Package 'odr'

May 9, 2026

Type Package

Title Optimal Design and Statistical Power for Experimental Studies Investigating Main, Mediation, and Moderation Effects

Version 1.8.3

Date 2026-2-11

Description Calculate the optimal sample size allocation that uses the minimum resources to achieve targeted statistical power in experiments.
Perform power analyses with and without accommodating costs and budget. The designs cover single-level and multilevel experiments detecting main, mediation, and moderation effects (and some combinations).
The references for the proposed methods include:
(1) Shen, Z., & Kelcey, B. (2020).
Optimal sample allocation under unequal costs in cluster-randomized trials.
Journal of Educational and Behavioral Statistics, 45(4): 446-474.
<doi:10.3102/1076998620912418>.
(2) Shen, Z., & Kelcey, B. (2022b). Optimal sample allocation for three-level multisite cluster-randomized trials.
Journal of Research on Educational Effectiveness, 15 (1), 130-150.
<doi:10.1080/19345747.2021.1953200>.
(3) Shen, Z., & Kelcey, B. (2022a). Optimal sample allocation in multisite randomized trials. The Journal of Experimental Education, 90(3), 693-711. <doi:10.1080/00220973.2020.1830361>.
(4) Shen, Z., Leite, W., Zhang, H., Quan, J., & Kuang, H. (2025).
Using ant colony optimization to identify optimal sample allocations in cluster-randomized trials.
The Journal of Experimental Education, 93(1), 167-185.
<doi:10.1080/00220973.2024.2306392>.
(5) Shen, Z., Li, W., & Leite, W. (in press). Statistical power and optimal design for randomized controlled trials investigating mediation effects. Psychological Methods.
<doi:10.1037/met0000698>.
(6) Champely, S. (2020). pwr: Basic functions for power analysis

(Version 1.3-0) [Software]. Available from <https://CRAN.R-project.org/package=pwr>.

Language en-US

Depends R (>= 3.3.0), stats (>= 3.0.0), graphics (>= 3.0.0), base(>= 3.0.0)

License GPL-3

Encoding UTF-8

RoxygenNote 7.3.3

Suggests rmarkdown, knitr, markdown

VignetteBuilder rmarkdown, knitr, markdown

NeedsCompilation no

Author Zuchao Shen [aut, cre] (ORCID: <https://orcid.org/0000-0003-3483-0451>), Benjamin Kelcey [aut]

Maintainer Zuchao Shen <zuchao.shen@gmail.com>

Repository CRAN

Date/Publication 2026-02-11 16:10:03 UTC

Contents

`odr-package` *Optimal Design and Statistical Power for Experimental Studies Investigating Main, Mediation, and Moderation Effects*

Description

This package is to help researchers design cost-efficient and well-powered experimental studies investigating main, mediation, and moderation effects (and some combinations). Specifically, this package can (a) identify optimal sample allocations, (b) compare design efficiency between different sample allocations, and (c) perform power analyses with and without accommodating costs and budget.

Details

The package covers seven types of experiments aiming to detect main, moderation, and mediation effects. These experiments are individual randomized controlled trials (RCTs), two-, three-, and four-level cluster-randomized trials (CRTs), two-, three-, and four-level multisite randomized trials (MRTs). The two categorical functions are 'od' and 'power'. The 'od' function can calculate the optimal sample allocation with and without constraint for each type of experiments. The 'power' function by default can calculate required budget (and required sample size) for desired power, minimum detectable effect size (MDES) under a fixed budget, statistical power under a fixed budget. The 'power' function can perform conventional power analyses (e.g., required sample size, power, MDES calculation). The uniform function 're' (or 'rpe') is to compare the relative (precision and) efficiency between two designs with different sample allocations (limited to main effects).

Author(s)

Zuchao Shen, Benjamin Kelcey

Maintainer: Zuchao Shen zuchao.shen@gmail.com (University of Georgia)

`gen.design.pars` *Generate optimal design parameters using ant colony optimization*

Description

This function can generate a set of optimal design parameters based on given distributions of the rank of optimization target (or budget).

Usage

```
gen.design.pars(
  dist.mean,
  dist.rank,
  n.of.ants,
  nl,
  q = 1e-04,
  n.of.archive = 100,
  xi = 0.5
)
```

Arguments

`dist.mean`	List of means - coordinates
`dist.rank`	Rank of the archived values of the objective function(s)
`n.of.ants`	The number of artificial ants in the search.
`nl`	Neighborhood of the search area
`q`	The locality of the search (0, 1)
`n.of.archive`	The number of the solution archive.
`xi`	The convergence pressure (0, Inf)

Value

Generated optimal design parameter value(s) (i.e., a matrix with n.of.ants rows and n.of.design.pars columns)

References

Socha, K., & Dorigo, M. (2008). Ant colony optimization for continuous domains. European Journal of Operational Research, 185(3), 1155-1173.

We thank Dr. Krzysztof Socha for providing us the original code (https://iridia.ulb.ac.be/supp/IridiaSupp2008-001/) for this function.

od.1 *Optimal sample allocation calculation for single-level experiments detecting main effects*

Description

The optimal design of single-level experiments detecting main effects is to choose the optimal sample allocation that minimizes the variance of a treatment effect under a fixed budget, which is approximately the optimal sample allocation that maximizes statistical power under a fixed budget. The optimal design parameter is the proportion of individuals to be assigned to treatment (p).

Usage

```
od.1(
  p = NULL,
  r12 = NULL,
  c1 = NULL,
  c1t = NULL,
  m = NULL,
  plots = TRUE,
  plim = NULL,
  varlim = NULL,
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  verbose = TRUE
)
```

Arguments

p	The proportion of individuals to be assigned to treatment.
r12	The proportion of outcome variance explained by covariates.
c1	The cost of sampling one unit in control condition.
c1t	The cost of sampling one unit in treatment condition.
m	Total budget, default value is the total costs of sampling 60 individuals across treatment conditions.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
plab	The plot label for p , default value is "Proportion of Individuals in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
vartitle	The title of variance plot, default value is NULL.
verbose	Logical; print the value of p if TRUE, otherwise not; default value is TRUE.

Value

Unconstrained or constrained optimal sample allocation (p). The function also returns the variance of the treatment effect, function name, design type, and parameters used in the calculation.

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.1(r12 = 0.5, c1 = 1, c1t = 5, varlim = c(0, 0.2))
  myod1$out # output

# Constrained p, no calculation performed #---------
  myod2 <- od.1(r12 = 0.5, c1 = 1, c1t = 5, varlim = c(0, 0.2), p = 0.5)
  myod2$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.87

# When sampling costs are equal, a balanced  design with p = 0.5 is the best #---------
  myod3 <- od.1(r12 = 0.5, c1 = 1, c1t = 1, varlim = c(0, 0.2))
  myod3$out # output
```

---

od.1.111 *Optimal sample allocation calculation for single-level randomized controlled trials (RCTs) investigating mediation effects (1-1-1)*

Description

The optimal design of single-level RCTs probing mediation effects is to identify the optimal sample allocation that use the minimum budget to achieve a fixed level of statistical power. The optimal design parameter is the proportion of individuals/units to be assigned to the experimental condition. This function identifies the optimal p.

Usage

```
od.1.111(
  a = NULL,
  b = NULL,
  c1 = NULL,
  c1t = NULL,
  m = NULL,
  r.yx = 0,
  r.mx = 0,
  r.mw = 0,
  q.a = 0,
  q.b = 0,
  test = "joint",
  p = NULL,
```

```
  n = NULL,
  tol = 1e-11,
  power = 0.8,
  d.p = c(0.1, 0.5),
  sig.level = 0.05,
  two.tailed = TRUE,
  plim = c(0.01, 0.99),
  varlim = c(0, 0.001),
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  nlim = c(6, 1e+06),
  verbose = TRUE,
  max.value = Inf,
  max.iter = 300,
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 20,
  q = 1e-04,
  xi = 0.5
)
```

Arguments

a	The treatment effect on the mediator.
b	The within-treatment correlation between the outcome and the mediator.
c1	The cost of sampling an individual in the control group.
c1t	The cost of sampling an individual in the treated group.
m	Total budget.
r.yx	The within-treatment correlation between the outcome and the covariate(s) in the outcome model.
r.mx	The within-treatment correlation between the mediator and the covariate(s) in the outcome model.
r.mw	The within-treatment correlation between the mediator and the covariate(s) in the mediator model.
q.a	The number of covariates at the mediator model (except the treatment indicator), the default value is zero.
q.b	The number of covariates in the outcome model (except the treatment indicator and the mediator), the default value is zero.
test	The type of test will be used to detect mediation effects. The default is the joint significance test (i.e., test = "joint", "Joint","JOINT"). Another choice is the Sobel test by specifying the argument as test = "sobel", "Sobel", or "SOBEL".
p	The proportion of level-4 clusters/units to be assigned to treatment.
n	Total number of individuals in the experimental study, the default value is NULL.
tol	convergence tolerance.

power	Statistical power specified, default is .80.
d.p	The initial sampling domains for p. Default is c(0.10, 0.50).
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Two tailed test, the default value is TRUE.
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
plab	The plot label for p , default value is "Proportion of Individuals in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
vartitle	The title of variance plot, default value is NULL.
nlim	The interval/range used to numerically solve for n, the default values are c(6, 1e7).
verbose	Print out evaluation process if TRUE, default is TRUE.
max.value	Maximal value of optimization when used as the stopping criterion. Default is infinite.
max.iter	Maximal number of function evaluations when used as the stopping criterion. Default is 300.
e	Maximum error value used when solution quality used as the stopping criterion, default is 1e-10.
n.of.ants	Number of ants used in each iteration after the initialization stage, the default value is 10.
n.of.archive	Size of the solution archive, default is 20.
q	Locality of the search (0,1), default is 0.0001.
xi	Convergence pressure (0, Inf), suggested: (0, 1), default is 0.5.

Value

Unconstrained or constrained optimal sample allocation p). The function also returns statistical power, function name, design type, and parameters used in the calculation.

Examples

