

Package ‘kko’

May 8, 2026

Title Kernel Knockoffs Selection for Nonparametric Additive Models

Version 1.0.1

Description A variable selection procedure, dubbed KKO, for nonparametric additive model with finite-sample false discovery rate control guarantee. The method integrates three key components: knockoffs, subsampling for stability, and random feature mapping for nonparametric function approximation. For more information, see the accompanying paper: Dai, X., Lyu, X., & Li, L. (2021). “Kernel Knockoffs Selection for Nonparametric Additive Models”. arXiv preprint <[doi:10.48550/arXiv.2105.11659](https://doi.org/10.48550/arXiv.2105.11659)>.

License GPL (>= 2)

Depends R (>= 3.6.3)

Imports grpreg, knockoff, doParallel, parallel, foreach, ExtDist

Suggests knitr, rmarkdown, ggplot2

Encoding UTF-8

LazyData false

RoxygenNote 7.1.2

VignetteBuilder knitr

Author Xiaowu Dai [aut],
Xiang Lyu [aut, cre],
Lexin Li [aut]

Maintainer Xiang Lyu <xianglyu@berkeley.edu>

NeedsCompilation no

Repository CRAN

Date/Publication 2022-02-01 09:10:05 UTC

Contents

generate_data	2
kko	3
KO_evaluation	4
rk_fit	5
rk_subsample	7
rk_tune	8

Index**11**

generate_data	<i>generate response from nonparametric additive model</i>
---------------	--

Description

The function generate response from additive models of various components.

Usage

```
generate_data(X, reg_coef, model = "linear", err_sd = 1)
```

Arguments

X	design matrix of additive model; rows are observations and columns are variables.										
reg_coef	regression coefficient vector.										
model	types of components. Default is "linear". Other choices are <table> <tr> <td>linear</td> <td>linear regression.</td> </tr> <tr> <td>poly</td> <td>polynomial of degree sampled from 2 to 4.</td> </tr> <tr> <td>sinpoly</td> <td>sum of polynomial of sin and cos.</td> </tr> <tr> <td>sinratio</td> <td>ratio of sin.</td> </tr> <tr> <td>sinmix</td> <td>sampled from poly and sinratio.</td> </tr> </table>	linear	linear regression.	poly	polynomial of degree sampled from 2 to 4.	sinpoly	sum of polynomial of sin and cos.	sinratio	ratio of sin.	sinmix	sampled from poly and sinratio.
linear	linear regression.										
poly	polynomial of degree sampled from 2 to 4.										
sinpoly	sum of polynomial of sin and cos.										
sinratio	ratio of sin.										
sinmix	sampled from poly and sinratio.										
err_sd	standard deviation of regression error.										

Value

reponse vector

Author(s)

Xiaowu Dai, Xiang Lyu, Lexin Li

Examples

```
p=5 # number of predictors
s=2 # sparsity, number of nonzero component functions
sig_mag=100 # signal strength
n= 200 # sample size
model="poly" # component function type
X=matrix(rnorm(n*p),n,p) %%%chol(toeplitz(0.3^(0:(p-1)))) # generate design
reg_coef=c(rep(1,s),rep(0,p-s)) # regression coefficient
reg_coef=reg_coef*(2*(rnorm(p)>0)-1)*sig_mag
y=generate_data(X,reg_coef,model) # reponse vector
```

kko *variable selection for additive model via KKO*

Description

The function applies KKO to compute importance scores of components.

Usage

```
kko(  
  X,  
  y,  
  X_k,  
  rfn_range = c(2, 3, 4),  
  n_stb_tune = 50,  
  n_stb = 100,  
  cv_folds = 10,  
  frac_stb = 1/2,  
  nCores_para = 4,  
  rkernl = c("laplacian", "gaussian", "cauchy"),  
  rk_scale = 1  
)
```

Arguments

X	design matrix of additive model; rows are observations and columns are variables.
y	response of additive model.
X_k	knockoffs matrix of design; the same size as X.
rfn_range	a vector of random feature expansion numbers to be tuned.
n_stb_tune	number of subsampling for tuning random feature numbers.
n_stb	number of subsampling for computing importance scores.
cv_folds	the folds of cross-validation for tuning group lasso penalty.
frac_stb	fraction of subsample size.
nCores_para	number of cores for parallelizing subsampling.
rkernl	kernel choices. Default is "laplacian". Other choices are "cauchy" and "gaussian".
rk_scale	scale parameter of sampling distribution for random feature expansion. For gaussian kernel, it is standard deviation of gaussian sampling distribution.

Value

a list of selection results.

importance_score	importance scores of variables for knockoff filtering.
selection_frequency	a 0/1 matrix of selection results on subsamples. Rows are subsamples, and columns are variables. T
rfn_tune	tuned optimal random feature number.
rfn_range	range of random feature numbers.
tune_result	a list of tuning results.

