

# Package ‘funStatTest’

May 8, 2026

**Title** Statistical Testing for Functional Data

**Version** 1.0.3

**Description** Implementation of two sample comparison procedures based on median-based statistical tests for functional data, introduced in Smida et al (2022) <[doi:10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997)>. Other competitive state-of-the-art approaches proposed by Chakraborty and Chaudhuri (2015) <[doi:10.1093/biomet/asu072](https://doi.org/10.1093/biomet/asu072)>, Horvath et al (2013) <[doi:10.1111/j.1467-9868.2012.01032.x](https://doi.org/10.1111/j.1467-9868.2012.01032.x)> or Cuevas et al (2004) <[doi:10.1016/j.csda.2003.10.021](https://doi.org/10.1016/j.csda.2003.10.021)> are also included in the package, as well as procedures to run test result comparisons and power analysis using simulations.

**License** AGPL (>= 3)

**URL** <https://plmlab.math.cnrs.fr/gdurif/funStatTest/>,  
<https://gdurif.pages.math.cnrs.fr/funStatTest/>

**Imports** checkmate, distr, dplyr, ggplot2, magrittr, Matrix, pbapply, stats, stringr, tibble, tidyr, tidyselect, utils

**Suggests** knitr, rmarkdown, testthat, vdiff

**VignetteBuilder** knitr

**Config/fusen/version** 0.6.0

**Date/Publication** 2024-05-23 09:00:03 UTC

**Encoding** UTF-8

**RoxygenNote** 7.3.1

**NeedsCompilation** no

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**Repository** CRAN

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comp_stat	<i>Compute multiple statistics</i>
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---

### Description

Computation of the different statistics defined in the package. See Smida et al (2022) for more details.

### Usage

```
comp_stat(MatX, MatY, stat = c("mo", "med"))
```

### Arguments

MatX	numeric matrix of dimension $n\_point \times n$ containing $n$ trajectories (in columns) of size $n\_point$ (in rows).
MatY	numeric matrix of dimension $n\_point \times m$ containing $m$ trajectories (in columns) of size $n\_point$ (in rows).
stat	character string or vector of character string, name of the statistics for which the p-values will be computed, among "mo", "med", "wmw", "hkr", "cff".

### Details

For HKR statistics, only the values of the two statistics, namely HKR1 and HKR2 and not the eigen values (see [stat\\_hkr\(\)](#) for more details).

### Value

list of named numeric value corresponding to the statistic values listed in `stat` input.

### Author(s)

Zaineb Smida, Ghislain DURIF, Lionel Cucala

## References

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:10.1080/10485252.2022.2064997, hal-03658578

## See Also

[stat\\_mo\(\)](#), [stat\\_med\(\)](#), [stat\\_wmw\(\)](#), [stat\\_hkr\(\)](#), [stat\\_cff\(\)](#)

## Examples

```
simu_data <- simul_data(
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 10,
  delta_shape = "constant", distrib = "normal"
)

MatX <- simu_data$mat_sample1
MatY <- simu_data$mat_sample2

res <- comp_stat(MatX, MatY, stat = c("mo", "med", "wmw", "hkr", "cff"))
res
```

---

permut\_pval

*Permutation-based computation of p-values*

---

## Description

Computation of the p-values associated to any statistics described in the package with the permutation methods. See Smida et al (2022) for more details.

## Usage

```
permut_pval(MatX, MatY, n_perm = 100, stat = c("mo", "med"), verbose = FALSE)
```

## Arguments

MatX	numeric matrix of dimension $n\_point \times n$ containing $n$ trajectories (in columns) of size $n\_point$ (in rows).
MatY	numeric matrix of dimension $n\_point \times m$ containing $m$ trajectories (in columns) of size $n\_point$ (in rows).
n_perm	integer, number of permutation to compute the p-values.
stat	character string or vector of character string, name of the statistics for which the p-values will be computed, among "mo", "med", "wmw", "hkr", "cff".
verbose	boolean, if TRUE, enable verbosity.

## Value

list of named numeric value corresponding to the p-values for each statistic listed in the stat input.