```
myod <- od.1.111(a = .3, b = .5, c1 = 10, c1t = 100)
myod
```

od.1.111m *Jointly optimal sample allocation identification for single-level randomized controlled trials (RCTs) investigating main and moderation effects (1-1-1m)*

Description

The optimal design of single-level RCTs probing main and moderation effects is to identify the jointly optimal sample allocation that use the minimum budget to achieve targeted statistical power for both effects. The optimal design parameter is the proportion of individuals/units assigned to the experimental condition. This function uses the ant colony optimization algorithm to identify the optimal p.

Usage

```
od.1.111m(
  d = NULL,
  gamma = NULL,
  n = NULL,
  Q = 0.5,
  p = NULL,
  binary = TRUE,
  c1 = NULL,
  c1t = NULL,
  r12 = NULL,
  r.yx = 0,
  r.mx = 0,
  r.ym = 0,
  m = NULL,
  q.main = 1,
  q.mod = 1,
  power.mod = 0.8,
  power.main = 0.8,
  d.p = c(0.1, 0.5),
  sig.level = 0.05,
  two.tailed = TRUE,
  verbose = TRUE,
  nlim = c(20, 1e+07),
  max.value = Inf,
  max.iter = 300,
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 50,
  q = 1e-04,
  xi = 0.5
)
```

Arguments

d	The standardized main effect size.
gamma	Moderated treatment effect.
n	Total number of individuals.
Q	The proportion of individuals in one group the binary moderator. Default value is 0.5, which requires the minimum number of individuals to achieve a targeted power. Change it as necessary.
p	The proportion of individuals assigned to the experimental group.
binary	Logical. The moderator is binary if TRUE and continuous if FALSE. Default is TRUE.
c1	The cost of sampling one unit in control condition.
c1t	The cost of sampling one unit in treatment condition.
r12	The proportion of within-treatment outcome variance explained by covariates in the model that estimated the main effect.
r.yx	Within-treatment correlation between the outcome (y) and the covariate (x) for continuous moderators. Within-treatment within-moderator correlation between the outcome (y) and the covariate (x) for binary moderators.
r.mx	Within-treatment correlation between the moderator (m) and the covariate (x), if specified, for continuous moderators.
r.ym	Within-treatment correlation between the outcome (y) and the moderator (m), if specified, for continuous moderators.
m	Total budget.
q.main	The number of covariates in the model estimating the main effect (besides the treatment, moderator). The default value is 1.
q.mod	The number of covariates in the moderation model (besides the treatment, moderator, and their interaction term). The default value is 1.
power.mod	Statistical power specified for the moderation effect. The default value is .80.
power.main	Statistical power specified for the main effect. The default value is .80.
d.p	The initial sampling domains for p. Default is c(0.10, 0.50).
sig.level	Significance level, default value is .05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
verbose	Print out evaluation process if TRUE. The default value is TRUE.
nlim	The range for identifying the root of sample size (n) numerically. Default is c(20, 1e7).
max.value	Maximal value of optimization when used as the stopping criterion. Default is infinite.
max.iter	Maximal number of function evaluations when used as the stopping criterion. The default value is 300.
e	Maximum error value used when solution quality used as the stopping criterion. The default value is 1e-10.

n.of.ants	Number of ants used in each iteration after the initialization stage. The default value is 10.
n.of.archive	Size of the solution archive, default is 20.
q	Locality of the search (0,1). The default value is 0.0001.
xi	Convergence pressure (0, Inf), suggested: (0, 1). The default value is 0.5.

Value

Unconstrained or constrained optimal sample allocation p). The function also returns statistical power for main and moderation effects, function name, design type, and parameters used in the calculation.

Examples

```
myod <- od.1.111m(d =.1, gamma = .2, r12 = .50,
                  c1 = 10, c1t = 100)
myod
```

---

od.2 *Optimal sample allocation calculation for two-level CRTs detecting main effects*

Description

The optimal design of two-level cluster randomized trials (CRTs) detecting main effects is to calculate the optimal sample allocation that minimizes the variance of a treatment effect under a fixed budget, which is approximately the optimal sample allocation that maximizes statistical power under a fixed budget. The optimal design parameters include the level-1 sample size per level-2 unit (n) and the proportion of level-2 clusters/groups to be assigned to treatment (p). This function solves the optimal n and/or p with and without constraints.

Usage

```
od.2(
  n = NULL,
  p = NULL,
  icc = NULL,
  r12 = NULL,
  r22 = NULL,
  c1 = NULL,
  c2 = NULL,
  c1t = NULL,
  c2t = NULL,
  m = NULL,
  plots = TRUE,
  plot.by = NULL,
  nlim = NULL,
```

```
  plim = NULL,
  varlim = NULL,
  nlab = NULL,
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  verbose = TRUE
)
```

Arguments

n	The level-1 sample size per level-2 unit.
p	The proportion of level-2 clusters/units to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
m	Total budget, default value is the total costs of sampling 60 level-2 units across treatment conditions.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
plot.by	Plot the variance by n and/or p; default value is plot.by = list(n = "n", p = "p").
nlim	The plot range for n, default value is c(2, 50).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
plab	The plot label for p, default value is "Proportion Level-2 Units in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
vartitle	The title of variance plot, default value is NULL.
verbose	Logical; print the values of n and p if TRUE, otherwise not; default value is TRUE.

Value

Unconstrained or constrained optimal sample allocation (n and p). The function also returns the variance of the treatment effect, function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2020). Optimal sample allocation under unequal costs in cluster-randomized trials. Journal of Educational and Behavioral Statistics, 45(4): 446–474. <https://doi.org/10.3102/1076998620912418>

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              varlim = c(0.01, 0.02))
  myod1$out # output
# Plot by p
  myod1 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              varlim = c(0.01, 0.02), plot.by = list(p = 'p'))

# Constrained optimal design with n = 20 #---------
  myod2 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              n = 20, varlim = c(0.005, 0.025))
  myod2$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.88

# Constrained optimal design with p = 0.5 #---------
  myod3 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
             p = 0.5, varlim = c(0.005, 0.025))
  myod3$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod3)
  myre$re # RE = 0.90

# Constrained n and p, no calculation performed #---------
  myod4 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              n = 20, p = 0.5, varlim = c(0.005, 0.025))
  myod4$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod4)
  myre$re # RE = 0.83
```

`od.2.221` *Optimal sample allocation calculation for two-level CRTs probing mediation effects with cluster-level mediators*

Description

The optimal design of two-level cluster randomized trials (CRTs) probing mediation effects with cluster-level mediators is to to identify the optimal sample allocation that requires the minimum budget to achieve certain power level. The optimal design parameters include the level-1 sample size per level-2 unit (`n`) and the proportion of level-2 clusters/groups to be assigned to treatment (`p`). This function solves the optimal `n` and/or `p` with and without a constraint.

Usage

```
od.2.221(
```

```
  a = NULL,
  b = NULL,
  d = NULL,
  n = NULL,
  p = NULL,
  icc = NULL,
  c1 = NULL,
  c1t = NULL,
  c2 = NULL,
  c2t = NULL,
  r12 = NULL,
  r22 = NULL,
  m = NULL,
  r2m = 0,
  r.yx = 0,
  r.mw = 0,
  r.yw = 0,
  q.main = 0,
  q.a = 0,
  q.b = 0,
  test = "joint",
  tol = 1e-11,
  power.med = 0.8,
  power.main = 0.8,
  d.p = c(0.1, 0.5),
  d.n = c(2, 100),
  sig.level = 0.05,
  two.tailed = TRUE,
  Jlim = NULL,
  verbose = TRUE,
  nrange = c(1.5, 10000),
  max.value = Inf,
  max.iter = 300,
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 50,
  q = 1e-04,
  xi = 0.5
)
```

Arguments

a	The treatment effect on the mediator.
b	The within treatment correlation between the outcome and the mediator at the cluster level.
d	Effect size.
n	The level-1 sample size per level-2 unit.
p	The proportion of level-2 clusters/units to be assigned to treatment.

icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
c1	The cost of sampling one level-1 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2	The cost of sampling one level-2 unit in control condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
m	Total budget.
r2m	The proportion of mediator variance explained by covariates in the mediator model.
r.yx	The correlation between the outcome and the covariate at the individual level.
r.mw	The correlation between the mediator and the covariate at the cluster level.
r.yw	The correlation between the outcome and the covariate at the cluster level.
q.main	The number of covariates in the outcome model testing main effects
q.a	The number of covariates in the mediator model (except the treatment indicator).
q.b	The number of covariates in the outcome model at the cluster level (except the treatment indicator and the mediator).
test	The type of test will be used to detect mediation effects. Default is the joint significance test (i.e., test = "joint"). The other choice is the Sobel test by specifying the argument as test = "sobel".
tol	convergence tolerance.
power.med	Statistical power specified for mediation. The default value is .80.
power.main	Statistical power specified for the total/main effect. The default value is .80.
d.p	The initial sampling domains for p. Default is c(0.1, 0.5).
d.n	The initial sampling domain for n. Default is c(2, 100).
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Two tailed test, the default value is TRUE.
Jlim	The range for J to solve for a numerical solution. Default is c(max(q.a, q.b)+4, 1e6).
verbose	Print out evaluation process if TRUE, default is TRUE.
nrange	The range of the individual-level sample size per cluster that used to exclude unreasonable values. Default value is c(1.5, 10000).
max.value	Maximal value of optimization when used as the stopping criterion. Default is -Inf.
max.iter	Maximal number of function evaluations when used as the stopping criterion.
e	Maximum error value used when solution quality used as the stopping criterion, default is 1e-10.
n.of.ants	Number of ants used in each iteration after the initialization of power analysis for calculating required budget, default value is 10.
n.of.archive	Size of the solution archive, default is 100.
q	Locality of the search (0,1), default is 0.0001.
xi	Convergence pressure (0, Inf), suggested: (0, 1), default is 0.5.

