continuous and endogenous regressor, normally distributed.

**Author(s)**

Raluca Gui <raluca.gui@business.uzh.ch>

**Examples**

```
data("dataHigherMoments")
# to recover the parameters,
# on average over many simulations
higherMomentsIV(formula = y ~ X1 + X2 + P|P|IIV(iiv=yp),
                 data=dataHigherMoments)
```

---

dataLatentIV

*Simulated Dataset with One Endogenous Continuous Regressor*

---

**Description**

A dataset with one endogenous regressor P, an instrument Z used to build P, an intercept and a dependent variable, y. The true parameter values for the coefficients are:  $b_0 = 3$  for the intercept and  $a_1 = -1$  for P.

**Usage**

```
data("dataLatentIV")
```

**Format**

A data frame with 2500 observations on 3 variables:

y a numeric vector representing the dependent variable.

P a numeric vector representing the endogenous variable.

Z a numeric vector used in the construction of the endogenous variable, P.

**Author(s)**

Raluca Gui <raluca.gui@business.uzh.ch>

---

`dataMultilevelIV`*Multilevel Simulated Dataset - Three Levels*

---

**Description**

A dataset simulated to exemplify the use of the `multilevelIV()` function. It has 2767 observations, clustered into 40 level-three variables and 1347 observations at level two. The endogenous regressor is X15 with a true coefficient value of -1.

**Usage**

```
data("dataMultilevelIV")
```

**Format**

A data frame with 2767 observations clustered into 40 level-three variables and 1347 level-two variables.

`y` a numeric vector representing the dependent variable.

X11 a level-one numeric vector representing a categorical exogenous variable with true parameter value equal to 3.

X12 a level-one numeric vector representing a binomial distributed exogenous variable with true parameter value equal to 9.

X13 a level-one numeric vector representing a binomial distributed exogenous variable with true parameter value equal to -2.

X14 a level-two numeric vector representing a normally distributed exogenous variable with true parameter value equal to 2.

X15 a level-two numeric vector representing a normally distributed endogenous variable, correlated with the level-two errors. Its true parameter value equals to  $-1$  and it has a correlation with the level two errors equal to 0.7.

X21 a level-two numeric vector representing a binomial distributed exogenous variable with true parameter value equal to -1.5.

X22 a level-two numeric vector representing a binomial distributed exogenous variable with true parameter value equal to -4.

X23 a level-two numeric vector representing a binomial distributed exogenous variable with true parameter value equal to -3.

X24 a level-two numeric vector representing a normally distributed exogenous variable with true parameter value equal to 6.

X31 a level-three numeric vector representing a normally distributed exogenous variable with true parameter value equal to 0.5.

X32 a level-three numeric vector representing a truncated normally distributed exogenous variable with true parameter value equal to 0.1.

X33 a level-three numeric vector representing a truncated normally distributed exogenous variable with true parameter value equal to -0.5.

SID a numeric vector identifying each level-three observations.

CID a numeric vector identifying each level-two observations.

**Author(s)**

Raluca Gui <raluca.gui@business.uzh.ch>

---

hetErrorsIV

*Fitting Linear Models with Endogenous Regressors using Heteroskedastic Covariance Restrictions*

---

**Description**

This function estimates the model parameters and associated standard errors for a linear regression model with one endogenous regressor. Identification is achieved through heteroscedastic covariance restrictions within the triangular system as proposed in Lewbel(2012).

**Usage**

```
hetErrorsIV(formula, data, verbose = TRUE)
```

**Arguments**

formula	A symbolic description of the model to be fitted. See the "Details" section for the exact notation.
data	A data.frame containing the data of all parts specified in the formula parameter.
verbose	Show details about the running of the function.

**Details**

**Method:** The method proposed in Lewbel(2012) identifies structural parameters in regression models with endogenous regressors by means of variables that are uncorrelated with the product of heteroskedastic errors. The instruments are constructed as simple functions of the model's data. The method can be applied when no external instruments are available or to supplement external instruments to improve the efficiency of the IV estimator. Consider the model in the equation:

$$y_t = \beta_0 + \beta_1 P_t + \beta_2 X_t + \epsilon_t$$

where  $t = 1, \dots, T$  indexes either time or cross-sectional units. The endogeneity problem arises from the correlation of  $P_t$  and  $\epsilon_t$ . As such:

$$P_t = \gamma Z_t + \nu_t,$$

where  $Z_t$  is a subset of variables in  $X_t$ .

The errors,  $\epsilon$  and  $\nu$ , may be correlated with each other. Structural parameters are identified by an ordinary two-stage least squares regression of  $Y$  on  $X$  and  $P$ , using  $X$  and  $[Z - E(Z)]\nu$  as instruments. A vital assumption for identification is that  $cov(Z, \nu^2) \neq 0$ . The strength of the instrument is proportional to the covariance of  $(Z - \bar{Z})\nu$  with  $\nu$ , which corresponds to the degree of heteroskedasticity of  $\nu$  with respect to  $Z$  (Lewbel 2012).

The assumption that the covariance between  $Z$  and the squared error is different from zero can be empirically tested (this is checked in the background when calling the function). If it is zero or close to zero, the instrument is weak, producing imprecise estimates, with large standard errors.

**Formula parameter:** The formula argument follows a four part notation:

A two-sided formula describing the model (e.g.  $y \sim X1 + X2 + P$ ), a single endogenous regressor (e.g.  $P$ ), and the exogenous variables from which the internal instrumental variables should be built (e.g.  $IIV(X1) + IIV(X2)$ ), each part separated by a single vertical bar ( $|$ ).

The instrumental variables that should be built are specified as (multiple) functions, one for each instrument. This function is `IIV` and uses the following arguments:

... The exogenous regressors to build the internal instruments from. If more than one is given, separate instruments are built for each.