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**References**

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:[10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997), [hal-03658578](https://hal.archives-ouvertes.fr/hal-03658578)

**See Also**

[stat\\_mo\(\)](#), [stat\\_med\(\)](#), [stat\\_wmw\(\)](#), [stat\\_hkr\(\)](#), [stat\\_cff\(\)](#), [comp\\_stat\(\)](#)

**Examples**

```
# simulate small data for the example
simu_data <- simul_data(
  n_point = 20, n_obs1 = 4, n_obs2 = 5, c_val = 10,
  delta_shape = "constant", distrib = "normal"
)

MatX <- simu_data$mat_sample1
MatY <- simu_data$mat_sample2
res <- permut_pval(
  MatX, MatY, n_perm = 100, stat = c("mo", "med", "wmw", "hkr", "cff"),
  verbose = TRUE)
res
```

---

plot\_simu

*Graphical representation of simulated data*

---

**Description**

Graphical representation of simulated data

**Usage**

```
plot_simu(simu)
```

**Arguments**

simu            list, output of [simul\\_data\(\)](#)

**Value**

the ggplot2 graph of simulated trajectories.

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**See Also**

[simul\\_data\(\)](#)

**Examples**

```
# constant delta
simu_data <- simul_data(
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 5,
  delta_shape = "constant", distrib = "normal"
)
plot_simu(simu_data)
# linear delta
simu_data <- simul_data(
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 5,
  delta_shape = "linear", distrib = "normal"
)
plot_simu(simu_data)
# quadratic delta
simu_data <- simul_data(
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 5,
  delta_shape = "quadratic", distrib = "normal"
)
plot_simu(simu_data)
```

---

power\_exp

*Simulation-based experiment for power analysis*

---

**Description**

Computation of the statistical power (i.e. risk to reject the null hypothesis when it is false) associated to any statistics described in the package based on simulation permutation-based p-values computations. See Smida et al (2022) for more details.

**Usage**

```
power_exp(
  n_simu = 100,
  alpha = 0.05,
  n_perm = 100,
  stat = c("mo", "med"),
  n_point = 100,
  n_obs1 = 50,
  n_obs2 = 50,
  c_val = 1,
```

```

    delta_shape = "constant",
    distrib = "normal",
    max_iter = 10000,
    verbose = FALSE
  )

```

### Arguments

n_simu	integer value, number of simulations to compute the statistical power.
alpha	numerical value, between 0 and 1, type I risk level to reject the null hypothesis in the simulation. Default value is 5%.
n_perm	integer, number of permutation to compute the p-values.
stat	character string or vector of character string, name of the statistics for which the p-values will be computed, among "mo", "med", "wmw", "hkr", "cff".
n_point	integer value, number of points in the trajectory.
n_obs1	integer value, number of trajectories in the first sample.
n_obs2	integer value, number of trajectories in the second sample.
c_val	numeric value, level of divergence between the two samples.
delta_shape	character string, shape of the divergence between the two samples, among "constant", "linear", "quadratic".
distrib	character string, type of probability distribution used to simulate the data among "normal", "cauchy", "dexp", "student".
max_iter	integer, maximum number of iteration for the iterative simulation process.
verbose	boolean, if TRUE, enable verbosity.

### Details

The `c_val` input argument should be strictly positive so that the null hypothesis is not verified when simulating the data (i.e. so that the two sample are not generated from the same probability distribution).

### Value

a list with the following elements:

- `power_res`: a list of named numeric value corresponding to the power values for each statistic listed in `stat` input.
- `pval_res`: a list of named numeric values corresponding to the p-values for each simulation and each statistic listed in the `stat` input.
- `simu_config`: information about input parameters used for simulation, including `n_simu`, `c_val`, `distrib`, `delta_shape`, `n_point`, `n_obs1`, `n_obs2`.

### Author(s)

Zaineb Smida, Ghislain DURIF, Lionel Cucala

## References

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:[10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997), [hal-03658578](https://hal.archives-ouvertes.fr/hal-03658578)

## See Also

[stat\\_mo\(\)](#), [stat\\_med\(\)](#), [stat\\_wmw\(\)](#), [stat\\_hkr\(\)](#), [stat\\_cff\(\)](#), [comp\\_stat\(\)](#)

## Examples

```
# simulate a few small data for the example
res <- power_exp(
  n_simu = 20, alpha = 0.05, n_perm = 100,
  stat = c("mo", "med", "wmw", "hkr", "cff"),
  n_point = 25, n_obs1 = 4, n_obs2 = 5, c_val = 10, delta_shape = "constant",
  distrib = "normal", max_iter = 10000, verbose = FALSE
)
res$power_res
```

---

simul_data	<i>Simulation of trajectories from two samples diverging by a delta function</i>
------------	--

---

## Description

Simulate `n_obs1` trajectories of length `n_point` in the first sample and `n_obs2` trajectories of length `n_point` in the second sample.