Value

Unconstrained or constrained optimal sample allocation (n and p). The function also returns the variance of a mediation effect or statistical power, function name, design type, and parameters used in the calculation.

---

od.2.mod *Optimal sample allocation calculation for two-level CRTs probing moderation effects with cluster-level moderators*

Description

The optimal design of two-level cluster randomized trials (CRTs) probing moderation effects with cluster-level moderators is to to identify the optimal sample allocation that requires the minimum budget to achieve a certain power level. The optimal design parameters include the level-1 sample size per level-2 unit (n) and the proportion of level-2 clusters/groups to be assigned to treatment (p). This function solves the optimal n and/or p with and without a constraint.

Usage

```
od.2.mod(
  d = NULL,
  gamma = NULL,
  n = NULL,
  Q = 0.5,
  p = NULL,
  icc = NULL,
  c1 = NULL,
  c1t = NULL,
  c2 = NULL,
  c2t = NULL,
  r12 = NULL,
  r22 = NULL,
  r12m = NULL,
  r22m = NULL,
  m = NULL,
  binary = TRUE,
  q.main = 1,
  q.mod = 1,
  power.mod = 0.8,
  power.main = 0.8,
  d.p = c(0.1, 0.5),
  d.n = c(2, 1000),
  sig.level = 0.05,
  two.tailed = TRUE,
  Jlim = NULL,
  verbose = TRUE,
```

```
  nrange = c(1.5, 10000),
  max.value = Inf,
  max.iter = 300,
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 50,
  q = 1e-04,
  xi = 0.5
)
```

Arguments

d	The standardized main or average treatment effect.
gamma	The standardized moderated treatment effect (i.e., regression coefficient of the interaction term of moderator and treatment).
n	The level-1 sample size per level-2 unit.
Q	The proportion of binary moderator that coded as 1. Default is 0.50.
p	The proportion of level-2 clusters/units to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
c1	The cost of sampling one level-1 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2	The cost of sampling one level-2 unit in control condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r12m	The proportion of outcome variance at the individual level explained by covariates in the model with the moderator.
r22m	The proportion of outcome variance at the cluster level explained by covariates in the model with the moderator.
m	Total budget.
binary	Logical; The moderator is binary if TRUE, and continuous if FALSE. The default is TRUE.
q.main	The number of covariates in the outcome model testing main effects (except the treatment indicator). The default value is 1.
q.mod	The number of cluster-level covariates in the model (except the treatment indicator, moderator, and the interaction term). The default value is 1.
power.mod	Statistical power specified for moderation. The default value is .80.
power.main	Statistical power specified for the total/main effect. The default value is .80.
d.p	The initial sampling domain for p. Default is c(0.1, 0.5).
d.n	The initial sampling domain for n. Default is c(2, 100).
sig.level	Significance level or type I error rate, default value is 0.05.

two.tailed	Two tailed test, the default value is TRUE.
Jlim	The range for J to solve for a numerical solution. Default is c(max(q.mod, q.main)+7, 1e6).
verbose	Print out evaluation process if TRUE, default is TRUE.
nrange	The range of the individual-level sample size per cluster that used to exclude unreasonable values. Default value is c(1.5, 10000).
max.value	Maximal value of optimization when used as the stopping criterion. Default is -Inf.
max.iter	Maximal number of function evaluations when used as the stopping criterion.
e	Maximum error value used when solution quality used as the stopping criterion, default is 1e-10.
n.of.ants	Number of ants used in each iteration after the initialization of power analysis for calculating required budget, default value is 10.
n.of.archive	Size of the solution archive, default is 100.
q	Locality of the search (0,1), default is 0.0001.
xi	Convergence pressure (0, Inf), suggested: (0, 1), default is 0.5.

Value

Unconstrained or constrained optimal sample allocation (n and p). The function also returns function name, design type, and parameters used in the calculation.

`od.2m` *Optimal sample allocation calculation for two-level MRTs detecting main effects*

Description

The optimal design of two-level multisite randomized trials (MRTs) detecting main effects is to calculate the optimal sample allocation that minimize the budget to achieve a fixed statistical power (e.g., 80 algorithm. Alternatively, the function can calculate the allocation that minimizes the variance of a treatment effect under a fixed budget, which is less precise than the ACO algorithm. The optimal design parameters include the level-one sample size per site (n) and the proportion of level-one unit to be assigned to treatment (p). This function solves the optimal n and/or p with and without a constraint.

Usage

```
od.2m(
  n = NULL,
  p = NULL,
  icc = NULL,
  r12 = NULL,
  r22m = NULL,
```

```
  c1 = NULL,
  c2 = NULL,
  c1t = NULL,
  omega = NULL,
  m = NULL,
  plots = TRUE,
  plot.by = NULL,
  nlim = NULL,
  plim = NULL,
  varlim = NULL,
  nlab = NULL,
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  verbose = TRUE,
  iter = 100,
  tol = 1e-10,
  q = 1,
  d = 0.1,
  power = 0.8,
  aco = TRUE,
  d.p = c(0.1, 0.5),
  d.n = c(2, 1000),
  sig.level = 0.05,
  two.tailed = TRUE,
  Jlim = c(2.5, 1e+10),
  nrange = c(2, 10000),
  max.value = Inf,
  max.iter = 300,
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 50,
  q.aco = 1e-04,
  xi = 0.5
)
```

Arguments

n	The level-1 sample size per level-2 unit.
p	The proportion of level-1 units within each level 2 unit to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22m	The proportion of variance of site-specific treatment effect explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.

c2	The cost of sampling one level-2 unit (site).
c1t	The cost of sampling one level-1 unit in treatment condition.
omega	The standardized variance of site-specific treatment effect.
m	Total budget, default is the total costs of sampling 60 sites.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
plot.by	Plot the variance by n and/or p; default value is plot.by = list(n = "n", p = "p").
nlim	The plot range for n, default value is c(2, 50).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
plab	The plot label for p, default value is "Proportion Level-1 Units in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
vartitle	The title of variance plot, default value is NULL.
verbose	Logical; print the values of n and p if TRUE, otherwise not; default value is TRUE.
iter	Number of iterations; default value is 100.
tol	Tolerance for convergence; default value is 1e-10.
q	The number of covariates at level 2. Default is 1.
d	Standardized effect size, default is 0.1.
power	Statistical power.
aco	Logic. If TRUE, the function will use the ant colony optimization (ACO) algorithm to identify optimal allocations. If FALSE, the function will use the first-order derivative method to identify optimal allocations. Default is TRUE.
d.p	The initial sampling domains for p. Default is c(0.1, 0.5).
d.n	The initial sampling domain for n. Default is c(2, 1000).
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
Jlim	The range for solving the root of level-2 sample size (J) numerically. Change the default values to a larger range (e.g., starting with a smaller value) if f() values at end points are not of opposite sign. For example, use Jlim = c(1.5, 1e10).
nrange	The range of the level-1 sample size per level-2 unit that used to exclude unreasonable values. Default value is c(2, 10000).
max.value	Maximal value of optimization when used as the stopping criterion. Default is -Inf.
max.iter	Maximal number of function evaluations when used as the stopping criterion.
e	Maximum error value used when solution quality used as the stopping criterion, default is 1e-10.
n.of.ants	Number of ants used in each iteration after the initialization of power analysis for calculating required budget, default value is 10.
n.of.archive	Size of the solution archive, default is 100.
q.aco	Locality of the ACO search (0,1), default is 0.0001.
xi	Convergence pressure (0, Inf), suggested: (0, 1), default is 0.5.

Value

Unconstrained or constrained optimal sample allocation (n and p). The function also returns the variance of the treatment effect, function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2022). Optimal sample allocation in multisite randomized trials. The Journal of Experimental Education, 90(3), 693-711. <https://doi.org/10.1080/00220973.2020.1830361>

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.2m(icc = 0.2, omega = 0.02, r12 = 0.5, r22m = 0.5,
              c1 = 1, c2 = 10, c1t = 10,
              varlim = c(0, 0.005))
  myod1$out

# Constrained optimal design with p = 0.5 #---------
  myod2 <- od.2m(icc = 0.2, omega = 0.02,
              r12 = 0.5, r22m = 0.5,
              c1 = 1, c2 = 10, c1t = 10,
              varlim = c(0, 0.005), p = 0.5)
  myod2$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.81

# Constrained optimal design with n = 5 #---------
  myod3 <- od.2m(icc = 0.2, omega = 0.02,
              r12 = 0.5, r22m = 0.5, c1 = 1, c2 = 10,
              c1t = 10, varlim = c(0, 0.005), n = 5)
  myod3$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod3)
  myre$re # RE = 0.78

# Constrained n and p, no calculation performed #---------
  myod4 <- od.2m(icc = 0.2, omega = 0.02, r12 = 0.5, r22m = 0.5,
              c1 = 1, c2 = 10, c1t = 10,
              varlim = c(0, 0.005), p = 0.5, n = 10)
  myod4$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod4)
  myre$re # RE = 0.79
```

---

od.2m.111 *Optimal sample allocation calculation for two-level multisite-randomized trials investigating mediation effects with individual-level mediators (1-1-1)*

Description

The optimal design of two-level multisite-randomized trials (MRTs) probing mediation effects with individual-level mediators, for the Sobel test, is to calculate the optimal sample allocation that minimizes the variance of a mediation effect under a fixed budget. For the joint significance test, it is to identify the optimal sample allocation that requires the minimum budget to achieve certain power level. The optimal design parameters include the level-1 sample size per level-2 unit (n) and the proportion of level-1 individuals/units to be assigned to treatment (p). This function solves the optimal n and/or p with and without a constraint.

Usage