Note that no argument to `IIV` is to be supplied as character but as symbols without quotation marks.

Optionally, additional external instrumental variables to also include in the instrumental variable regression can be specified. These external instruments have to be already present in the data and are provided as the fourth right-hand side part of the formula, again separated by a vertical bar.

See the example section for illustrations on how to specify the formula parameter.

## Value

Returns an object of classes `rendo.ivreg` and `ivreg`. It extends the object returned from function `ivreg` of package `AER` and slightly modifies it by adapting the `call` and `formula` components. The summary function prints additional diagnostic information as described in documentation for [summary.ivreg](#).

All generic accessor functions for `ivreg` such as `anova`, `hatvalues`, or `vcov` are available.

## References

Lewbel, A. (2012). Using Heteroskedasticity to Identify and Estimate Mismeasured and Endogenous Regressor Models, *Journal of Business & Economic Statistics*, 30(1), 67-80.

Angrist, J. and Pischke, J.S. (2009). *Mostly Harmless Econometrics: An Empiricists Companion*, Princeton University Press.

## See Also

[ivreg](#)

## Examples

```
data("dataHetIV")
# P is the endogenous regressor in all examples
# X1 generates a weak instrument but for the examples
# this is ignored

# 2 IVs, one from X1, one from X2
het <- hetErrorsIV(y~X1+X2+P|P|IIV(X1)+IIV(X2), data=dataHetIV)
# same as above
het <- hetErrorsIV(y~X1+X2+P|P|IIV(X1,X2), data=dataHetIV)

# use X2 as an external IV
het <- hetErrorsIV(y~X1+P|P|IIV(X1)|X2, data=dataHetIV)
```

summary(het)

---

higherMomentsIV      *Fitting Linear Models with Endogenous Regressors using Lewbel's Higher Moments Approach*

---

### Description

Fits linear models with one endogenous regressor using internal instruments built using the approach described in Lewbel A. (1997). This is a statistical technique to address the endogeneity problem where no external instrumental variables are needed. The implementation allows the incorporation of external instruments if available. An important assumption for identification is that the endogenous variable has a skewed distribution.

### Usage

higherMomentsIV(formula, data, verbose = TRUE)

### Arguments

formula	A symbolic description of the model to be fitted. See the "Details" section for the exact notation.
data	A data.frame containing the data of all parts specified in the formula parameter.
verbose	Show details about the running of the function.

### Details

#### Method:

Consider the model:

$$Y_t = \beta_0 + \beta_1 X_t + \alpha P_t + \epsilon_t \quad (1)$$

$$P_t = \gamma Z_t + \nu_t \quad (2)$$

The observed data consist of  $Y_t$ ,  $X_t$  and  $P_t$ , while  $Z_t$ ,  $\epsilon_t$ , and  $\nu_t$  are unobserved. The endogeneity problem arises from the correlation of  $P_t$  with the structural error  $\epsilon_t$ , since  $E(\epsilon\nu) \neq 0$ . The requirement for the structural and measurement error is to have mean zero, but no restriction is imposed on their distribution.

Let  $\bar{S}$  be the sample mean of a variable  $S_t$  and  $G_t = G(X_t)$  for any given function  $G$  that has finite third own and cross moments. Lewbel(1997) proves that the following instruments can be constructed and used with two-stage least squares to obtain consistent estimates:

$$q_{1t} = (G_t - \bar{G}) \quad (3a)$$

$$q_{2t} = (G_t - \bar{G})(P_t - \bar{P}) \quad (3b)$$

$$q_{3t} = (G_t - \bar{G})(Y_t - \bar{Y}) \quad (3c)$$

$$q_{4t} = (Y_t - \bar{Y})(P_t - \bar{P}) \quad (3d)$$

$$q_{5t} = (P_t - \bar{P})^2 \quad (3e)$$

$$q_{6t} = (Y_t - \bar{Y})^2 \quad (3f)$$

Instruments in equations 3e and 3f can be used only when the measurement and the structural errors are symmetrically distributed. Otherwise, the use of the instruments does not require any distributional assumptions for the errors. Given that the regressors  $G(X) = X$  are included as instruments,  $G(X)$  should not be linear in  $X$  in equation 3a.

Let small letter denote deviation from the sample mean:  $s_i = S_i - \bar{S}$ . Then, using as instruments the variables presented in equations 3 together with 1 and  $X_t$ , the two-stage-least-squares estimation will provide consistent estimates for the parameters in equation 1 under the assumptions exposed in Lewbel(1997).

### Formula parameter:

The formula argument follows a four part notation:

A two-sided formula describing the model (e.g.  $y \sim X1 + X2 + P$ ), a single endogenous regressor (e.g.  $P$ ), and the exogenous variables from which the internal instrumental variables should be build (e.g.  $IIV(iiv=y2)$ ), each part separated by a single vertical bar ( $|$ ).

The instrumental variables that should be built are specified as (multiple) functions, one for each instrument. This function is IIV and uses the following arguments:

- `iiv` Which internal instrument to build. One of `g`, `gp`, `gy`, `yp`, `p2`, `y2` can be chosen.
- `g` Which function `g` represents in `iiv`. One of `x2`, `x3`, `lnx`, `1/x` can be chosen. Only required if the type of internal instrument demands it.
- `...` The exogenous regressors to build the internal instrument. If more than one is given, separate instruments are built for each. Only required if the type of internal instrument demands it.

Note that no argument to IIV is to be supplied as character but as symbols without quotation marks.

Optionally, additional external instrumental variables to also include in the instrumental variable regression can be specified. These external instruments have to be already present in the data and are provided as the fourth right-hand side part of the formula, again separated by a vertical bar.

See the example section for illustrations on how to specify the formula parameter.

