

# Package ‘monotone’

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**Type** Package

**Title** Performs Monotone Regression

**Version** 0.1.2

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**Description** The monotone package contains a fast up-and-down-blocks implementation for the pool-adjacent-violators algorithm for simple linear ordered monotone regression, including two spin-off functions for unimodal and bivariate monotone regression (see <[doi:10.18637/jss.v102.c01](https://doi.org/10.18637/jss.v102.c01)>).

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bimonotone

*Bivariate Monotone Regression Function*

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### Description

bimonotone performs bivariate monotone regression. The function uses the up-and-down-blocks implementation (Kruskal, 1964) of the pool-adjacent-violators algorithm (Ayer, Brunk, Ewing, Reid, and Silverman, 1955), with additional lookaheads, repeatedly, for both rows and columns, until convergence.

### Usage

```
bimonotone(  
  x,  
  w = matrix(1, nrow(x), ncol(x)),  
  maxiter = 65536,  
  eps = 1.49011611938477e-08  
)
```

### Arguments

x	a real-valued matrix.
w	a real-valued matrix with positive weights (default a matrix with ones).
maxiter	maximum number of iterations (default = 65536)
eps	precision of estimates (default = 1.4901161193847656e-08)

### Details

Error checking on x, w, maxiter, or eps is not present.

### Value

Returns a real-valued matrix with values of both rows and columns of x in monotone order.

### References

Bril G, Dykstra R, Pillers C, Robertson T (1984). Algorithm AS 206: isotonic regression in two independent variables. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 33(3), 352-357. URL <https://www.jstor.org/stable/pdf/2347723.pdf>.

Busing, F.M.T.A. (2022). Monotone Regression: A Simple and Fast O(n) PAVA Implementation. *Journal of Statistical Software, Code Snippets*, 102 (1), pp. 1-25. (<doi:10.18637/jss.v102.c01>)

Dykstra R.L., Robertson T. (1982). An algorithm for isotonic regression for two or more independent variables. *The Annals of Statistics*, 10(3), 708-716. URL [https://projecteuclid.org/download/pdf\\_1/euclid.aos/11763458](https://projecteuclid.org/download/pdf_1/euclid.aos/11763458)

Turner, T.R. (2019). Iso: Functions to Perform Isotonic Regression. R package version 0.0-18. URL <https://cran.r-project.org/package=Iso>

## Examples

```
G <- matrix( c( 1, 5.2, 0.1, 0.1, 5, 0, 6, 2, 3, 5.2, 5, 7, 4, 5.5, 6, 6 ), 4, 4 )
print( G )
H <- bimonotone( G )
print( H )
y <- c( 8, 4, 8, 2, 2, 0, 8 )
x <- bimonotone( as.matrix( y ) )
print( x )
x <- bimonotone( t( as.matrix( y ) ) )
print( x )
```

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legacy

*Monotone Regression Legacy Function*

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## Description

legacy provides some functions for monotone regression from the past. Current implementations have been translated into C for proper comparison in Busing (2022).

## Usage

```
legacy(x, w = rep(1, length(x)), number = 0)
```

## Arguments

x	a real-valued vector.
w	a real-valued vector with positive weights (default a vector with ones).
number	function number (specifications below).

## Details

Legacy implementations by number, function, author, and year:

- 0 = default (do nothing)
- 1 = fitm() by Kruskal (1964).
- 2 = wmrnh() by van Waning (1976).
- 3 = amalgm() by Cran (1980).
- 4 = pav() by Brill (1984).
- 5 = isoreg() by Gupta (1995).
- 6 = iso\_pava() by Turner (1997).
- 7 = isotonic() by Kincaid (2001).
- 8 = isomean() by Strimmer (2008).
- 9 = pooled\_pava() by Pedregosa (2011).

- 10 = linear\_pava() by Tulloch (2014).
- 11 = inplace\_pava() by Varoquaux (2016).
- 12 = md\_pava() by Danish (2016).
- 13 = reg\_1d\_l2() by Xu (2017).
- 14 = jbkpava() by de Leeuw (2017).

Error checking on w or x is not present.

### Value

Returns a real-valued vector with values of x in increasing order.