```
od.2m.111(
  a = NULL,
  b = NULL,
  icc.m = NULL,
  icc = NULL,
  c1 = NULL,
  c1t = NULL,
  c2 = NULL,
  m = NULL,
  r12m = 0,
  r22m = 0,
  r12 = 0,
  omega = 0.01,
  q.a = 0,
  q.b = 3,
  test = "joint",
  n = NULL,
  p = NULL,
  iter = 100,
  tol = 1e-11,
  power = 0.8,
  d.p = c(0.1, 0.5),
  d.n = c(5, 50),
  sig.level = 0.05,
  two.tailed = TRUE,
  plots = TRUE,
  nlim = c(4, 100),
  plim = c(0.01, 0.99),
  varlim = c(0, 0.001),
  nlab = NULL,
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  Jlim = c(3, 1e+05),
  verbose = TRUE,
  max.value = Inf,
  max.iter = 300,
```

```
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 50,
  q = 1e-04,
  xi = 0.5,
  plot.by = list(n = "n", p = "p")
)
```

Arguments

a	The treatment effect on the mediator.
b	The within treatment correlation between the outcome and the mediator.
icc.m	The intraclass correlation coefficient for the mediator.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
c1	The cost of sampling one level-1 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2	The cost of sampling one level-2 unit (site).
m	Total budget.
r12m	The proportion of within treatment mediator variance at the level one explained by covariates.
r22m	The proportion of treatment-by-site variance explained by covariates.
r12	The proportion of within treatment individual-level outcome variance explained by covariates.
omega	The treatment-by-site variance of the outcome.
q.a	The number of covariates at the individual level of the mediator model (except the treatment indicator).
q.b	The number of covariates in the outcome model (except the treatment indicator and the mediator).
test	The type of test will be used to detect mediation effects. Default is the joint significance test (i.e., test = "joint"). Another choice is the Sobel test by specifying the argument as test = "sobel".
n	The level-1 sample size per level-2 unit.
p	The proportion of level-1 units to be assigned to treatment.
iter	number of iteration used for solving roots in the Sobel test.
tol	convergence tolerance.
power	Statistical power specified, default is .80.
d.p	The initial sampling domains for p. Default is c(0.10, 0.50).
d.n	The initial sampling domain for n. Default is c(4, 500).
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.

plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
nlim	The plot range for n, default value is c(2, 50).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
plab	The plot label for p, default value is "Proportion Level-3 Units in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
vartitle	The title of variance plot, default value is NULL.
Jlim	The range for J to search for a numerical solution. Default is c(3, 10e4).
verbose	Print out evaluation process if TRUE, default is TRUE.
max.value	Maximal value of optimization when used as the stopping criterion. Default is infinite.
max.iter	Maximal number of function evaluations when used as the stopping criterion. Default is 200.
e	Maximum error value used when solution quality used as the stopping criterion, default is 1e-10.
n.of.ants	Number of ants used in each iteration after the initialization of power analysis for calculating required budget, default value is 10.
n.of.archive	Size of the solution archive, default is 100.
q	Locality of the search (0,1), default is 0.0001.
xi	Convergence pressure (0, Inf), suggested: (0, 1), default is 0.5.
plot.by	Plot the variance by n, J, K and/or p; default value is plot.by = list(n = "n", J = "J", K = 'K', p = "p").

Value

Unconstrained or constrained optimal sample allocation (n and p). The function also returns statistical power, function name, design type, and parameters used in the calculation.

---

od.2m.mod *Optimal sample allocation identification for two-level multisite randomized trials (MRTs) investigating main and moderation effects*

Description

The optimal design of two-level MRTs probing main and moderation effects identify the optimal sample allocations. The optimal design parameters include the level-1 sample size per level-2 unit (n) and the proportion of level-1 individuals/units assigned to the experimental group (p). This function solves the optimal n and/or p with and without a constraint.

Usage

```
od.2m.mod(
  n = NULL,
  p = NULL,
  icc = NULL,
  r12 = NULL,
  r22m = NULL,
  c1 = NULL,
  c2 = NULL,
  c1t = NULL,
  omega = NULL,
  m = NULL,
  plots = TRUE,
  plot.by = list(n = "n", p = "p"),
  verbose = TRUE,
  iter = 100,
  tol = 1e-10,
  q = 1,
  q.mod = 1,
  d = 0.1,
  gamma = 0.1,
  power = 0.8,
  power.mod = 0.8,
  mod.level = 2,
  d.p = c(0.1, 0.5),
  d.n = c(2, 1000),
  sig.level = 0.05,
  two.tailed = TRUE,
  Jlim = c(2.5, 1e+10),
  binary = TRUE,
  nrange = c(2, 10000),
  Q = 0.5,
  max.value = Inf,
  max.iter = 300,
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 50,
  q.aco = 1e-04,
  xi = 0.5
)
```

Arguments

n	The level-1 sample size per level-2 unit.
p	The proportion of level-1 units within each level 2 unit to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.

r12	The proportion of level-1 variance explained by covariates.
r22m	The proportion of variance of site-specific treatment effect explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit (site).
c1t	The cost of sampling one level-1 unit in treatment condition.
omega	The treatment-by-site variance of the outcome.
m	Total budget, default is the total costs of sampling 60 sites.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
plot.by	Plot the variance by n and/or p; default value is plot.by = list(n = "n", p = "p").
verbose	Logical; print the values of n and p if TRUE, otherwise not; default value is TRUE.
iter	Number of iterations; default value is 100.
tol	Tolerance for convergence; default value is 1e-10.
q	The number of covariates in the model detecting the main/average treatment effect. The default is 1.
q.mod	The number of predictors at the moderator level in the moderation model.
d	Standardized effect size, default is 0.1.
gamma	The standardized moderated treatment effect.
power	Statistical power specified for the main effect. The default is .80.
power.mod	Statistical power for the moderation effect. The default is .80.
mod.level	The level of the moderator is at. The moderator is at level 1 if mod.level is 1, and at level 2 if mod.level is 2. The default is mod.level = 1.
d.p	The initial sampling domains for p. Default is c(0.1, 0.5).
d.n	The initial sampling domain for n. Default is c(2, 1000).
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
Jlim	The range for solving the root of level-2 sample size (J) numerically. Change the default values to a larger range (e.g., starting with a smaller value) if f() values at end points are not of opposite sign. For example, use Jlim = c(1.5, 1e10).
binary	Logical; The moderator is binary if TRUE, and continuous if FALSE. The default is TRUE.
nrange	The range of the level-1 sample size per level-2 unit that used to exclude unreasonable values. Default value is c(2, 10000).
Q	The proportion of units in one group for the binary moderator. Default is 0.5.
max.value	Maximal value of optimization when used as the stopping criterion. Default is -Inf.
max.iter	Maximal number of function evaluations when used as the stopping criterion.

e	Maximum error value used when solution quality used as the stopping criterion, default is 1e-10.
n.of.ants	Number of ants used in each iteration after the initialization of power analysis for calculating required budget, default value is 10.
n.of.archive	Size of the solution archive, default is 100.
q.aco	The locality of the search (0, 1)
xi	Convergence pressure (0, Inf), suggested: (0, 1), default is 0.5.

Value

Unconstrained or constrained optimal sample allocation (n and p). The function also returns statistical power formulas, function name, design type, and parameters used in the calculation.

Examples

```
myod <- od.2m.mod(icc = .2, r12 = .5, r22m = .5,
                  c1 = 10, c1t = 100, c2 = 50, omega = .01,
                  gamma = 0.1)
myod$out
```

---

od.2m.only.mod *Using the first-order derivative method to identify the optimal sample allocations for moderation effects in two-level multisite randomized trials (MRTs)*

Description

The optimal design of two-level MRTs probing moderation effects identify the optimal sample allocations. The optimal design parameters include the level-1 sample size per level-2 unit (n) and the proportion of level-1 individuals/units assigned to the experimental group (p). This function solves the optimal n and/or p with and without a constraint using the first-order derivative method to minimize the variance of the moderation effect estimator. It includes binary or continuous moderators at level 2 or level 1.

Usage

```
od.2m.only.mod(
  icc = NULL,
  r12 = NULL,
  r22m = NULL,
  c1 = NULL,
  c1t = NULL,
  c2 = NULL,
  omega = 0.01,
  Q = 0.5,
  n = NULL,
```

```
  p = NULL,
  m = NULL,
  iter = 300,
  binary = TRUE,
  mod.level = 2,
  nlim = c(2, 300),
  plim = c(0.01, 0.99),
  varlim = c(0, 0.005),
  by = c("n", "p"),
  varlab = "Variance",
  nlab = "Level-One Sample Size (n)",
  plab = "Proportion (p)",
  vartitle = ""
)
```

Arguments

icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22m	The proportion of variance of site-specific treatment effect explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2	The cost of sampling one level-2 unit (site).
omega	The standardized variance of site-specific treatment effect.
Q	The proportion of units in one group for the binary moderator. Default is 0.5.
n	The level-1 sample size per level-2 unit.
p	The proportion of level-1 units within each level 2 unit to be assigned to treatment.
m	The total cost to plot the variance curve. The default value is the total cost of sampling 60 sites at the optimal allocation.
iter	Number of iterations; default value is 100.
binary	Logical; The moderator is binary if TRUE, and continuous if FALSE. The default is TRUE.
mod.level	The level of the moderator is at. The moderator is at level 1 if mod.level is 1, and at level 2 if mod.level is 2. The default is mod.level = 1.
nlim	The plot range for n, default value is c(2, 50).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
by	Dimensions to plot power curves by the optimal design parameters. The default value is by all optimal design parameters for a type of design. For example, default values are by = "p" for single-level designs, by = c("n", "p") for two-level designs, and by = c("n", "p", "J") for three-level designs.

varlab	The plot label for variance, default value is "Variance".
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
plab	The plot label for p, default value is "Proportion Level-1 Units in Treatment: p".
vartitle	The title of variance plot, default value is NULL.

Examples