### Value

Returns an object of classes `rendo.ivreg` and `ivreg`, It extends the object returned from function `ivreg` of package `AER` and slightly modifies it by adapting the `call` and `formula` components. The summary function prints additional diagnostic information as described in documentation for [summary.ivreg](#).

All generic accessor functions for `ivreg` such as `anova`, `hatvalues`, or `vcov` are available.

### References

Lewbel A (1997). "Constructing Instruments for Regressions with Measurement Error When No Additional Data are Available, With an Application to Patents and R&D." *Econometrica*, 65(5), 1201–1213.

**See Also**[ivreg](#)**Examples**

```

data("dataHigherMoments")
# P is the endogenous regressor in all examples

# 2 IVs with g*p, g=x^2, separately for each regressor X1 and X2.
hm <- higherMomentsIV(y~X1+X2+P|P|IIV(iiv=gp, g=x2, X1, X2),
  data = dataHigherMoments)

# same as above
hm <- higherMomentsIV(y~X1+X2+P|P|IIV(iiv=gp, g=x2, X1) +
  IIV(iiv=gp, g=x2, X2),
  data = dataHigherMoments)

# 3 different IVs
hm <- higherMomentsIV(y~X1+X2+P|P|IIV(iiv=y2) + IIV(iiv=yp) +
  IIV(iiv=g,g=x3,X1),
  data = dataHigherMoments)

# use X2 as external IV
hm <- higherMomentsIV(y~X1+P|P|IIV(iiv=y2)+IIV(iiv=g,g=lnx,X1)| X2,
  data = dataHigherMoments)

summary(hm)

```

latentIV

---

*Fitting Linear Models with one Endogenous Regressor using Latent Instrumental Variables*

---

**Description**

Fits linear models with one endogenous regressor and no additional explanatory variables using the latent instrumental variable approach presented in Ebbes, P., Wedel, M., Böckenholt, U., and Steerneman, A. G. M. (2005). This is a statistical technique to address the endogeneity problem where no external instrumental variables are needed. The important assumption of the model is that the latent variables are discrete with at least two groups with different means and the structural error is normally distributed.

**Usage**

```

latentIV(
  formula,
  data,
  start.params = c(),
  optimx.args = list(),
  verbose = TRUE
)

```

### Arguments

formula	A symbolic description of the model to be fitted. Of class "formula".
data	A data.frame containing the data of all parts specified in the formula parameter.
start.params	A named vector containing a set of parameters to use in the first optimization iteration. The names have to correspond exactly to the names of the components specified in the formula parameter. If not provided, a linear model is fitted to derive them.
optimx.args	A named list of arguments which are passed to <code>optimx</code> . This allows users to tweak optimization settings to their liking.
verbose	Show details about the running of the function.

### Details

Let's consider the model:

$$Y_t = \beta_0 + \alpha P_t + \epsilon_t$$

$$P_t = \pi' Z_t + \nu_t$$

where  $t = 1, \dots, T$  indexes either time or cross-sectional units,  $Y_t$  is the dependent variable,  $P_t$  is a  $k \times 1$  continuous, endogenous regressor,  $\epsilon_t$  is a structural error term with mean zero and  $E(\epsilon^2) = \sigma_\epsilon^2$ ,  $\alpha$  and  $\beta_0$  are model parameters.  $Z_t$  is a  $1 \times 1$  vector of instruments, and  $\nu_t$  is a random error with mean zero and  $E(\nu^2) = \sigma_\nu^2$ . The endogeneity problem arises from the correlation of  $P$  and  $\epsilon_t$  through  $E(\epsilon\nu) = \sigma_{\epsilon\nu}$

latentIV considers  $Z_t'$  to be a latent, discrete, exogenous variable with an unknown number of groups  $m$  and  $\pi$  is a vector of group means. It is assumed that  $Z$  is independent of the error terms  $\epsilon$  and  $\nu$  and that it has at least two groups with different means. The structural and random errors are considered normally distributed with mean zero and variance-covariance matrix  $\Sigma$ :

$$\Sigma = \begin{pmatrix} \sigma_\epsilon^2 & \sigma_{\epsilon\nu} \\ \sigma_{\epsilon\nu} & \sigma_\nu^2 \end{pmatrix}$$

The identification of the model lies in the assumption of the non-normality of  $P_t$ , the discreteness of the unobserved instruments and the existence of at least two groups with different means.

The method has been implemented such that the latent variable has two groups. Ebbes et al.(2005) show in a Monte Carlo experiment that even if the true number of the categories of the instrument is larger than two, estimates are approximately consistent. Besides, overfitting in terms of the number of groups/categories reduces the degrees of freedom and leads to efficiency loss. For a model with additional explanatory variables a Bayesian approach is needed, since in a frequentist approach identification issues appear.

Identification of the parameters relies on the distributional assumptions of the latent instruments as well as that of the endogenous regressor  $P_t$ . Specifically, the endogenous regressor should have a non-normal distribution while the unobserved instruments,  $Z$ , should be discrete and have at least two groups with different means Ebbes, Wedel, and Böckenholt (2009). A continuous distribution for the instruments leads to an unidentified model, while a normal distribution of the endogenous regressor gives rise to inefficient estimates.

