

Package ‘cquad’

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Title Conditional Maximum Likelihood for Quadratic Exponential Models
for Binary Panel Data

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Description Estimation, based on conditional maximum likelihood, of the quadratic exponential model proposed by Bartolucci, F. & Nigro, V. (2010, *Econometrica*) <[DOI:10.3982/ECTA7531](https://doi.org/10.3982/ECTA7531)> and of a simplified and a modified version of this model. The quadratic exponential model is suitable for the analysis of binary longitudinal data when state dependence (further to the effect of the covariates and a time-fixed individual intercept) has to be taken into account. Therefore, this is an alternative to the dynamic logit model having the advantage of easily allowing conditional inference in order to eliminate the individual intercepts and then getting consistent estimates of the parameters of main interest (for the covariates and the lagged response). The simplified version of this model does not distinguish, as the original model does, between the last time occasion and the previous occasions. The modified version formulates in a different way the interaction terms and it may be used to test in a easy way state dependence as shown in Bartolucci, F., Nigro, V. & Pigini, C. (2018, *Econometric Reviews*) <[DOI:10.1080/07474938.2015.1060039](https://doi.org/10.1080/07474938.2015.1060039)>.

The package also includes estimation of the dynamic logit model by a pseudo conditional estimator based on the quadratic exponential model, as proposed by Bartolucci, F. & Nigro, V. (2012, *Journal of Econometrics*) <[DOI:10.1016/j.jeconom.2012.03.004](https://doi.org/10.1016/j.jeconom.2012.03.004)>.

For large time dimensions of the panel, the computation of the proposed models involves a recursive function from Krailo M. D., & Pike M. C. (1984, *Journal of the Royal Statistical Society. Series C (Applied Statistics)*) and Bartolucci F., Valentini, F. & Pigini C. (2021, *Computational Economics*) <[DOI:10.1007/s10614-021-10218-2](https://doi.org/10.1007/s10614-021-10218-2)>.

License GPL (>= 2)

Depends R (>= 2.0.0), MASS, plm, Formula

URL <https://cran.r-project.org/package=cquad> (CRAN releases),
https://github.com/fravale/cquad_dev/ (development repository)

BugReports https://github.com/fravale/cquad_dev/issues

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cquad-package	<i>Conditional Maximum Likelihood for Quadratic Exponential Models for Binary Panel Data</i>
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Description

Estimation, based on conditional maximum likelihood, of the quadratic exponential model proposed by Bartolucci & Nigro (2010) and of a simplified and a modified version of this model. The quadratic exponential model is suitable for the analysis of binary longitudinal data when state dependence (further to the effect of the covariates and a time-fixed individual intercept) has to be taken into account. Therefore, this is an alternative to the dynamic logit model having the advantage of easily allowing conditional inference in order to eliminate the individual intercepts and then getting consistent estimates of the parameters of main interest (for the covariates and the lagged response). The simplified version of this model does not distinguish, as the original model does, between the last time occasion and the previous occasions. The modified version formulates in a different way the interaction terms and it may be used to test in a easy way state dependence as shown in Bartolucci, Nigro & Pignini (2018). The package also includes estimation of the dynamic logit model by a pseudo conditional estimator based on the quadratic exponential model, as proposed by Bartolucci & Nigro (2012).

Details

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Author(s)

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References

Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator, *Econometrica*, **78**, 719-733.

Bartolucci, F. and Nigro, V. (2012). Pseudo conditional maximum likelihood estimation of the dynamic logit model for binary panel data, *Journal of Econometrics*, **170**, 102-116.

Bartolucci, F. and Pigni, C. (2017). cquad: An R and Stata package for conditional maximum likelihood estimation of dynamic binary panel data models, *Journal of Statistical Software*, **78**, 1-26, doi:10.18637/jss.v078.i07.

Bartolucci, F., Nigro, V., & Pigni, C. (2018). Testing for state dependence in binary panel data with individual covariates by a modified quadratic exponential model. *Econometric Reviews*, **37(1)**, 61-88.

Bartolucci, F., Valentini, F., & Pigni, C. (2021), Recursive Computation of the Conditional Probability Function of the Quadratic Exponential Model for Binary Panel Data, *Computational Economics*, <https://doi.org/10.1007/s10614-021-10218-2>.