```
myod <- od.2m.only.mod(icc = .2, r12 = .5, r22m = .5,
                       c1 = 10, c1t = 100, c2 = 50, omega = .01)
myod$out
```

---

od.3 *Optimal sample allocation calculation for three-level CRTs detecting main effects*

Description

The optimal design of three-level cluster randomized trials (CRTs) is to calculate the optimal sample allocation that minimizes the variance of treatment effect under fixed budget, which is approximately the optimal sample allocation that maximizes statistical power under a fixed budget. The optimal design parameters include the level-1 sample size per level-2 unit (n), the level-2 sample size per level-3 unit (J), and the proportion of level-3 clusters/groups to be assigned to treatment (p). This function solves the optimal n, J and/or p with and without constraints.

Usage

```
od.3(
  n = NULL,
  J = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32 = NULL,
  c1 = NULL,
  c2 = NULL,
  c3 = NULL,
  c1t = NULL,
  c2t = NULL,
  c3t = NULL,
  m = NULL,
  plots = TRUE,
  plot.by = NULL,
  nlim = NULL,
  Jlim = NULL,
```

```
  plim = NULL,
  varlim = NULL,
  nlab = NULL,
  Jlab = NULL,
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  verbose = TRUE,
  iter = 100,
  tol = 1e-10
)
```

Arguments

n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
p	The proportion of level-3 clusters/units assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32	The proportion of level-3 variance explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c3	The cost of sampling one level-3 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
c3t	The cost of sampling one level-3 unit in treatment condition.
m	Total budget, default is the total costs of sampling 60 level-3 units across treatment conditions.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
plot.by	Plot the variance by n, J and/or p; default is plot.by = list(n = "n", J = "J", p = "p").
nlim	The plot range for n, default value is c(2, 50).
Jlim	The plot range for J, default value is c(2, 50).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
Jlab	The plot label for J, default value is "Level-2 Sample Size: J".
plab	The plot label for p, default is "Proportion Level-3 Units in Treatment: p".
varlab	The plot label for variance, default value is "Variance".

vartitle	The title of variance plot, default value is NULL.
verbose	Logical; print the values of n, J, and p if TRUE, otherwise not; default is TRUE.
iter	Number of iterations; default value is 100.
tol	Tolerance for convergence; default value is 1e-10.

Value

Unconstrained or constrained optimal sample allocation (n, J, and p). The function also returns the variance of the treatment effect, function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2020). Optimal sample allocation under unequal costs in cluster-randomized trials. Journal of Educational and Behavioral Statistics, 45(4): 446–474. <https://doi.org/10.3102/1076998620912418>

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
              varlim = c(0.005, 0.025))
  myod1$out # output
# Plots by p and J
  myod1 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
              varlim = c(0.005, 0.025), plot.by = list(p = 'p', J = 'J'))

# Constrained optimal design with J = 20 #---------
  myod2 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5, J = 20,
              c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
              varlim = c(0, 0.025))
  myod2$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.53

# Constrained optimal design with p = 0.5 #---------
  myod3 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5, p = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
              varlim = c(0.005, 0.025))
  myod3$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod3)
  myre$re # RE = 0.84

# Constrained n, J and p, no calculation performed #---------
 myod4 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5, n = 10, J = 10, p = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
              varlim = c(0, 0.025))
  myod4$out
# Relative efficiency (RE)
```

```
myre <- re(od = myod1, subod= myod4)
myre$re # RE = 0.61
```

---

od.3m *Optimal sample allocation calculation for three-level MRTs detecting main effects*

Description

The optimal design of two-level multisite randomized trials (MRTs) detecting main effects is to calculate the optimal sample allocation that minimize the budget to achieve a fixed statistical power (e.g., 80 algorithm. Alternatively, the function can calculate the optimal allocation that minimizes the variance of a treatment effect under a fixed budget, which is less precise than the ACO algorithm. The optimal design parameters include the level-1 sample size per level-2 unit (n), the level-2 sample size per level-3 unit (J), and the proportion of level-2 unit to be assigned to treatment (p). This function solves the optimal n, J and/or p with and without constraints.

Usage

```
od.3m(
  n = NULL,
  J = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32m = NULL,
  c1 = NULL,
  c2 = NULL,
  c3 = NULL,
  c1t = NULL,
  c2t = NULL,
  omega = NULL,
  m = NULL,
  plots = TRUE,
  plot.by = NULL,
  nlim = NULL,
  Jlim = NULL,
  plim = NULL,
  varlim = NULL,
  nlab = NULL,
  Jlab = NULL,
  plab = NULL,
  varlab = NULL,
  Klim = c(6, 1e+10),
```

```
  q = 1,
  d = 0.1,
  vartitle = NULL,
  verbose = TRUE,
  iter = 100,
  tol = 1e-10,
  power = 0.8,
  ACO = TRUE,
  d.p = c(0.5, 0.9),
  d.n = c(2, 100),
  d.J = c(2, 100),
  sig.level = 0.05,
  two.tailed = TRUE,
  nrange = c(2, 1000),
  Jrange = c(2, 1000),
  max.value = Inf,
  max.iter = 300,
  e = 1e-10,
  n.of.ants = 10,
  n.of.archive = 50,
  q.aco = 1e-04,
  xi = 0.5
)
```

Arguments

n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
p	The proportion of level-2 units within each level-3 site to be assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32m	The proportion of variance of site-specific treatment effect explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit (site).
c3	The cost of sampling one level-3 unit (site).
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
omega	The standardized variance of site-specific treatment effect.
m	Total budget, default is the total costs of sampling 60 level-3 units.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.

plot.by	Plot the variance by n, J and/or p; default value is plot.by = list(n = "n", J = "J", p = "p").
nlim	The plot range for n, default value is c(2, 50).
Jlim	The range for solving the root of level-2 sample size (J) numerically. Change the default values to a larger range (e.g., starting with a smaller value) if f() values at end points are not of opposite sign. For example, use Jlim = c(1.5, 1e10).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
Jlab	The plot label for J, default value is "Level-2 Sample Size: J".
plab	The plot label for p, default value is "Proportion Level-2 Units in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
Klim	The plot range for K, default value is c(2, 50).
q	The number of covariates at level 2. Default is 1.
d	Standardized effect size, default is 0.1.
vartitle	The title of variance plot, default value is NULL.
verbose	Logical; print the values of n, J, and p if TRUE, otherwise not; default value is TRUE.#'
iter	Number of iterations; default value is 100.
tol	Tolerance for convergence; default value is 1e-10.
power	Statistical power.
ACO	Logic. If TRUE, the function will use the ant colony optimization (ACO) algorithm to identify optimal allocations. If FALSE, the function will use the first-order derivative method to identify optimal allocations. Default is TRUE.
d.p	The initial sampling domains for p. Default is c(0.1, 0.5).
d.n	The initial sampling domain for n. Default is c(2, 1000).
d.J	The initial sampling domain for J. Default is c(2, 1000).
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
nrange	The range of the level-1 sample size per level-2 unit that used to exclude unreasonable values. Default value is c(2, 10000).
Jrange	The range of the level-2 sample size per level-3 unit that used to exclude unreasonable values. Default value is c(2, 10000).
max.value	Maximal value of optimization when used as the stopping criterion. Default is -Inf.
max.iter	Maximal number of function evaluations when used as the stopping criterion.
e	Maximum error value used when solution quality used as the stopping criterion, default is 1e-10.

n.of.ants	Number of ants used in each iteration after the initialization of power analysis for calculating required budget, default value is 10.
n.of.archive	Size of the solution archive, default is 100.
q.aco	Locality of the ACO search (0,1), default is 0.0001.
xi	Convergence pressure (0, Inf), suggested: (0, 1), default is 0.5.

Value

Unconstrained or constrained optimal sample allocation (n, J, and p). The function also returns the variance of the treatment effect, function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2022). Optimal sampling ratios in three-level multisite experiments. Journal of Research on Educational Effectiveness, 15(1), 130-150. <https://doi.org/10.1080/19345747.2021.1953200>

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.3m(icc2 = 0.2, icc3 = 0.1, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32m = 0.5,
              c1 = 1, c2 = 5,
              c1t = 1, c2t = 200, c3 = 200,
              varlim = c(0, 0.005))
  myod1$out # output
# Plots by p and J
  myod1 <- od.3m(icc2 = 0.2, icc3 = 0.1, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32m = 0.5,
              c1 = 1, c2 = 5,
              c1t = 1, c2t = 200, c3 = 200,
              varlim = c(0, 0.005), plot.by = list(p = 'p', J = 'J'))

# Constrained optimal design with p = 0.5 #---------
  myod2 <- od.3m(icc2 = 0.2, icc3 = 0.1, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32m = 0.5,
              c1 = 1, c2 = 5,
              c1t = 1, c2t = 200, c3 = 200,
              varlim = c(0, 0.005), p = 0.5)
  myod2$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.81

# Constrained optimal design with n = 5 #---------
  myod3 <- od.3m(icc2 = 0.2, icc3 = 0.1, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32m = 0.5,
              c1 = 1, c2 = 5,
              c1t = 1, c2t = 200, c3 = 200,
              varlim = c(0, 0.005), n = 5)
  myod3$out
# Relative efficiency (RE)
```

```
  myre <- re(od = myod1, subod= myod3)
  myre$re # RE = 0.89

# Constrained n, J and p, no calculation performed #---------
  myod4 <- od.3m(icc2 = 0.2, icc3 = 0.1, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32m = 0.5,
              c1 = 1, c2 = 5,
              c1t = 1, c2t = 200, c3 = 200,
              varlim = c(0, 0.005), p = 0.5, n = 15, J = 20)
  myod4$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod4)
  myre$re # RE = 0.75
```

---

od.4 *Optimal sample allocation calculation for four-level CRTs detecting main effects*

Description

The optimal design of four-level cluster randomized trials (CRTs) is to calculate the optimal sample allocation that minimizes the variance of treatment effect under fixed budget, which is approximately the optimal sample allocation that maximizes statistical power under a fixed budget. The optimal design parameters include the level-1 sample size per level-2 unit (n), the level-2 sample size per level-3 unit (J), the level-3 sample size per level-4 unit (K), and the proportion of level-4 clusters/groups to be assigned to treatment (p). This function solves the optimal n, J, K and/or p with and without constraints.

Usage