Additional parameters used during model fitting and printed in summary are:

- pi1** The instrumental variables  $Z$  are assumed to be divided into two groups. pi1 represents the estimated group mean of the first group.
- pi2** The estimated group mean of the second group of the instrumental variables  $Z$ .
- theta5** The probability of being in the first group of the instruments.
- theta6** The variance,  $\sigma_\epsilon^2$
- theta7** The covariance,  $\sigma_{\epsilon\nu}$
- theta8** The variance,  $\sigma_\nu^2$

### Value

An object of classes `rendo.latent.IV` and `rendo.base` is returned which is a list and contains the following components:

<code>formula</code>	The formula given to specify the fitted model.
<code>terms</code>	The terms object used for model fitting.
<code>model</code>	The <code>model.frame</code> used for model fitting.
<code>coefficients</code>	A named vector of all coefficients resulting from model fitting.
<code>names.main.coefs</code>	a vector specifying which coefficients are from the model. For internal usage.
<code>start.params</code>	A named vector with the initial set of parameters used to optimize the log-likelihood function.
<code>res.optimx</code>	The result object returned by the function <code>optimx</code> after optimizing the log-likelihood function.
<code>hessian</code>	A named, symmetric matrix giving an estimate of the Hessian at the found solution.
<code>m.delta.diag</code>	A diagonal matrix needed when deriving the <code>vcov</code> to apply the delta method on <code>theta5</code> which was transformed during the LL optimization.
<code>fitted.values</code>	Fitted values at the found optimal solution.
<code>residuals</code>	The residuals at the found optimal solution.

The function summary can be used to obtain and print a summary of the results. The generic accessor functions `coefficients`, `fitted.values`, `residuals`, `vcov`, `confint`, `logLik`, `AIC`, `BIC`, `case.names`, and `nobs` are available.

### References

- Ebbes, P., Wedel, M., Böckenholt, U., and Steerneman, A. G. M. (2005). 'Solving and Testing for Regressor-Error (in)Dependence When no Instrumental Variables are Available: With New Evidence for the Effect of Education on Income'. *Quantitative Marketing and Economics*, 3:365–392.
- Ebbes P., Wedel M., Böckenholt U. (2009). "Frugal IV Alternatives to Identify the Parameter for an Endogenous Regressor." *Journal of Applied Econometrics*, 24(3), 446–468.

### See Also

- [summary](#) for how fitted models are summarized
- [optimx](#) for possible elements of parameter `optimx.arg`

**Examples**

```

data("dataLatentIV")

# function call without any initial parameter values
l1 <- latentIV(y ~ P, data = dataLatentIV)
summary(l1)

# function call with initial parameter values given by the user
l11 <- latentIV(y ~ P, start.params = c("(Intercept)"=2.5, P=-0.5),
               data = dataLatentIV)
summary(l11)

# use own optimization settings (see optimx())
# set maximum number of iterations to 50'000
l12 <- latentIV(y ~ P, optimx.args = list(itnmax = 50000),
               data = dataLatentIV)

# print detailed tracing information on progress
l13 <- latentIV(y ~ P, optimx.args = list(control = list(trace = 6)),
               data = dataLatentIV)

# use method L-BFGS-B instead of Nelder-Mead and print report every 50 iterations
l14 <- latentIV(y ~ P, optimx.args = list(method = "L-BFGS-B", control=list(trace = 2, REPORT=50)),
               data = dataLatentIV)

# read out all coefficients, incl auxiliary coefs
lat.all.coefs <- coef(l14)
# same as above
lat.all.coefs <- coef(l14, complete = TRUE)
# only main model coefs
lat.main.coefs <- coef(l14, complete = FALSE)

```

**Description**

Estimates multilevel models (max. 3 levels) employing the GMM approach presented in Kim and Frees (2007). One of the important features is that, using the hierarchical structure of the data, no external instrumental variables are needed, unlike traditional instrumental variable techniques. Specifically, the approach controls for endogeneity at higher levels in the data hierarchy. For example, for a three-level model, endogeneity can be handled either if present at level two, at level three or at both levels. Level one endogeneity, where the regressors are correlated with the structural errors (errors at level one), is not addressed. Moreover, if considered, random slopes cannot be endogenous. Also, the dependent variable has to have a continuous distribution. The function returns the coefficient estimates obtained with fixed effects, random effects and the GMM estimator

proposed by Kim and Frees (2007), such that a comparison across models can be done. Asymptotically, the multilevel GMM estimators share the same properties of corresponding fixed effects estimators, but they allow the estimation of all the variables in the model, unlike the fixed effects counterpart.

To facilitate the choice of the estimator to be used for the given data, the function also conducts omitted variable test based on the Hausman-test for panel data (Hausman, 1978). It allows to compare a robust estimator and an estimator that is efficient under the null hypothesis of no omitted variables, and to compare two robust estimators at different levels. The results of these tests are returned when calling `summary()` on a fitted model.

### Usage

```
multilevelIV(
  formula,
  data,
  lmer.control = lmerControl(optimizer = "Nelder_Mead", optCtrl = list(maxfun = 1e+05)),
  verbose = TRUE
)
```

### Arguments

<code>formula</code>	A symbolic description of the model to be fitted. See the "Details" section for the exact notation.
<code>data</code>	A <code>data.frame</code> containing the data of all parts specified in the formula parameter.
<code>lmer.control</code>	An output from <code>lmerControl</code> that will be used to fit the <code>lmer</code> model from which the variance and correlation are obtained.
<code>verbose</code>	Show details about the running of the function.