## Usage

```
simul_data(
  n_point,
  n_obs1,
  n_obs2,
  c_val = 0,
  delta_shape = "constant",
  distrib = "normal",
  max_iter = 10000
)
```

## Arguments

<code>n_point</code>	integer value, number of points in the trajectory.
<code>n_obs1</code>	integer value, number of trajectories in the first sample.
<code>n_obs2</code>	integer value, number of trajectories in the second sample.
<code>c_val</code>	numeric value, level of divergence between the two samples.

delta_shape	character string, shape of the divergence between the two samples, among "constant", "linear", "quadratic".
distrib	character string, type of probability distribution used to simulate the data among "normal", "cauchy", "dexp", "student".
max_iter	integer, maximum number of iteration for the iterative simulation process.

### Value

A list with the following elements

- `mat_sample1`: numeric matrix of dimension `n_point` x `n_obs1` containing `n_obs1` trajectories (in columns) of size `n_point` (in rows) corresponding to sample 1.
- `mat_sample2`: numeric matrix of dimension `n_point` x `n_obs2` containing `n_obs2` trajectories (in columns) of size `n_point` (in rows) corresponding to sample 2.

### Author(s)

Zaineب Smida, Ghislain DURIF, Lionel Cucala

### References

Zaineب Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:[10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997), hal-03658578

### See Also

[plot\\_simu\(\)](#), [simul\\_traj\(\)](#)

### Examples

```
simu_data <- simul_data(  
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 10,  
  delta_shape = "constant", distrib = "normal"  
)  
str(simu_data)
```

---

simul\_traj

*Single trajectory simulation*

---

### Description

Simulate a trajectory of length `n_point` using a random generator associated to different probability distribution.

### Usage

```
simul_traj(n_point, distrib = "normal", max_iter = 10000)
```

**Arguments**

n_point	integer value, number of points in the trajectory.
distrib	character string, type of probability distribution used to simulate the data among "normal", "cauchy", "dexp", "student".
max_iter	integer, maximum number of iteration for the iterative simulation process.

**Value**

Vector of size n\_point with the trajectory values.

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**References**

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:[10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997), [hal-03658578](https://hal.archives-ouvertes.fr/hal-03658578)

**See Also**

[simul\\_data\(\)](#)

**Examples**

```
simu_vec <- simul_traj(100)
plot(simu_vec, xlab = "point", ylab = "value")
```

---

stat\_cff

*Cuevas-Febrero-Fraiman statistic*

---

**Description**

The Cuevas-Febrero-Fraiman statistics defined in Cuevas et al (2004) (and noted CFF in Smida et al 2022) is computed to compare two sets of functional trajectories.

**Usage**

```
stat_cff(MatX, MatY)
```

**Arguments**

MatX	numeric matrix of dimension n_point x n containing n trajectories (in columns) of size n_point (in rows).
MatY	numeric matrix of dimension n_point x m containing m trajectories (in columns) of size n_point (in rows).

**Value**

numeric value corresponding to the WMW statistic value

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**References**

Cuevas, A, Febrero, M, and Fraiman, R (2004) An anova test for functional data. Computational Statistics & Data Analysis, 47(1): 111–122. doi:[10.1016/j.csda.2003.10.021](https://doi.org/10.1016/j.csda.2003.10.021)

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, Journal of Nonparametric Statistics, 34:2, 520-553, doi:[10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997), hal-03658578

**See Also**

[comp\\_stat\(\)](#), [permut\\_pval\(\)](#)

**Examples**

```
simu_data <- simul_data(  
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 10,  
  delta_shape = "constant", distrib = "normal"  
)  
  
MatX <- simu_data$mat_sample1  
MatY <- simu_data$mat_sample2  
  
stat_cff(MatX, MatY)
```

---

stat\_hkr

*Horváth-Kokoszka-Reeder statistics*

---

**Description**

The Horváth-Kokoszka-Reeder statistics defined in Chakraborty & Chaudhuri (2015) (and noted HKR1 and HKR2 in Smida et al 2022) are computed to compare two sets of functional trajectories.

**Usage**

```
stat_hkr(MatX, MatY)
```

**Arguments**

MatX	numeric matrix of dimension $n\_point \times n$ containing $n$ trajectories (in columns) of size $n\_point$ (in rows).
MatY	numeric matrix of dimension $n\_point \times m$ containing $m$ trajectories (in columns) of size $n\_point$ (in rows).