Cox, D. R. (1972), The Analysis of multivariate binary data, *Applied Statistics*, **21**, 113-120.

Examples

```
# example based on simulated data
data(data_sim)
data_sim = data_sim[1:500,] # to speed up the example, remove otherwise
# static model
out1 = cquad(y~X1+X2,data_sim)
# dynamic model
out2 = cquad(y~X1+X2,data_sim,dyn=TRUE)
```

 cquad

Interface for functions fitting different versions of cquad

Description

Fit by conditional maximum likelihood each of the models in cquad package.

Usage

```
cquad(formula, data, index = NULL, model = c("basic", "equal", "extended", "pseudo"),
      w = rep(1, n), dyn = FALSE, Ttol=10)
```

Arguments

formula	formula with the same syntax as in plm package
data	data.frame or pdata.frame
index	to denote panel structure as in plm package
model	type of model = "basic", "equal", "extended", "pseudo"
w	vector of weights (optional)
dyn	TRUE if in the dynamic version; FALSE for the static version (by default)
Ttol	Threshold individual observations that activates the recursive algorithm (default=10)

Value

formula	formula defining the model
lk	conditional log-likelihood value
coefficients	estimate of the regression parameters
vcov	asymptotic variance-covariance matrix for the parameter estimates
scv	matrix of individual scores
J	Hessian of the log-likelihood function
se	standard errors
ser	robust standard errors
Tv	number of time occasions for each unit

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Ancona "Politecnica delle Marche"), Francesco Valentini (University of Ancona "Politecnica delle Marche")

Examples

```

# example based on simulated data
data(data_sim)
data_sim = data_sim[1:500,] # to speed up the example, remove otherwise
# basic (static) model
out1 = cquad(y~X1+X2,data_sim)
summary(out1)
# basic (dynamic) model
out2 = cquad(y~X1+X2,data_sim,dyn=TRUE)
summary(out2)
# equal model
out3 = cquad(y~X1+X2,data_sim,model="equal")
summary(out3)
# extended model
out4 = cquad(y~X1+X2,data_sim,model="extended")
summary(out4)
# psuedo CML for dynamic model
out5 = cquad(y~X1+X2,data_sim,model="pseudo")
summary(out5)

```

cquad_basic

Conditional maximum likelihood estimation of the basic quadratic exponential model

Description

Fit by conditional maximum likelihood a simplified version of the model for binary longitudinal data proposed by Bartolucci & Nigro (2010); see also Cox (1972).

Usage

```

cquad_basic(id, yv, X = NULL, be = NULL, w = rep(1, n), dyn =
FALSE, Ttol=10)

```

Arguments

id	list of the reference unit of each observation
yv	corresponding vector of response variables
X	corresponding matrix of covariates (optional)
be	initial vector of parameters (optional)
w	vector of weights (optional)
dyn	TRUE if in the dynamic version; FALSE for the static version (by default)
Ttol	Threshold individual observations that activates the recursive algorithm (default=10)

Value

formula	formula defining the model
lk	conditional log-likelihood value
coefficients	estimate of the regression parameters (including for the lag-response)
vcov	asymptotic variance-covariance matrix for the parameter estimates
scv	matrix of individual scores
J	Hessian of the log-likelihood function
se	standard errors
ser	robust standard errors
Tv	number of time occasions for each unit

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigini (University of Ancona "Politecnica delle Marche"), Francesco Valentini (University of Ancona "Politecnica delle Marche")

References

- Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator, *Econometrica*, **78**, pp. 719-733.
- Cox, D. R. (1972), The Analysis of multivariate binary data, *Applied Statistics*, **21**, 113-120.

Examples

```
# example based on simulated data
data(data_sim)
data_sim = data_sim[1:500,] # to speed up the example, remove otherwise
id = data_sim$id; yv = data_sim$y; X = cbind(X1=data_sim$X1,X2=data_sim$X2)
# static model
out1 = cquad_basic(id,yv,X,Ttol=10)
summary(out1)
# dynamic model
out2 = cquad_basic(id,yv,X,dyn=TRUE,Ttol=10)
summary(out2)
```

cquad_equ

Conditional maximum likelihood estimation for the modified version of the quadratic exponential model (to test for state dependence)

Description

Fit by conditional maximum likelihood a modified version of the model for binary longitudinal data proposed by Bartolucci & Nigro (2010), in which the interaction terms have an extended form. This modified version is used to test for state dependence as described in Bartolucci et al. (2018).