### Details

**Method:** Multilevel modeling is a generalization of regression methods that recognize the existence of such data hierarchies by allowing for residual components at each level in the hierarchy. For example, a three-level multilevel model which allows for grouping of students within classrooms, over time, would include time, student and classroom residuals (see equation below). Thus, the residual variance is partitioned into four components: between-classroom (the variance of the classroom-level residuals), within-classroom (the variance of the student-level residuals), between student (the variance of the student-level residuals) and within-student (the variance of the time-level residuals). The classroom residuals represent the unobserved classroom characteristics that affect student's outcomes. These unobserved variables lead to correlation between outcomes for students from the same classroom. Similarly, the unobserved time residuals lead to correlation between a student's outcomes over time. A three-level model can be described as follows:

$$\begin{aligned}
 y_{cst} &= Z_{cst}^1 \beta_{cs}^1 + X_{cst}^1 \beta_1 + \epsilon_{cst}^1 \\
 \beta_{cs}^1 &= Z_{cs}^2 \beta_c^2 + X_{cs}^2 \beta_2 + \epsilon_{cs}^2 \\
 \beta_c^2 &= X_c^3 \beta_3 + \epsilon_c^3
 \end{aligned}$$

Like in single-level regression, in multilevel models endogeneity is also a concern. The additional problem is that in multilevel models there are multiple independent assumptions involving various random components at different levels. Any moderate correlation between some predictors and a random component or error term, can result in a significant bias of the coefficients and of the variance components. The multilevel GMM approach for addressing endogeneity uses both the between and within variations of the exogenous variables, but only the within variation of the variables assumed endogenous. The assumptions in the multilevel generalized moment of moments model is that the errors at each level are normally distributed and independent of each other. Moreover, the slope variables are assumed exogenous. Since the model does not handle "level 1 dependencies", an additional assumption is that the level 1 structural error is uncorrelated with any of the regressors. If this assumption is not met, additional, external instruments are necessary. The coefficients of the explanatory variables appear in the vectors  $\beta_1$ ,  $\beta_2$  and  $\beta_3$ . The term  $\beta_{cs}^1$  captures latent, unobserved characteristics that are classroom and student specific while  $\beta_c^2$  captures latent, unobserved characteristics that are classroom specific. For identification, the disturbance term  $\epsilon_{cst}$  is assumed independent of the other variables,  $Z_{cst}^1$  and  $X_{cst}^1$ . When all model variables are assumed exogenous, the GMM estimator is the usual GLS estimator, denoted as REF. When all variables (except the variables used as slope) are assumed endogenous, the fixed-effects estimator is used, FE. While REF assumes all explanatory variables are uncorrelated with the random intercepts and slopes in the model, FE allows for endogeneity of all effects but sweeps out the random components as well as the explanatory variables at the same levels. The more general estimator GMM proposed by Kim and Frees (2007) allows for some of the explanatory variables to be endogenous and uses this information to build instrumental variables. The multilevel GMM estimator uses both the between and within variations of the exogenous variables, but only the within variation of the variables assumed endogenous. When all variables are assumed exogenous, GMM estimator equals REF. When all covariates are assume endogenous, GMM equals FE.

#### Formula parameter:

The formula argument follows a two part notation:

In the first part, the model is specified while in the second part, the endogenous regressors are indicated. These two parts are separated by a single vertical bar (|).

The first RHS follows the exact same model specification as required by the `lmer` function of package `lme4` and internally will be used to fit a `lmer` model. In the second part, one or multiple endogenous regressors are indicated by passing them to the special function `endo` (e.g. `endo(X1, X2)`). Note that no argument to `endo()` is to be supplied as character but as symbols without quotation marks.

See the example section for illustrations on how to specify the formula parameter.

#### Value

`multilevelIV` returns an object of class "rendo.multilevel".

The generic accessor functions `coef`, `fitted`, `residuals`, `vcov`, `confint`, and `nobs`, are available. Note that an additional argument `model` with possible values "REF", "FE\_L2", "FE\_L3", "GMM\_L2", or "GMM\_L3" is available for `summary`, `fitted`, `residuals`, `confint`, and `vcov` to extract the features for the specified model.

Note that the obtained coefficients are rounded with `round(x, digits=getOption("digits"))`.

An object of class `rendo.multilevel` is returned that is a list and contains the following components:

<code>formula</code>	the formula given to specify the model to be fitted.
<code>num.levels</code>	the number of levels detected from the model.
<code>dt.model.data</code>	a <code>data.table</code> of model data including data for slopes and level group ids
<code>coefficients</code>	a matrix of rounded coefficients, one column per model.
<code>coefficients.se</code>	a matrix of coefficients' SE, one column per model.
<code>l.fitted</code>	a named list which contains the fitted values per model sorted as the input data
<code>l.residuals</code>	a named list which contains the residuals per model sorted as the input data
<code>l.vcov</code>	a list of variance-covariance matrix, named per model.
<code>V</code>	the variance-covariance matrix $V$ of the disturbance term.
<code>W</code>	the weight matrix $W$ , such that $W=V^{(-1/2)}$ per highest level group.
<code>l.ovt</code>	a list of results of the Hausman OVT, named per model.

## References

- Hausman J (1978). "Specification Tests in Econometrics." *Econometrica*, 46(6), 1251–1271.
- Kim, Jee-Seon and Frees, Edward W. (2007). "Multilevel Modeling with Correlated Effects". *Psychometrika*, 72(4), 505-533.

## See Also

- [lmer](#) for more details on how to specify the formula parameter
- [lmerControl](#) for more details on how to provide the `lmer.control` parameter
- [summary](#) for how fitted models are summarized