Author(s)

Xiaowu Dai, Xiang Lyu, Lexin Li

Examples

```
library(knockoff)
p=4 # number of predictors
sig_mag=100 # signal strength
n= 100 # sample size
rkern="laplacian" # kernel choice
s=2 # sparsity, number of nonzero component functions
rk_scale=1 # scaling paramtere of kernel
rfn_range=c(2,3,4) # number of random features
cv_folds=15 # folds of cross-validation in group lasso
n_stb=10 # number of subsampling for importance scores
n_stb_tune=5 # number of subsampling for tuning random feature number
frac_stb=1/2 # fraction of subsample
nCores_para=2 # number of cores for parallelization
X=matrix(rnorm(n*p),n,p)%%chol(toeplitz(0.3^(0:(p-1)))) # generate design
X_k = create.second_order(X) # generate knockoff
reg_coef=c(rep(1,s),rep(0,p-s)) # regression coefficient
reg_coef=reg_coef*(2*(rnorm(p)>0)-1)*sig_mag
y=X%% reg_coef + rnorm(n) # response

kko(X,y,X_k,rfn_range,n_stb_tune,n_stb,cv_folds,frac_stb,nCores_para,rkern,rk_scale)
```

KO_evaluation

evaluate performance of KKO selection

Description

The function computes {FDP, FPR, TPR} of selection by knockoff filtering on importance scores of KKO.

Usage

```
KO_evaluation(W, reg_coef, fdr_range = 0.2, offset = 1)
```

Arguments

W importance scores of variables.
 reg_coef true regression coefficient.
 fdr_range FDR control levels of knockoff filter.
 offset 0/1. If 1, knockoff+ filter. Otherwise, knockoff filter.

Value

FDP, FPR, TPR of knockoff filtering at fdr_range.

Author(s)

Xiaowu Dai, Xiang Lyu, Lexin Li

Examples

```
library(knockoff)
p=5 # number of predictors
sig_mag=100 # signal strength
n= 100 # sample size
rkkernel="laplacian" # kernel choice
s=2 # sparsity, number of nonzero component functions
rk_scale=1 # scaling paramtere of kernel
rfn_range=c(2,3,4) # number of random features
cv_folds=15 # folds of cross-validation in group lasso
n_stb=10 # number of subsampling for importance scores
n_stb_tune=5 # number of subsampling for tuning random feature number
frac_stb=1/2 # fraction of subsample
nCores_para=2 # number of cores for parallelization
X=matrix(rnorm(n*p),n,p)%*%chol(toeplitz(0.3^(0:(p-1)))) # generate design
X_k = create.second_order(X) # generate knockoff
reg_coef=c(rep(1,s),rep(0,p-s)) # regression coefficient
reg_coef=reg_coef*(2*(rnorm(p)>0)-1)*sig_mag
y=X%*% reg_coef + rnorm(n) # response

kko_fit=kko(X,y,X_k,rfn_range,n_stb_tune,n_stb,cv_folds,frac_stb,nCores_para,rkernel,rk_scale)
W=kko_fit$importance_score
fdr_range=c(0.2,0.3,0.4,0.5)
KO_evaluation(W,reg_coef,fdr_range,offset=1)
```

rk_fit

nonparametric additive model seleciton via random kernel

Description

The function selects additive components via applying group lasso on random feature expansion of data and knockoffs.

Usage

```
rk_fit(
  X,
  y,
  X_k,
  rfn,
  cv_folds,
  rkernel = "laplacian",
  rk_scale = 1,
  rseed = NULL
)
```

Arguments

X	design matrix of additive model; rows are observations and columns are variables.
y	response of additive model.
X_k	knockoffs matrix of design; the same size as X.
rfn	random feature expansion number.
cv_folds	the folds of cross-validation for tuning group lasso penalty.
rkernel	kernel choices. Default is "laplacian". Other choices are "cauchy" and "gaussian".
rk_scale	scaling parameter of sampling distribution for random feature expansion. For gaussian kernel, it is standard deviation of gaussian sampling distribution.
rseed	seed for random feature expansion.

Value

a 0/1 vector indicating selected components.

Author(s)

Xiaowu Dai, Xiang Lyu, Lexin Li

Examples

```
library(knockoff)
p=5 # number of predictors
sig_mag=100 # signal strength
n= 200 # sample size
rkernel="laplacian" # kernel choice
s=2 # sparsity, number of nonzero component functions
rk_scale=1 # scaling parameter of kernel
rfn= 3 # number of random features
cv_folds=15 # folds of cross-validation in group lasso
X=matrix(rnorm(n*p),n,p)%*%chol(toeplitz(0.3^(0:(p-1)))) # generate design
X_k = create.second_order(X) # generate knockoff
reg_coef=c(rep(1,s),rep(0,p-s)) # regression coefficient
```

```

reg_coef=reg_coef*(2*(rnorm(p)>0)-1)*sig_mag
y=X%% reg_coef + rnorm(n) # response

# the first half is variables of design X, and the latter is knockoffs X_k
rk_fit(X,y,X_k,rfn,cv_folds,rkernel,rk_scale)

```

rk_subsample *compute selection frequency of rk_fit on subsamples*

Description

The function applies rk_fit on subsamples and record selection results.