Usage

```
cquad_equ(id, yv, X = NULL, be = NULL, w = rep(1, n), Ttol=10)
```

Arguments

id	list of the reference unit of each observation
yv	corresponding vector of response variables
X	corresponding matrix of covariates (optional)
be	initial vector of parameters (optional)
w	vector of weights (optional)
Ttol	Threshold individual observations that activates the recursive algorithm (default=10)

Value

formula	formula defining the model
lk	conditional log-likelihood value
coefficients	estimate of the regression parameters (including for the lag-response)
vcov	asymptotic variance-covariance matrix for the parameter estimates
scv	matrix of individual scores
J	Hessian of the log-likelihood function
se	standard errors
ser	robust standard errors
Tv	number of time occasions for each unit

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Perugia), Francesco Valentini (University of Ancona "Politecnica delle Marche")

References

Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator, *Econometrica*, **78**, 719-733.

Bartolucci, F., Nigro, V., & Pigni, C. (2018). Testing for state dependence in binary panel data with individual covariates by a modified quadratic exponential model. *Econometric Reviews*, **37(1)**, 61-88.

Examples

```
# example based on simulated data
data(data_sim)
data_sim = data_sim[1:500,] # to speed up the example, remove otherwise
id = data_sim$id; yv = data_sim$y; X = cbind(X1=data_sim$X1,X2=data_sim$X2)

out = cquad_equ(id,yv,X,Ttol=10)
```

cquad_ext	<i>Conditional maximum likelihood estimation of the quadratic exponential model for panel data</i>
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Description

Fit by conditional maximum likelihood the model for binary longitudinal data proposed by Bartolucci & Nigro (2010).

Usage

```
cquad_ext(id, yv, X = NULL, be = NULL, w = rep(1, n), Ttol=10)
```

Arguments

id	list of the reference unit of each observation
yv	corresponding vector of response variables
X	corresponding matrix of covariates (optional)
be	initial vector of parameters (optional)
w	vector of weights (optional)
Ttol	Threshold individual observations that activates the recursive algorithm (default=10)

Value

formula	formula defining the model
lk	conditional log-likelihood value
coefficients	estimate of the regression parameters (including for the lag-response)
vcov	asymptotic variance-covariance matrix for the parameter estimates
scv	matrix of individual scores
J	Hessian of the log-likelihood function
se	standard errors
ser	robust standard errors
Tv	number of time occasions for each unit

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Ancona "Politecnica delle Marche"), Francesco Valentini (University of Ancona "Politecnica delle Marche")

References

Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator. *Econometrica*, **78**, pp. 719-733.

Examples

```
# example based on simulated data
data(data_sim)
data_sim = data_sim[1:500,] # to speed up the example, remove otherwise
id = data_sim$id; yv = data_sim$y; X = cbind(X1=data_sim$X1,X2=data_sim$X2)
# static model
out = cquad_ext(id,yv,X,Ttol=10)
summary(out)
```

cquad_pseudo	<i>Pseudo conditional maximum likelihood estimation of the dynamic logit model</i>
--------------	--

Description

Estimate the dynamic logit model for binary longitudinal data by the pseudo conditional maximum likelihood method proposed by Bartolucci & Nigro (2012).

Usage

```
cquad_pseudo(id, yv, X = NULL, be = NULL, w = rep(1,n), Ttol=10)
```

Arguments

id	list of the reference unit of each observation
yv	corresponding vector of response variables
X	corresponding matrix of covariates (optional)
be	initial vector of parameters (optional)
w	vector of weights (optional)
Ttol	Threshold individual observations that activates the recursive algorithm (default=10)

Value

formula	formula defining the model
lk	conditional log-likelihood value
coefficients	estimate of the regression parameters (including for the lag-response)
vcov	asymptotic variance-covariance matrix for the parameter estimates
scv	matrix of individual scores
J	Hessian of the log-likelihood function
se	standard errors
se2	robust standard errors that also take into account the first step
Tv	number of time occasions for each unit

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Ancona "Politecnica delle Marche"), Francesco Valentini (University of Ancona "Politecnica delle Marche")

References

Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator, *Econometrica*, **78**, 719-733.