## Examples

```
data("dataMultilevelIV")

# Two levels
res.ml.L2 <- multilevelIV(y ~ X11 + X12 + X13 + X14 + X15 + X21 + X22 + X23 + X24 + X31 +
  X32 + X33 + (1|SID) | endo(X15),
  data = dataMultilevelIV, verbose = FALSE)

# Three levels
res.ml.L3 <- multilevelIV(y ~ X11 + X12 + X13 + X14 + X15 + X21 + X22 + X23 + X24 + X31 +
  X32 + X33 + (1|CID) + (1|SID) | endo(X15),
  data = dataMultilevelIV, verbose = FALSE)

# L2 with multiple endogenous regressors
res.ml.L2 <- multilevelIV(y ~ X11 + X12 + X13 + X14 + X15 + X21 + X22 + X23 + X24 + X31 +
  X32 + X33 + (1|SID) | endo(X15, X21, X22),
  data = dataMultilevelIV, verbose = FALSE)
```

```

# same as above
res.ml.L2 <- multilevelIV(y ~ X11 + X12 + X13 + X14 + X15 + X21 + X22 + X23 + X24 + X31 +
  X32 + X33 + (1|SID) | endo(X15, X21) + endo(X22),
  data = dataMultilevelIV, verbose = FALSE)

# Fit above model with different settings for lmer()
lmer.control <- lme4::lmerControl(optimizer="nloptwrap",
  optCtrl=list(algorithm="NLOPT_LN_COBYLA",
  xtol_rel=1e-6))
res.ml.L2.cob <- multilevelIV(y ~ X11 + X12 + X13 + X14 + X15 + X21 + X22 + X23 + X24 +
  X31 + X32 + X33 + (1|SID) | endo(X15, X21) + endo(X22),
  data = dataMultilevelIV, verbose = FALSE,
  lmer.control = lmer.control) # use different controls for lmer

# specify argument "model" in the S3 methods to obtain results for the respective model
# default is "REF" for all methods

summary(res.ml.L3)
# same as above
summary(res.ml.L3, model = "REF")

# complete pval table for L3 fixed effects
L3.FE.p <- coef(summary(res.ml.L3, model = "FE_L3"))

# variance covariance matrix
L2.FE.var <- vcov(res.ml.L2, model = "FE_L2")
L2.GMM.var <- vcov(res.ml.L2, model = "GMM_L2")
# residuals
L3.REF.resid <- resid(res.ml.L3, model = "REF")

```

---

predict.rendo.copula.correction

*Predict method for Models using the Gaussian Copula Approach*

---

## Description

Predicted values based on linear models with endogenous regressors estimated using the gaussian copula.

## Usage

```

## S3 method for class 'rendo.copula.correction'
predict(object, newdata, ...)

```

**Arguments**

object	Object of class inheriting from "rendo.copula.correction"
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are returned.
...	ignored, for consistency with the generic function.

**Value**

predict.copula.correction produces a vector of predictions

**See Also**

The model fitting function [copulaCorrection](#)

**Examples**

```
## Not run:
data("dataCopCont")

c1 <- copulaCorrection(y~X1+X2+P|continuous(P), num.boots=10,
                      data=dataCopCont)

# returns the fitted values
predict(c1)

# using the data used for fitting also for predicting,
# correctly results in fitted values
all.equal(predict(c1, dataCopCont), fitted(c1)) # TRUE

## End(Not run)
```

---

predict.rendo.ivreg     *Predict method for fitted Regression Models with Internal Instrumental Variables*

---

**Description**

Predicted values based on model objects fitted using the instrumental variables regression fitted with IVs generated from the data.

**Usage**

```
## S3 method for class 'rendo.ivreg'
predict(object, newdata, ...)
```

**Arguments**

object	Object of class inheriting from "rendo.ivreg"
newdata	An optional data frame without any instrumental variables in which to look for variables with which to predict. If omitted, the fitted values are returned.
...	ignored, for consistency with the generic function.

**Value**

predict.rendo.ivreg produces a vector of predictions

**See Also**

The model fitting functions [hetErrorsIV](#), [higherMomentsIV](#).

**Examples**

```
data("dataHetIV")

het <- hetErrorsIV(y~X1+X2+P|P|IIV(X1, X2),
                  data = dataHetIV)

# returns the fitted values
predict(het)

# using the data used for fitting also for predicting,
# correctly results in fitted values
all.equal(predict(het, dataHetIV), fitted(het)) # TRUE
```

---

predict.rendo.latent.IV

*Predict method for Models using the Latent Instrumental Variables approach*

---

**Description**

Predicted values based on linear models estimated using the latent instrumental variables approach for a single endogenous regressor.

**Usage**

```
## S3 method for class 'rendo.latent.IV'
predict(object, newdata, ...)
```

**Arguments**

object            Object of class inheriting from "rendo.latent.IV"  
 newdata          An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are returned.  
 ...              ignored, for consistency with the generic function.

**Value**

predict.rendo.latent.IV produces a vector of predictions

**See Also**

The model fitting function [latentIV](#)

**Examples**

```
data("dataLatentIV")

lat <- latentIV(y ~ P, data = dataLatentIV)

# returns the fitted values
predict(lat)

# using the data used for fitting also for predicting,
# correctly results in fitted values
all.equal(predict(lat, dataLatentIV), fitted(lat)) # TRUE
```

---

predict.rendo.multilevel

*Predict method for Multilevel GMM Estimations*

---

**Description**

Predicted values based on multilevel models employing the GMM approach for hierarchical data with endogenous regressors.

**Usage**

```
## S3 method for class 'rendo.multilevel'
predict(
  object,
  newdata,
  model = c("REF", "FE_L2", "FE_L3", "GMM_L2", "GMM_L3"),
  ...
)
```

**Arguments**

object	Object of class inheriting from "rendo.multilevel"
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values for the specified model are returned.
model	character string to indicate for which fitted model predictions are made. Possible values are: "REF", "FE_L2", "FE_L3", "GMM_L2", or "GMM_L3".
...	ignored, for consistency with the generic function.

**Value**

predict.rendo.multilevel produces a vector of predictions

**See Also**

The model fitting function [multilevelIV](#)

**Examples**

```
data("dataMultilevelIV")

# Two levels
res.ml.L2 <- multilevelIV(y ~ X11 + X12 + X13 + X14 + X15 + X21 + X22 + X23 + X24 + X31 +
                        X32 + X33 + (1|SID) | endo(X15),
                        data = dataMultilevelIV, verbose = FALSE)
predict(res.ml.L2, model = "FE_L2")

# using the data used for fitting also for predicting,
# correctly results in fitted values
all.equal(predict(res.ml.L2, dataMultilevelIV, model = "GMM_L2"),
          fitted(res.ml.L2, model = "GMM_L2")) # TRUE
```

---

 REndo

*Fitting Linear Models with Endogenous Regressors using Latent Instrumental Variables*

---

**Description**

Fits linear models with endogenous regressor using latent instrumental variable approaches.