```
od.4(
  n = NULL,
  J = NULL,
  K = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  icc4 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32 = NULL,
  r42 = NULL,
  c1 = NULL,
  c2 = NULL,
  c3 = NULL,
  c4 = NULL,
  c1t = NULL,
```

```
  c2t = NULL,
  c3t = NULL,
  c4t = NULL,
  m = NULL,
  plots = TRUE,
  plot.by = NULL,
  nlim = NULL,
  Jlim = NULL,
  Klim = NULL,
  plim = NULL,
  varlim = NULL,
  nlab = NULL,
  Jlab = NULL,
  Klab = NULL,
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  verbose = TRUE,
  iter = 100,
  tol = 1e-10
)
```

Arguments

n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
K	The level-3 sample size per level-4 unit.
p	The proportion of level-4 clusters/units to be assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
icc4	The unconditional intraclass correlation coefficient (ICC) at level 4.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32	The proportion of level-3 variance explained by covariates.
r42	The proportion of level-4 variance explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c3	The cost of sampling one level-3 unit in control condition.
c4	The cost of sampling one level-4 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
c3t	The cost of sampling one level-3 unit in treatment condition.
c4t	The cost of sampling one level-4 unit in treatment condition.

m	Total budget, default value is the total costs of sampling 60 level-4 units across treatment conditions.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
plot.by	Plot the variance by n, J, K and/or p; default value is plot.by = list(n = "n", J = "J", K = 'K', p = "p").
nlim	The plot range for n, default value is c(2, 50).
Jlim	The plot range for J, default value is c(2, 50).
Klim	The plot range for K, default value is c(2, 50).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
Jlab	The plot label for J, default value is "Level-2 Sample Size: J".
Klab	The plot label for K, default value is "Level-3 Sample Size: K".
plab	The plot label for p, default value is "Proportion Level-4 Units in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
vartitle	The title of variance plot, default value is NULL.
verbose	Logical; print the values of n, J, K, and p if TRUE, otherwise not; default value is TRUE.
iter	Number of iterations; default value is 100.
tol	Tolerance for convergence; default value is 1e-10.

Value

Unconstrained or constrained optimal sample allocation (n, J, K, and p). The function also returns the variance of the treatment effect, function name, design type, and parameters used in the calculation.

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
  myod1$out # output
# Plots by p and K
  myod1 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01), plot.by = list(p = 'p', K = 'K'))

# Constrained optimal design with p = 0.5 #---------
  myod2 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, p = 0.5,
```

```
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
  myod2$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.78

# Constrained optimal design with K = 20 #---------
  myod3 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05,  K = 20,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
  myod3$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod3)
  myre$re # RE = 0.67

# Constrained n, J, K and p, no calculation performed #---------
  myod4 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05,
              r12 = 0.5, n = 10, J = 10, K = 20, p = 0.5,
              r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
  myod4$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod4)
  myre$re # RE = 0.27
```

`od.4m` *Optimal sample allocation calculation for four-level MRTs detecting main effects*

Description

The optimal design of four-level multisite randomized trials (MRTs) is to calculate the optimal sample allocation that minimizes the variance of treatment effect under fixed budget, which is approximately the optimal sample allocation that maximizes statistical power under a fixed budget. The optimal design parameters include the level-1 sample size per level-2 unit (`n`), the level-2 sample size per level-3 unit (`J`), the level-3 sample size per level-4 unit (`K`), and the proportion of level-3 units to be assigned to treatment (`p`). This function solves the optimal `n`, `J`, `K` and/or `p` with and without constraints.

Usage

```
od.4m(
```

```
  n = NULL,
  J = NULL,
  K = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  icc4 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32 = NULL,
  r42m = NULL,
  c1 = NULL,
  c2 = NULL,
  c3 = NULL,
  c4 = NULL,
  c1t = NULL,
  c2t = NULL,
  c3t = NULL,
  omega = NULL,
  m = NULL,
  plots = TRUE,
  plot.by = NULL,
  nlim = NULL,
  Jlim = NULL,
  Klim = NULL,
  plim = NULL,
  varlim = NULL,
  nlab = NULL,
  Jlab = NULL,
  Klab = NULL,
  plab = NULL,
  varlab = NULL,
  vartitle = NULL,
  verbose = TRUE,
  iter = 100,
  tol = 1e-10
)
```

Arguments

n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
K	The level-3 sample size per level-4 unit.
p	The proportion of level-3 units to be assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
icc4	The unconditional intraclass correlation coefficient (ICC) at level 4.

r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32	The proportion of level-3 variance explained by covariates.
r42m	The proportion of variance of site-specific treatment effect explained by covariates.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c3	The cost of sampling one level-3 unit in control condition.
c4	The cost of sampling one level-4 unit (site).
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
c3t	The cost of sampling one level-3 unit in treatment condition.
omega	The standardized variance of site-specific treatment effect.
m	Total budget, default is the total costs of sampling 60 level-4 units.
plots	Logical, provide variance plots if TRUE, otherwise not; default value is TRUE.
plot.by	Plot the variance by n, J, K and/or p; default value is plot.by = list(n = "n", J = "J", K = 'K', p = "p").
nlim	The plot range for n, default value is c(2, 50).
Jlim	The plot range for J, default value is c(2, 50).
Klim	The plot range for K, default value is c(2, 50).
plim	The plot range for p, default value is c(0, 1).
varlim	The plot range for variance, default value is c(0, 0.05).
nlab	The plot label for n, default value is "Level-1 Sample Size: n".
Jlab	The plot label for J, default value is "Level-2 Sample Size: J".
Klab	The plot label for K, default value is "Level-3 Sample Size: K".
plab	The plot label for p, default value is "Proportion Level-3 Units in Treatment: p".
varlab	The plot label for variance, default value is "Variance".
vartitle	The title of variance plot, default value is NULL.
verbose	Logical; print the values of n, J, K, and p if TRUE, otherwise not; default value is TRUE.
iter	Number of iterations; default value is 100.
tol	Tolerance for convergence; default value is 1e-10.

Value

Unconstrained or constrained optimal sample allocation (n, J, K, and p). The function also returns the variance of the treatment effect, function name, design type, and parameters used in the calculation.

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.4m(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
              c1 = 1, c2 = 5, c3 = 25,
              c1t = 1, c2t = 50, c3t = 250, c4 = 500,
              varlim = c(0, 0.005))
  myod1$out # output
# Plots by p and K
  myod1 <- od.4m(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
              c1 = 1, c2 = 5, c3 = 25,
              c1t = 1, c2t = 50, c3t = 250, c4 = 500,
              varlim = c(0, 0.005), plot.by = list(p = 'p', K = 'K'))

# Constrained optimal design with p = 0.5 #---------
  myod2 <- od.4m(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
              c1 = 1, c2 = 5, c3 = 25,
              c1t = 1, c2t = 50, c3t = 250, c4 = 500,
              varlim = c(0, 0.005), p = 0.5)
  myod2$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.88

# Constrained optimal design with J = 20 #---------
  myod3 <- od.4m(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
              c1 = 1, c2 = 5, c3 = 25,
              c1t = 1, c2t = 50, c3t = 250, c4 = 500,
              varlim = c(0, 0.005), J = 20)
  myod3$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod3)
  myre$re # RE = 0.58

# Constrained n, J, K and p, no calculation performed #---------
  myod4 <- od.4m(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
              c1 = 1, c2 = 5, c3 = 25,
              c1t = 1, c2t = 50, c3t = 250, c4 = 500,
              varlim = c(0, 0.005), p = 0.5, n = 15, J = 20, K = 5)
  myod4$out
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod4)
  myre$re # RE = 0.46
```

plot.power *Plot statistical power curves under a fixed budget across optimal design parameters*

Description

This function plots statistical power curves (for main, moderation, and/or mediation effects) under a fixed budget across optimal design parameters.

Usage

```
## S3 method for class 'power'
plot(
  expr = NULL,
  nlim = c(2, 300),
  plim = c(0.01, 0.99),
  Jlim = c(3, 300),
  n = NULL,
  p = NULL,
  J = NULL,
  powerlim = c(0, 1),
  plot.title = NULL,
  m = NULL,
  d = NULL,
  gamma = NULL,
  omega = NULL,
  power = 0.8,
  q = NULL,
  by = c("n", "p", "J"),
  legend = TRUE,
  nlab = "Level-One Sample Size (n)",
  plab = "Proportion (p)",
  Jlab = "Level-Two Sample Size (J)",
  powerlab = "Statistical Power"
)
```

Arguments

expr	Returned objects from an od function (e.g., od.2m, od.2m.mod).
nlim	The limits of the level-1 sample size (n) for calculating and plotting power curves.
plim	The limits of the proportion to the treated (p) for calculating and plotting power curves.
Jlim	The limits of the level-2 sample size (J) for calculating and plotting power curves.
n	The sample size at level 1 per level-2 unit.
p	The proportion of units in the treatment condition.

J	Level-2 sample size.
powerlim	The power limit for plotting power curves.
plot.title	The title of the plot (e.g., plot.title = "Power Curves"). The default is NULL.
m	Total budget, default is the total costs of sampling 60 sites.
d	Standardized effect size for a main/average effect.
gamma	Standardized effect size for a moderation effect.
omega	The treatment-by-site variance.
power	Statistical power.
q	The number of covariates.
by	Dimensions to plot power curves by the optimal design parameters. The default value is by all optimal design parameters for a type of design. For example, default values are by = "p" for single-level designs, by = c("n", "p") for two-level designs, and by = c("n", "p", "J") for three-level designs.
legend	Logical; present plot legend if TRUE. The default is TRUE.
nlab	Label for the x-axis when the plot is by the optimal design parameter "n".
plab	Label for the x-axis when the plot is by the optimal design parameter "p".
Jlab	Label for the x-axis when the plot is by the optimal design parameter "J".
powerlab	The label for the statistical power.

`power.1` *Budget and/or sample size, power, MDES calculation for single-level experiments detecting main effects*

Description

This function can calculate required budget for desired power, power or minimum detectable effect size (MDES) under fixed budget for single-level experiments. It also can perform conventional power analyses (e.g., required sample size, power, and MDES calculation).

Usage