Bartolucci, F. and Nigro, V. (2012), Pseudo conditional maximum likelihood estimation of the dynamic logit model for binary panel data, *Journal of Econometrics*, **170**, 102-116.

Examples

```
## Not run:
# example based on simulated data
data(data_sim)
data_sim = data_sim[1:500,] # to speed up the example, remove otherwise
id = data_sim$id; yv = data_sim$y; X = cbind(X1=data_sim$X1,X2=data_sim$X2)
# estimate dynamic logit model
out = cquad_pseudo(id,yv,X, Ttol=10)
summary(out)

## End(Not run)
```

data_sim

Simulated dataset

Description

It contains a dataset simulated from the dynamic logit model

Usage

```
data(data_sim)
```

Format

The observations are for 1000 sample units at 5 five time occasions:

id list of the reference unit of each observation
time number of the time occasion
X1 first covariate
X2 second covariate
y response

Examples

```
data(data_sim)
head(data_sim)
```

print.cquad	<i>Print output for class cquad</i>
-------------	-------------------------------------

Description

Print output for class cquad and output provided by cquad_basic, cquad_equ, cquad_ext, cquad_pseudo

Usage

```
## S3 method for class 'cquad'
print(x, ...)
```

Arguments

x	output of class cquad
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigini (University of Ancona "Politecnica delle Marche")

quasi_sym	<i>Recursive computation of the conditional likelihood for the Quadratic Exponential Model proposed in Bartolucci & Nigro (2010)</i>
-----------	--

Description

Recursively compute the denominator of the individual conditional likelihood function for the Quadratic Exponential Model, adapted from Krailo & Pike (1984).

Usage

```
quasi_sym(eta, s, dyn=FALSE, y0=NULL)
```

Arguments

eta	individual vector of products between covariate and parameters
s	total score of the individual
dyn	TRUE if in the dynamic version; FALSE for the static version (by default)
y0	Individual initial observation for dynamic models

Value

f	value of the denominator
d1	first derivative of the recursive function
d11	a component of the score function
D2	second derivative of the recursive function
D12	a component of the Hessian matrix

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Ancona "Politecnica delle Marche"), Francesco Valentini (University of Ancona "Politecnica delle Marche")

References

Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator, *Econometrica*, **78**, 719-733.

Bartolucci, F., Valentini, F., & Pigni, C. (2021), Recursive Computation of the Conditional Probability Function of the Quadratic Exponential Model for Binary Panel Data, *Computational Economics*, <https://doi.org/10.1007/s10614-021-10218-2>.

Krailo, M. D., & Pike, M. C. (1984). Algorithm AS 196: conditional multivariate logistic analysis of stratified case-control studies, *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, **33(1)**, 95-103.

quasi_sym_equ	<i>Recursive computation of the conditional likelihood for the Modified Quadratic Exponential Model proposed in Bartolucci et al. (2018)</i>
---------------	--

Description

Recursively compute the denominator of the individual conditional likelihood function for the Modified Quadratic Exponential Model recursively, adapted from Krailo & Pike (1984).

Usage

```
quasi_sym_equ(eta, s, y0=NULL)
```

Arguments

eta	individual vector of products between covariate and parameters
s	total score of the individual
y0	Individual initial observation for dynamic models

Value

f	value of the denominator
d1	first derivative of the recursive function
d11	a component of the score function
D2	second derivative of the recursive function
D12	a component of the Hessian matrix

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Ancona "Politecnica delle Marche"), Francesco Valentini (University of Ancona "Politecnica delle Marche")

References

- Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator, *Econometrica*, **78**, 719-733.
- Bartolucci, F., Nigro, V., & Pigni, C. (2018). Testing for state dependence in binary panel data with individual covariates by a modified quadratic exponential model. *Econometric Reviews*, **37**(1), 61-88.
- Bartolucci, F., Valentini, F., & Pigni, C. (2021), Recursive Computation of the Conditional Probability Function of the Quadratic Exponential Model for Binary Panel Data, *Computational Economics*, <https://doi.org/10.1007/s10614-021-10218-2>.
- Krailo, M. D., & Pike, M. C. (1984). Algorithm AS 196: conditional multivariate logistic analysis of stratified case-control studies, *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, **33**(1), 95-103.