The methods included in the package are Lewbel's (1997) <doi:10.2307/2171884> higher moments approach as well as Lewbel's (2012) <doi:10.1080/07350015.2012.643126> heteroskedasticity approach, Park and Gupta's (2012) <doi:10.1287/mksc.1120.0718> joint estimation method that uses Gaussian copula and Kim and Frees's (2007) <doi:10.1007/s11336-007-9008-1> multilevel generalized method of moment approach that deals with endogeneity in a multilevel setting. These are statistical techniques to address the endogeneity problem where no external instrumental variables are needed.

The main functions to estimate models are:

latentIV() the latent instrumental variables method of Ebbes et al. (2005)  
copulaCorrection() copula correction method proposed by Paek and Gupta (2012)  
hetErrorsIV() heteroskedastic errors approach proposed by Lewbel(2012)  
higherMomentsIV() higher moments method proposed by Lewbel (1997)  
multilevelIV() multilevel GMM method proposed by Kim and Frees (2007)

### Differences between current (2.0.0) and previous version of REndo

Note that with version 2.0.0 sweeping changes were which greatly improve functionality but break backwards compatibility. Various bugs were fixed, performance improved, handling of S3 objects and methods across the package was harmonized, and a set of argument checks has been added. Starting with REndo 2.0, all functions support the use of transformations such as  $I(x^2)$  or  $\log(x)$  in the formulas. Moreover, the call of most of the functions (except latentIV() and multilevelIV()) changed from the previous versions, making use of the Formula package.

Check the NEWS file or our [github page](#) for the latest updates and for reporting issues.

See our publication in the Journal of Statistical Software for more details: [doi:10.18637/jss.v107.i03](https://doi.org/10.18637/jss.v107.i03).

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### References

Gui R, Meierer M, Schilter P, Algesheimer R (2023). “REndo: Internal Instrumental Variables to Address Endogeneity.” Journal of Statistical Software, 107 (3), 1-43. [doi:10.18637/jss.v107.i03](https://doi.org/10.18637/jss.v107.i03)

### See Also

Useful links:

- <https://github.com/mmeierer/REndo>
- Report bugs at <https://github.com/mmeierer/REndo/issues>

---

```
summary.rendo.copula.correction
```

*Summarizing Bootstrapped copulaCorrection Model Fits*

---

## Description

summary method for a model of class `rendo.copula.correction` resulting from fitting `copulaCorrection`.

## Usage

```
## S3 method for class 'rendo.copula.correction'
summary(object, ...)
```

## Arguments

<code>object</code>	an object of class <code>rendo.copula.correction</code> , a result of a call to <code>copulaCorrection</code> .
<code>...</code>	ignored, for consistency with the generic function.

## Details

For a single continuous endogenous regressor, the estimation is realized in two steps by first obtaining the empirical distribution of the endogenous regressor and then the likelihood function is built. Also for all other cases the estimation is realized in two steps and hence the standard errors reported by the fitted OLS model are not correct.

The standard errors and the confidence intervals are therefore obtained using bootstrapping with replacement as described in Efron (1979). The reported lower and upper boundaries are from the 95% bootstrapped percentile confidence interval. If there are too few bootstrapped estimates, no boundaries are reported.

For a single continuous endogenous regressor the model was fitted using maximum likelihood optimization. The related goodness of fit measures and convergence indicators are also reported here.

## Value

The function computes and returns a list of summary statistics which contains the following components:

<code>coefficients</code>	a $p \times 4$ matrix with columns for the estimated coefficients for the the original data, the standard error derived from the bootstrapped parameters, and the lower and upper boundaries of the 95% bootstrap confidence interval.
<code>num.boots</code>	the number of bootstraps performed.
<code>names.main.coefs</code>	a vector specifying which coefficients are from the model. For internal usage.
<code>start.params</code>	a named vector with the initial set of parameters used to optimize the log-likelihood function.
<code>vcov</code>	variance covariance matrix derived from the bootstrapped parameters.

names.vars.continuous  
                           the names of the continuous endogenous regressors.  
 names.vars.discrete  
                           the names of the discrete endogenous regressors.

For the case of a single continuous endogenous regressor, also the following components resulting from the log-likelihood optimization are returned:

AIC                  Akaike's An Information Criterion for the model fitted on the provided data.  
 BIC                  Schwarz's Bayesian Criterion for the model fitted on the provided data.  
 KKT1                first Kuhn, Karush, Tucker optimality condition as returned by optimx.  
 KKT2                second Kuhn, Karush, Tucker optimality condition as returned by optimx.  
 conv.code          the convergence code as returned by optimx.  
 log.likelihood     the value of the log-likelihood function at the found solution for the provided data.

## References

Effron, B.(1979). "Bootstrap Methods: Another Look at the Jackknife", The Annals of Statistics, 7(1), 1-26.

## See Also

The model fitting function [copulaCorrection](#)  
[confint](#) for how the confidence intervals are derived  
[vcov](#) for how the variance-covariance matrix is derived  
[optimx](#) for explanations about the returned conv.code and KKT.

Function coef will extract the coefficients matrix and function vcov will extract the component vcov from the returned summary object.