```
power.1(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  d = NULL,
  power = NULL,
  m = NULL,
  n = NULL,
  p = NULL,
  r12 = NULL,
```

```
  q = 1,
  c1 = NULL,
  c1t = NULL,
  dlim = NULL,
  powerlim = NULL,
  nlim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	Returned object from function od.1; default value is NULL; if expr is specified, parameter values of r12, c1, c1t, and p used or solved in function od.1 will be passed to the current function; only the value of p that specified or solved in function od.1 can be overwritten if constraint is specified.
constraint	Specify the constrained value of p in list format to overwrite that from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
d	Effect size.
power	Statistical power.
m	Total budget.
n	The total sample size.
p	The proportion of individuals to be assigned to treatment.
r12	The proportion of outcome variance explained by covariates.
q	The number of covariates. Default is 1.
c1	The cost of sampling one unit in control condition.
c1t	The cost of sampling one unit in treatment condition.
dlim	The range for solving the root of effect size (d) numerically, default value is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
nlim	The range for searching the root of sample size (n) numerically, default value is c(4, 10e10).
mlim	The range for searching the root of budget (m) numerically, default value is the costs sampling nlim units across treatment conditions or c(4 * ncost, 10e10 * ncost) with ncost = ((1 - p) * c1 + p * c1t).
rounded	Logical; round p that is from functions od.1 to two decimal places if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (or required sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

Examples

```
# Unconstrained optimal design
  myod1 <- od.1(r12 = 0.5, c1 = 1, c1t = 5, varlim = c(0, 0.2))
  myod1$out   # p = 0.31

# ------- Power analyses by default considering costs and budget -------
# Required budget and sample size
  mym.1 <- power.1(expr = myod1, d = 0.2, q = 1, power = 0.8)
  mym.1$out  # m = 1032 n = 461
  # mym.1$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  mym.1 <- power.1(d = 0.2, power = 0.8, c1 = 1, c1t = 5,
                   r12 = 0.5, p = 0.31, q = 1)
# Required budget and sample size with constrained p
  mym.2 <- power.1(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5))
  mym.2$out  # m = 1183, n = 394

# Power calculation
  mypower <- power.1(expr = myod1, q = 1, d = 0.2, m = 1032)
  mypower$out  # power = 0.80
# Power calculation under constrained p (p = 0.5)
  mypower.1 <- power.1(expr = myod1, q = 1, d = 0.2, m = 1032,
                constraint = list(p = 0.5))
  mypower.1$out  # power = 0.74

# MDES calculation
  mymdes <- power.1(expr = myod1, q = 1, power = 0.80, m = 1032)
  mymdes$out  # d = 0.20


# ------- Conventional power analyses with cost.model = FALSE-------
# Required sample size n
  myn <- power.1(cost.model = FALSE, expr = myod1, d = 0.2, q = 1, power = 0.8)
  myn$out  # n = 461
  # myn$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  myn <- power.1(cost.model = FALSE, d = 0.2, power = 0.8,
                   r12 = 0.5, p = 0.31, q = 1)

# Power calculation
  mypower1 <- power.1(cost.model = FALSE, expr = myod1, n = 461, d = 0.2, q = 1)
  mypower1$out  # power = 0.80

# MDES calculation
  mymdes1 <- power.1(cost.model = FALSE, expr = myod1, n = 461, power = 0.8, q = 1)
```

```
mymdes1$out  # d = 0.20
```

---

power.1.111 *Budget and/or sample size, power, MDES calculation for single-level randomized controlled trials (RCTs) investigating mediation effects*

Description

This function can calculate required budget for desired power and power under a fixed budget for RCTs probing mediation effects. It also can perform conventional power analyses (e.g., required sample size and power calculation).

Usage

```
power.1.111(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  a = NULL,
  b = NULL,
  power = NULL,
  m = NULL,
  test = NULL,
  n = NULL,
  p = NULL,
  c1 = NULL,
  c1t = NULL,
  r.yx = 0,
  r.mx = 0,
  r.mw = 0,
  q.a = 0,
  q.b = 0,
  max.iter = 300,
  alim = c(0, 4),
  blim = c(0.01, 0.99),
  powerlim = NULL,
  nlim = c(6, 1e+07),
  mlim = NULL
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for a desired power, power under fixed budget) if TRUE. Otherwise, conventional power analyses are performed (e.g., required sample size and power calculation); default value is TRUE.

expr	Returned object from function od.1.111; default value is NULL; if expr is specified, parameter values of a, b, c, ct, and p used or solved in function od.1.111 will be passed to the current function; only the values of p that specified or solved in function od.1.111 can be overwritten if constraint is specified.
constraint	If specified, the constrained value of p in a list format (e.g., constraint = list(p = 0.5)) will overwrite that from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Two tailed test, the default value is TRUE.
a	The treatment effect on the mediator.
b	The within-treatment correlation between the outcome and the mediator.
power	Statistical power specified, default is .80.
m	Total budget.
test	The type of test will be used to detect mediation effects. The default is the NULL or the one used in the expr Choices are the joint significance test (i.e., test = "joint", "Joint","JOINT") or the Sobel test (test = "sobel", "Sobel", or "SOBEL").
n	Total number of individuals in the experimental study, the default value is NULL.
p	The proportion of level-4 clusters/units to be assigned to treatment.
c1	The cost of sampling an individual in the control group.
c1t	The cost of sampling an individual in the treated group.
r.yx	The within-treatment correlation between the outcome and the covariate(s) in the outcome model.
r.mx	The within-treatment correlation between the mediator and the covariate(s) in the outcome model.
r.mw	The within-treatment correlation between the mediator and the covariate(s) in the mediator model.
q.a	The number of covariates at the mediator model (except the treatment indicator), the default value is zero.
q.b	The number of covariates in the outcome model (except the treatment indicator and the mediator), the default value is zero.
max.iter	Maximal number of function evaluations when used as the stopping criterion. Default is 300.
alim	The range for identifying the root of a path effect (a) numerically. Default value is c(0, 4).
blim	The range for identifying the root of b path within-treatment correlation between the mediator and outcome (b) numerically. Default value is (.01, .99), if (b) is negative, please re-code the outcome or mediator to make it positive.
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
nlim	The interval/range used to numerically solve for n, the default values are c(6, 1e7).
mlim	The range for identifying the root of budget (m) numerically, default value is the costs sampling nlim units.

Value

Required budget (or required sample size), statistical power, (a) , or (b) depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

Examples

```
# Optimal design and power analyses accommodating costs and budget
myod <- od.1.111(a = .3, b = .5, c1 = 10, c1t = 100)
# myod
mypower <- power.1.111(expr = myod, power = .8)
#mypower

# Conventional power analyses
mypower <- power.1.111(cost.model = FALSE, a = .3, b = .5, test = "joint",
                       power = .8, p =.5)
#mypower
mypower <- power.1.111(cost.model = FALSE, n = 350, b = .5, test = "joint",
                       power = .8, p =.5)
#mypower
```

---

power.1.111m *Budget and/or sample size, power, MDES calculation for single-level randomized controlled trials (RCTs) investigating moderation effects (1-1-1m)*

Description

This function can calculate required budget for desired power, the minimum detectable effect size, and statistical power under a fixed budget in randomized controlled trials (RCTs) probing moderation effects. It also can perform conventional power analyses (e.g., required sample size calculation, minimum detectable effect size calculation, and power calculation).

Usage

```
power.1.111m(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  gamma = NULL,
  binary = TRUE,
  power = NULL,
  m = NULL,
  n = NULL,
  p = NULL,
```

```
  Q = 0.5,
  c1 = NULL,
  c1t = NULL,
  r.yx = 0,
  r.mx = 0,
  r.ym = 0,
  q.mod = 1,
  gammalim = c(0.005, 5),
  powerlim = c(1e-04, 0.9999),
  nlim = c(20, 1e+07),
  mlim = NULL
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for a desired power, power under fixed budget) if TRUE. Otherwise, conventional power analyses are performed (e.g., required sample size and power calculation); default value is TRUE.
expr	Returned object from function od.1.111m; default value is NULL; if expr is specified, parameter values of a, b, c, ct, and p used or solved in function od.1.111m will be passed to the current function; only the values of p that specified or solved in function od.1.111m can be overwritten if constraint is specified.
constraint	If specified, the constrained value of p in a list format (e.g., constraint = list(p = 0.5)) will overwrite that from expr; default value is NULL.
sig.level	Significance level, default value is .05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
gamma	Moderated treatment effect.
binary	Logical. The moderator is binary if TRUE and continuous if FALSE. Default is TRUE.
power	Statistical power.
m	Total budget.
n	Total number of individuals.
p	The proportion of individuals assigned to the experimental group.
Q	The proportion of individuals in one group the binary moderator. Default value is 0.5, which requires the minimum number of individuals to achieve a targeted power. Change it as necessary.
c1	The cost of sampling one unit in control condition.
c1t	The cost of sampling one unit in treatment condition.
r.yx	Within-treatment correlation between the outcome (y) and the covariate (x) for continuous moderators. Within-treatment within-moderator correlation between the outcome (y) and the covariate (x) for binary moderators.

r.mx	Within-treatment correlation between the moderator (m) and the covariate (x), if specified, for continuous moderators.
r.ym	Within-treatment correlation between the outcome (y) and the moderator (m), if specified, for continuous moderators.
q.mod	The number of covariates in the moderation model (besides the treatment, moderator, and their interaction term). The default value is 1.
gammalim	The range for identifying the root of moderation effect size (gamma) numerically, default value is c(0.005, 5).
powerlim	The range for identifying the root of power (power) numerically, default value is c(0.0001, 0.9999).
nlim	The range for identifying the root of sample size (n) numerically. Default is c(20, 1e7).
mlim	The range for identifying the root of budget (m) numerically, default value is the costs sampling nlim units.