quasi_sym_pseudo	<i>Recursive computation of pseudo conditional maximum likelihood method proposed by Bartolucci & Nigro (2012).</i>
------------------	---

Description

Recursively compute the denominator of the individual conditional likelihood function for the pseudo conditional maximum likelihood method proposed by Bartolucci & Nigro (2012) recursively, adapted from Krailo & Pike (1984).

Usage

```
quasi_sym_pseudo(eta, qi, s, y0=NULL)
```

Arguments

eta	individual vector of products between covariate and parameters
s	total score of the individual
qi	Vector of quantities from first step estimation
y0	Individual initial observation for dynamic models

Value

f	value of the denominator
d1	first derivative of the recursive function
d11	a component of the score function
D2	second derivative of the recursive function
D12	a component for the Hessian matrix

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Ancona "Politecnica delle Marche"), Francesco Valentini (University of Ancona "Politecnica delle Marche")

References

Bartolucci, F. and Nigro, V. (2010), A dynamic model for binary panel data with unobserved heterogeneity admitting a root-n consistent conditional estimator, *Econometrica*, **78**, 719-733.

Bartolucci, F. and Nigro, V. (2012), Pseudo conditional maximum likelihood estimation of the dynamic logit model for binary panel data, *Journal of Econometrics*, **170**, 102-116.

Bartolucci, F., Valentini, F., & Pigni, C. (2021), Recursive Computation of the Conditional Probability Function of the Quadratic Exponential Model for Binary Panel Data, *Computational Economics*, <https://doi.org/10.1007/s10614-021-10218-2>.

Krailo, M. D., & Pike, M. C. (1984). Algorithm AS 196: conditional multivariate logistic analysis of stratified case-control studies, *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, **33(1)**, 95-103.

sim_panel_logit	<i>Simulate data from the dynamic logit model</i>
-----------------	---

Description

Simulate data from the dynamic logit model given a set of covariates and a vector of parameters.

Usage

```
sim_panel_logit(id, al, X = NULL, eta, dyn = FALSE)
```

Arguments

id	list of the reference unit of each observation
al	list of individual specific effects
X	corresponding matrix of covariates (optional)
eta	vector of parameters
dyn	TRUE if in the dynamic version; FALSE for the static version (by default)

Value

yv simulated vector of binary response variables
 pv vector of probabilities of "success"

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigni (University of Ancona "Politecnica delle Marche")

Examples

```
# simulate data from the static logit model
n = 1000; TT = 5                    # sample size, number of time occasions
id = (1:n)%x%rep(1,TT)            # vector of indices
al = rnorm(n)                      # simulate alpha
X = matrix(rnorm(2*n*TT),n*TT,2) # simulate two covariates
eta1 = c(1,-1)                    # vector of parameters
out = sim_panel_logit(id,al,X,eta1)
y1 = out$yv

# simulate data from the dynamic logit model
eta2 = c(1,-1,2)                  # vector of parameters including state dependence
out = sim_panel_logit(id,al,X,eta2,dyn=TRUE)
y2 = out$yv
```

sq *Generate binary sequences*

Description

Generate binary sequences of a certain length and with a certain sum.

Usage

```
sq(J, s = NULL)
```

Arguments

J length of the binary sequences
 s sum of the binary sequences (optional)

Value

M Matrix of binary configurations

Author(s)

Francesco Bartolucci (University of Perugia)

Examples

```
# generate all sequence of 5 binary variables
sq(5)
# generate all sequence of 5 binary variables, with sum equal 2
sq(5,2)
```

`summary.cquad`*Summary for class cquad*

Description

Summarize the output for class cquad provided by cquad_basic, cquad_equ, cquad_ext, cquad_pseudo

Usage

```
## S3 method for class 'cquad'
summary(object, ...)
```

Arguments

object	output of class cquad
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci (University of Perugia), Claudia Pigini (University of Ancona "Politecnica delle Marche")

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