Usage

```

rk_subsample(
  X,
  y,
  X_k,
  rfn,
  n_stb,
  cv_folds,
  frac_stb = 1/2,
  nCores_para,
  rkernel = "laplacian",
  rk_scale = 1
)

```

Arguments

X	design matrix of additive model; rows are observations and columns are variables.
y	response of additive model.
X_k	knockoffs matrix of design; the same size as X.
rfn	random feature expansion number.
n_stb	number of subsampling.
cv_folds	the folds of cross-validation for tuning group lasso.
frac_stb	fraction of subsample size.
nCores_para	number of cores for parallelizing subsampling.
rkernel	kernel choices. Default is "laplacian". Other choices are "cauchy" and "gaussian".
rk_scale	scaling parameter of sampling distribution for random feature expansion. For gaussian kernel, it is standard deviation of gaussian sampling distribution.

Value

a 0/1 matrix indicating selection results. Rows are subsamples, and columns are variables. The first half columns are variables of design X, and the latter are knockoffs X_k .

Author(s)

Xiaowu Dai, Xiang Lyu, Lexin Li

Examples

```
library(knockoff)
p=5 # number of predictors
sig_mag=100 # signal strength
n= 100 # sample size
rkern="laplacian" # kernel choice
s=2 # sparsity, number of nonzero component functions
rk_scale=1 # scaling paramtere of kernel
rfn= 3 # number of random features
cv_folds=15 # folds of cross-validation in group lasso
n_stb=10 # number of subsampling
frac_stb=1/2 # fraction of subsample
nCores_para=2 # number of cores for parallelization
X=matrix(rnorm(n*p),n,p)%%chol(toeplitz(0.3^(0:(p-1)))) # generate design
X_k = create.second_order(X) # generate knockoff
reg_coef=c(rep(1,s),rep(0,p-s)) # regression coefficient
reg_coef=reg_coef*(2*(rnorm(p)>0)-1)*sig_mag
y=X%% reg_coef + rnorm(n) # response

rk_subsample(X,y,X_k,rfn,n_stb,cv_folds,frac_stb,nCores_para,rkern,rk_scale)
```

rk_tune

tune random feature number for KKO.

Description

The function applys KKO with different random feature numbers to tune the optimal number.

Usage

```
rk_tune(
  X,
  y,
  X_k,
  rfn_range,
  n_stb,
  cv_folds,
  frac_stb = 1/2,
```

```

nCores_para = 1,
rkernel = "laplacian",
rk_scale = 1
)

```

Arguments

X	design matrix of additive model; rows are observations and columns are variables.
y	response of additive model.
X_k	knockoffs matrix of design; the same size as X.
rfn_range	a vector of random feature expansion numbers to be tuned.
n_stb	number of subsampling in KKO.
cv_folds	the folds of cross-validation for tuning group lasso.
frac_stb	fraction of subsample.
nCores_para	number of cores for parallelizing subsampling.
rkernel	kernel choices. Default is "laplacian". Other choices are "cauchy" and "gaussian".
rk_scale	scaling parameter of sampling distribution for random feature expansion. For gaussian kernel, it is standard deviation of gaussian sampling distribution.

Value

a list of tuning results.

rfn_tune	tuned optimal random feature number.
rfn_range	a vector of random feature expansion numbers to be tuned.
scores	scores of random feature numbers. rfn_tune has the maximal score.
Pi_list	a list of subsample selection results for each random feature number.

Author(s)

Xiaowu Dai, Xiang Lyu, Lexin Li

Examples

```

library(knockoff)
p=5 # number of predictors
sig_mag=100 # signal strength
n= 100 # sample size
rkernel="laplacian" # kernel choice
s=2 # sparsity, number of nonzero component functions
rk_scale=1 # scaling paramtere of kernel
rfn_range= c(2,3,4) # number of random features
cv_folds=15 # folds of cross-validation in group lasso
n_stb=10 # number of subsampling

```

```
frac_stb=1/2 # fraction of subsample
nCores_para=2 # number of cores for parallelization
X=matrix(rnorm(n*p),n,p)%*%chol(toeplitz(0.3^(0:(p-1)))) # generate design
X_k = create.second_order(X) # generate knockoff
reg_coef=c(rep(1,s),rep(0,p-s)) # regression coefficient
reg_coef=reg_coef*(2*(rnorm(p)>0)-1)*sig_mag
y=X%*% reg_coef + rnorm(n) # response

rk_tune(X,y,X_k,rfn_range,n_stb,cv_folds,frac_stb,nCores_para,rkernel,rk_scale)
```

Index

`generate_data`, 2

`kko`, 3

`KO_evaluation`, 4

`rk_fit`, 5

`rk_subsample`, 7

`rk_tune`, 8