### References

Busing, F.M.T.A. (2022). Monotone Regression: A Simple and Fast O(n) PAVA Implementation. *Journal of Statistical Software, Code Snippets*, 102 (1), pp. 1-25. (<doi:10.18637/jss.v102.c01>)

### Examples

```
y <- c( 8, 4, 8, 2, 2, 0, 8 )
x <- legacy( y, number = 1 )
print( x )
```

---

monotone

*Monotone Regression Function*

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### Description

monotone performs simple linear ordered monotone or isotonic regression. The function follows the up-and-down-blocks implementation (Kruskal, 1964) of the pool-adjacent-violators algorithm (Ayer, Brunk, Ewing, Reid, and Silverman, 1955) with additional lookaheads (Busing, 2022).

### Usage

```
monotone(x, w = rep(1, length(x)))
```

### Arguments

x                    a real-valued vector.  
w                    a real-valued vector with positive weights (default a vector with ones).

### Details

Error checking on x or w is not present.

**Value**

Returns a real-valued vector with values of  $x$  in increasing order.

**References**

Ayer M., H.D. Brunk, G.M. Ewing, W.T. Reid, and E. Silverman (1955). An empirical distribution function for sampling with incomplete information. *The Annals of Mathematical Statistics*, pp. 641-647. URL <https://www.jstor.org/stable/pdf/2236377.pdf>.

Busing, F.M.T.A. (2022). Monotone Regression: A Simple and Fast  $O(n)$  PAVA Implementation. *Journal of Statistical Software, Code Snippets*, 102 (1), pp. 1-25. (<doi:10.18637/jss.v102.c01>)

Kruskal, J.B. (1964). Nonmetric multidimensional scaling: a numerical method. *Psychometrika*, 29(2), pp. 115-129. URL [http://cda.psych.uiuc.edu/psychometrika\\_highly\\_cited\\_articles/kruskal\\_1964b.pdf](http://cda.psych.uiuc.edu/psychometrika_highly_cited_articles/kruskal_1964b.pdf).

**Examples**

```
y <- c( 8, 4, 8, 2, 2, 0, 8 )
x <- monotone( y )
print( x )
```

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unimonotone

*Unimodal Monotone Regression Function*


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**Description**

unimonotone performs unimodal monotone regression. The function follows the up-and-down-blocks implementation (Kruskal, 1964) of the pool-adjacent-violators algorithm (Ayer, Brunk, Ewing, Reid, and Silverman, 1955) for both isotonic and antitonic regression, and the prefix isotonic regression approach (Stout, 2008) with additional lookaheads and progressive error sum-of-squares computation.

**Usage**

```
unimonotone(x, w = rep(1, length(x)))
```

**Arguments**

$x$  a real-valued vector.  
 $w$  a real-valued vector with positive weights (default a vector with ones).

**Details**

Error checking on  $x$  or  $w$  is not present.

**Value**

Returns a real-valued vector with values of  $x$  in umbrella order.

## References

Bril G, Dykstra R, Pillers C, Robertson T (1984). Algorithm AS 206: isotonic regression in two independent variables. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 33(3), 352-357. URL <https://www.jstor.org/stable/pdf/2347723.pdf>.

Busing, F.M.T.A. (2022). Monotone Regression: A Simple and Fast O(n) PAVA Implementation. *Journal of Statistical Software, Code Snippets*, 102 (1), pp. 1-25. (<doi:10.18637/jss.v102.c01>)

Stout, Q.F. (2008). Unimodal Regression via Prefix Isotonic Regression. *Computational Statistics and Data Analysis*, 53, pp. 289-297. URL <https://doi:10.1016/j.csda.2008.08.005>

Turner, T.R. and Wollan, P.C. (1997). Locating a maximum using isotonic regression. *Computational statistics and data analysis*, 25(3), pp. 305-320. URL [https://doi.org/10.1016/S0167-9473\(97\)00009-1](https://doi.org/10.1016/S0167-9473(97)00009-1)

Turner, T.R. (2019). Iso: Functions to Perform Isotonic Regression. R package version 0.0-18. URL <https://cran.r-project.org/package=Iso>

## Examples

```
y <- c( 0.0,61.9,183.3,173.7,250.6,238.1,292.6,293.8,268.0,285.9,258.8,
297.4,217.3,226.4,170.1,74.2,59.8,4.1,6.1 )
x <- unimonotone( y )
print( x )
```

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