---

summary.rendo.latent.IV

*Summarizing latentIV Model Fits*

---

## Description

summary method for a model of class rendo.latent.IV resulting from fitting latentIV

## Usage

```
## S3 method for class 'rendo.latent.IV'
summary(object, ...)
```

**Arguments**

object            an object of class `rendo.latent.IV`, a result of a call to `latentIV`.  
 ...              ignored, for consistency with the generic function.

**Value**

The function `summary.rendo.latent.IV` computes and returns a list of summary statistics which contains the following components:

`coefficients`    a  $p \times 4$  matrix with columns for the estimated coefficients, its standard error, the t-statistic and corresponding (two-sided) p-value.  
`start.params`    a named vector with the initial set of parameters used to optimize the log-likelihood function.  
`names.main.coefs`  
                   a vector specifying which coefficients are from the model. For internal usage.  
`vcov`            variance covariance matrix derived from the hessian.  
`AIC`            Akaike's An Information Criterion for the model fitted on the provided data.  
`BIC`            Schwarz's Bayesian Criterion for the model fitted on the provided data.  
`KKT1`          first Kuhn, Karush, Tucker optimality condition as returned by `optimx`.  
`KKT2`          second Kuhn, Karush, Tucker optimality condition as returned by `optimx`.  
`conv.code`      the convergence code as returned by `optimx`.  
`log.likelihood` the value of the log-likelihood function at the found solution for the provided data.

**See Also**

The model fitting function [latentIV](#)

Function `coef` will extract the `coefficients` matrix and function `vcov` will extract the component `vcov` from the returned summary object.

---

summary.rendo.multilevel

*Summarizing Multilevel GMM Estimation with Endogenous Regressors Model Fits*

---

**Description**

summary method for class "rendo.multilevel".

**Usage**

```
## S3 method for class 'rendo.multilevel'
summary(object, model = c("REF", "FE_L2", "FE_L3", "GMM_L2", "GMM_L3"), ...)
```

**Arguments**

object	an object of class "rendo.multilevel", usually, a result of a call to <code>multilevelIV</code> .
model	character string to indicate which fitted model should be summarized. Possible values are: "REF", "FE_L2", "FE_L3", "GMM_L2", or "GMM_L3".
...	ignored, for consistency with the generic function.

**Details**

The `multilevelIV()` function estimates three models, namely: the usual random effects model (REF), the fixed effects model (FE) and the hierarchical GMM model (GMM) proposed by Kim and Frees (2007). The fixed effects and the GMM estimators are calculated at each level - so in the case of a three-level model, the function estimates, besides the random effects, fixed effects models at level two (FE\_L2) and at level three (FE\_L3). The same is true for the GMM estimators, the `multilevelIV()` function will return a GMM estimator at level-three (GMM\_L3) and a GMM estimator at level two (GMM\_L2).

In order to facilitate the choice of estimator to be used, the `summary()` function also returns an omitted variable test (OVT). This test is based on the Hausman test for panel data. The OVT allows the comparison of a robust estimator and an estimator which is efficient under the null hypothesis of no omitted variables. Moreover, it allows the comparison of two robust estimators at different levels.

For the model specified in argument `model`, the `summary()` function returns the summary statistics of the estimated coefficients, together with the results of the omitted variable test between the specified model and each other model.

**Value**

For the model specified in argument `model`, the function `summary.rendo.multilevel` computes and returns a list of summary statistics and the results of the omitted variable tests for the fitted `multilevel` object given in `object`.

An object of class `summary.rendo.multilevel` is returned that is a list using the component call of argument `object`, plus,

<code>summary.model</code>	the model parameter with which the summary function was called.
<code>coefficients</code>	a $p \times 4$ matrix with columns for the estimated coefficients, its standard error, the t-statistic and corresponding (two-sided) p-value.
<code>OVT.table</code>	results of the Hausman omitted variable test for the specified model compared to all other models.
<code>vcov</code>	variance covariance matrix derived from the GMM fit of this model.

**See Also**

The model fitting function [multilevelIV](#)

Function `coef` will extract the `coefficients` matrix and function `vcov` will extract the component `vcov`.

**Examples**

```

data("dataMultilevelIV")
# Fit two levels model
res.ml.L2 <- multilevelIV(y ~ X11 + X12 + X13 + X14 + X15 + X21 + X22 + X23 + X24 + X31 +
                        X32 + X33 + (1|SID) | endo(X15),
                        data = dataMultilevelIV, verbose = FALSE)

# Get summary for FE_L2 (does not print)
res.sum <- summary(res.ml.L2, model = "FE_L2")
# extract table with coefficients summary statistics
sum.stat.FE_L2 <- coef(res.sum)
# extract vcov of model FE_L2
FE_L2.vcov <- vcov(res.sum)
# same as above
FE_L2.vcov <- vcov(res.ml.L2, model = "FE_L2")

```

---

vcov.rendo.boots

*Calculate Variance-Covariance Matrix for Models Fitted with Bootstrapped Parameters*


---

**Description**

The variance-covariance matrix is derived from the bootstrapped parameter estimates stored in the object. It is based on Efron (1979) and calculates the result as follows:

$$\frac{1}{B-1} \sum_{b=1}^B (\theta_b - \bar{\theta})(\theta_b - \bar{\theta})$$

where B is the number of bootstraps and  $\bar{\theta}$  is the mean of the bootstrapped coefficients.

**Usage**

```

## S3 method for class 'rendo.boots'
vcov(object, ...)

```

**Arguments**

```

object      a fitted model object with bootstrapped parameters. Typically from copulaCorrection
...         ignored, for consistency with the generic function.

```

**Value**

A matrix of the estimated covariances between the parameter estimates of the model. The row and column names correspond to the parameter names given by the coef method.

**References**

Effron, B.(1979). "Bootstrap Methods: Another Look at the Jackknife", *The Annals of Statistics*, 7(1), 1-26.

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