**Value**

A list with the following elements

- T1: numeric value corresponding to the HKR1 statistic value
- T2: numeric value corresponding to the HKR2 statistic value
- eigenval: numeric vector of eigen values from the empirical pooled covariance matrix of MatX and MatY (see Smida et al, 2022, for more details)

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**References**

Horváth, L., Kokoszka, P., & Reeder, R. (2013). Estimation of the mean of functional time series and a two-sample problem. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, 75(1), 103–122. doi:10.1111/j.14679868.2012.01032.x

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:10.1080/10485252.2022.2064997, hal-03658578

**See Also**

`comp_stat()`, `permut_pval()`

**Examples**

```
simu_data <- simul_data(
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 10,
  delta_shape = "constant", distrib = "normal"
)

MatX <- simu_data$mat_sample1
MatY <- simu_data$mat_sample2

stat_hkr(MatX, MatY)
```

---

stat\_med

*MED median statistic*

---

**Description**

The MED median statistics defined in Smida et al (2022) is computed to compare two sets of functional trajectories.

**Usage**

```
stat_med(MatX, MatY)
```

**Arguments**

MatX	numeric matrix of dimension $n\_point \times n$ containing $n$ trajectories (in columns) of size $n\_point$ (in rows).
MatY	numeric matrix of dimension $n\_point \times m$ containing $m$ trajectories (in columns) of size $n\_point$ (in rows).

**Value**

numeric value corresponding to the MED median statistic value

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**References**

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:10.1080/10485252.2022.2064997, hal-03658578

**See Also**

[comp\\_stat\(\)](#), [permut\\_pval\(\)](#)

**Examples**

```
simu_data <- simul_data(  
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 10,  
  delta_shape = "constant", distrib = "normal"  
)  
  
MatX <- simu_data$mat_sample1  
MatY <- simu_data$mat_sample2  
  
stat_med(MatX, MatY)
```

---

stat\_mo

*MO median statistic*

---

**Description**

The MO median statistics defined in Smida et al (2022) is computed to compare two sets of functional trajectories.

**Usage**

```
stat_mo(MatX, MatY)
```

**Arguments**

MatX	numeric matrix of dimension $n\_point \times n$ containing $n$ trajectories (in columns) of size $n\_point$ (in rows).
MatY	numeric matrix of dimension $n\_point \times m$ containing $m$ trajectories (in columns) of size $n\_point$ (in rows).

**Value**

numeric value corresponding to the MO median statistic value

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**References**

Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:[10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997), [hal-03658578](https://hal.archives-ouvertes.fr/hal-03658578)

**See Also**

[comp\\_stat\(\)](#), [permut\\_pval\(\)](#)

**Examples**

```
simu_data <- simul_data(  
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 10,  
  delta_shape = "constant", distrib = "normal"  
)  
  
MatX <- simu_data$mat_sample1  
MatY <- simu_data$mat_sample2  
  
stat_mo(MatX, MatY)
```

---

stat\_wmw

*Wilcoxon-Mann-Whitney (WMW) statistic*

---

**Description**

The Wilcoxon-Mann-Whitney statistic defined in Chakraborty & Chaudhuri (2015) (and noted WMW in Smida et al 2022) is computed to compare two sets of functional trajectories.

**Usage**

```
stat_wmw(MatX, MatY)
```

**Arguments**

MatX	numeric matrix of dimension $n\_point \times n$ containing $n$ trajectories (in columns) of size $n\_point$ (in rows).
MatY	numeric matrix of dimension $n\_point \times m$ containing $m$ trajectories (in columns) of size $n\_point$ (in rows).

**Value**

numeric value corresponding to the WMW statistic value

**Author(s)**

Zaineb Smida, Ghislain DURIF, Lionel Cucala

**References**

- Anirvan Chakraborty, Probal Chaudhuri, A Wilcoxon–Mann–Whitney-type test for infinite-dimensional data, *Biometrika*, Volume 102, Issue 1, March 2015, Pages 239–246, doi:[10.1093/biomet/asu072](https://doi.org/10.1093/biomet/asu072)
- Zaineb Smida, Lionel Cucala, Ali Gannoun & Ghislain Durif (2022) A median test for functional data, *Journal of Nonparametric Statistics*, 34:2, 520-553, doi:[10.1080/10485252.2022.2064997](https://doi.org/10.1080/10485252.2022.2064997), hal-03658578

**See Also**

[comp\\_stat\(\)](#), [permut\\_pval\(\)](#)

**Examples**

```
simu_data <- simul_data(  
  n_point = 100, n_obs1 = 50, n_obs2 = 75, c_val = 10,  
  delta_shape = "constant", distrib = "normal"  
)  
  
MatX <- simu_data$mat_sample1  
MatY <- simu_data$mat_sample2  
  
stat_wmw(MatX, MatY)
```

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