Value

Required budget (m) or required sample size (n), statistical power(power), minimum detectable moderation effect size (gamma), depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

Examples

```
# Optimal design and power analyses accommodating costs and budget
myod <- od.1.111m(d =.1, gamma = .2, r12 = .50,
                  c1 = 10, c1t = 100)
myod
N <- power.1.111m(expr = myod, power = .8)
N$out

# Conventional power analyses
# Required sample size for a binary moderator
N <- power.1.111m(cost.model = FALSE, gamma = .2, power = .8, p =.5)
N

# Required sample size for a continuous moderator
N <- power.1.111m(cost.model = FALSE,
                   gamma = .2, power = .8, p =.5, binary = FALSE)
N
```

---

power.2 *Budget and/or sample size, power, MDES calculation for two-level CRTs detecting main effects*

Description

This function can calculate required budget for desired power, power or minimum detectable effect size (MDES) under fixed budget for two-level cluster randomized trials (CRTs). It also can perform conventional power analyses (e.g., required sample size, power, and MDES calculation).

Usage

```
power.2(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  d = NULL,
  power = NULL,
  m = NULL,
  n = NULL,
  J = NULL,
  p = NULL,
  icc = NULL,
  r12 = NULL,
  r22 = NULL,
  q = NULL,
  c1 = NULL,
  c2 = NULL,
  c1t = NULL,
  c2t = NULL,
  dlim = NULL,
  powerlim = NULL,
  Jlim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

`cost.model`	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
`expr`	Returned object from function `od.2`; default is NULL; if `expr` is specified, parameter values of `icc`, `r12`, `r22`, `c1`, `c2`, `c1t`, `c2t`, `n`, and `p` used or solved in function `od.2` will be passed to the current function; only the values of `n` and `p` that specified or solved in function `od.2` can be overwritten if `constraint` is specified.
`constraint`	Specify the constrained values of `n` and/or `p` in list format to overwrite those from `expr`; default is NULL.
`sig.level`	Significance level or type I error rate, default value is 0.05.

two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
d	Effect size.
power	Statistical power.
m	Total budget.
n	The level-1 sample size per level-2 unit.
J	The total level-2 sample size.
p	The proportion of level-2 clusters/units to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
q	The number of level-2 covariates.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
dlim	The range for solving the root of effect size (d) numerically, default value is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Jlim	The range for searching the root of level-2 sample size (J) numerically, default is c(4, 10e10).
mlim	The range for searching the root of budget (m) numerically, default is the costs sampling Jlim level-2 units across treatment conditions or c(4 * Jcost, 10e10 * Jcost), with Jcost = ((1 - p) * (c1 * n + c2) + p * (c1t * n + c2t)).
rounded	Logical; round n and p that are from functions od.2 to integer and two decimal places, respectively if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (and/or required level-2 sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2020). Optimal sample allocation under unequal costs in cluster-randomized trials. Journal of Educational and Behavioral Statistics, 45(4): 446–474. <https://doi.org/10.3102/1076998620912418>

Examples

```
# Unconstrained optimal design
  myod1 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50)
  myod1$out   # n = 8.9, p = 0.33

# ------- Power analyses by default considering costs and budget -------
# Required budget and sample size
  mym.1 <- power.2(expr = myod1, d = 0.2, q = 1, power = 0.8)
  mym.1$out  # m = 3755, J = 130.2
  #mym.1$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  mym.1 <- power.2(d = 0.2, power = 0.8, icc = 0.2,
                c1 = 1, c2 = 5, c1t = 1, c2t = 50,
                 r12 = 0.5, r22 = 0.5, n = 9, p = 0.33, q = 1)
# Required budget and sample size with constrained p
  mym.2 <- power.2(expr = myod1, d = 0.2, q = 1, power = 0.8,
              constraint = list(p = 0.5))
  mym.2$out  # m = 4210, J = 115.3
# Required budget and sample size with constrained p and n
  mym.3 <- power.2(expr = myod1, d = 0.2, q = 1, power = 0.8,
              constraint = list(p = 0.5, n = 20))
  mym.3$out  # m = 4568, J = 96.2

# Power calculation
  mypower <- power.2(expr = myod1, q = 1, d = 0.2, m = 3755)
  mypower$out  # power = 0.80
# Power calculation under constrained p (p = 0.5)
  mypower.1 <- power.2(expr = myod1, q = 1, d = 0.2, m = 3755,
              constraint = list(p = 0.5))
  mypower.1$out  # power = 0.75

# MDES calculation
  mymdes <- power.2(expr = myod1, q = 1, power = 0.80, m = 3755)
  mymdes$out  # d = 0.20


# ------- Conventional power analyses with cost.model = FALSE-------
# Required J
  myJ <- power.2(cost.model = FALSE, expr = myod1, d = 0.2, q = 1, power = 0.8)
  myJ$out  # J = 130.2
  #myJ$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  myJ <- power.2(cost.model = FALSE, d = 0.2, power = 0.8, icc = 0.2,
                 r12 = 0.5, r22 = 0.5, n = 9, p = 0.33, q = 1)

# Power calculation
  mypower1 <- power.2(cost.model = FALSE, expr = myod1, J = 130, d = 0.2, q = 1)
  mypower1$out  # power = 0.80

# MDES calculation
  mymdes1 <- power.2(cost.model = FALSE, expr = myod1, J = 130, power = 0.8, q = 1)
  mymdes1$out  # d = 0.20
```

power.2.221 *Budget and/or sample size, power calculation for CRTs probing mediation effects with cluster-level mediators*

Description

This function can calculate required budget for desired power and power under a fixed budget for experimental studies with group mediators probing mediation effects. It also can perform conventional power analyses (e.g., required sample size and power calculation).

Usage

```
power.2.221(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  a = NULL,
  b = NULL,
  test = "joint",
  n = NULL,
  p = NULL,
  power = NULL,
  J = NULL,
  m = NULL,
  c1 = NULL,
  c1t = NULL,
  c2 = NULL,
  c2t = NULL,
  r2m = r2m,
  r.yx = 0,
  r.mw = 0,
  r.yw = 0,
  icc = NULL,
  q = 0,
  q.a = 0,
  q.b = 0,
  powerlim = NULL,
  Jlim = NULL,
  mlim = NULL
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	returned object from function od.2.221; default value is NULL; if expr is specified, parameter values of a, b, c1, c1t, and p used or solved in function od.2.221 will be passed to the current function; only the values of p and n that specified or solved in function od.2.221 can be overwritten if constraint is specified.
constraint	specify the constrained value of p and/or n in a list format to overwrite that/those from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
a	The treatment effect on the mediator.
b	The within treatment correlation between the outcome and the mediator at the cluster level.
test	The type of test will be used to detect mediation effects. Default is the joint significance test (i.e., test = "joint"). The other choice is the Sobel test by specifying the argument as test = "sobel".
n	The level-1 sample size per level-2 unit.
p	The proportion of level-2 clusters/units to be assigned to treatment.
power	Statistical power.
J	The total level-2 sample size.
m	Total budget.
c1	The cost of sampling one level-1 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2	The cost of sampling one level-2 unit in control condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
r2m	The proportion of mediator variance explained by covariates in the mediator model.
r.yx	The correlation between the outcome and the covariate at the individual level.
r.mw	The correlation between the mediator and the covariate at the cluster level.
r.yw	The correlation between the outcome and the covariate at the cluster level.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
q	The number of level-2 covariates.
q.a	The number of covariates in the mediator model (except the treatment indicator).
q.b	The number of covariates in the outcome model at the cluster level (except the treatment indicator and the mediator).

powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Jlim	The range for searching the root of level-2 sample size (J) numerically, default is c(4, 10e10).
mlim	the range for searching the root of budget (m) numerically, default value is the costs sampling nlim units across treatment conditions or c(4 * ncost, 10e10 * ncost) with ncost = ((1 - p) * c1 + p * c1t)

Value

Required budget (or required sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

---

power.2.mod *Statistical power, sample size (and/or budget), minimum detectable moderator effect size calculation for two-level cluster-randomized trials (CRTs) detecting moderation effects*

Description

This function can calculate power, required sample size/budget for desired power, or minimum detectable moderation effect size (MDMES) under a fixed budget in two-level CRTs. It also can perform conventional power analyses (e.g., required sample size, power, and MDMES calculation).

Usage

```
power.2.mod(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  gamma = NULL,
  power.mod = NULL,
  m = NULL,
  n = NULL,
  J = NULL,
  p = NULL,
  icc = NULL,
  r12 = NULL,
  r22 = NULL,
  r12m = NULL,
  r22m = NULL,
  q.mod = 1,
  c1 = NULL,
```

```
  c2 = NULL,
  c1t = NULL,
  c2t = NULL,
  gammalim = c(0, 5),
  powerlim = c(1e-10, 1 - 1e-10),
  Jlim = c(5.5, 1e+10),
  binary = TRUE,
  mlim = NULL,
  rounded = TRUE,
  Q = 0.5
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	Returned objects from function od.2.mod; default is NULL; if expr is specified, parameter values of icc, r12, r22, r12m, r22m, c1, c2, c1t, c2t, p, and n used or solved in function od.2.mod will be passed to the current function; only the values of p and n that specified or solved in function od.2.mod can be overwritten if constraint is specified.
constraint	Specify the constrained values of p and/or n in list format to overwrite those from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Two tailed test, the default value is TRUE.
gamma	The standardized moderated treatment effect (i.e., regression coefficient of the interaction term of moderator and treatment).
power.mod	Statistical power specified for moderation. The default value is .80.
m	Total budget.
n	The level-1 sample size per level-2 unit.
J	The total level-2 sample size.
p	The proportion of level-2 clusters/units to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r12m	The proportion of outcome variance at the individual level explained by covariates in the model with the moderator.
r22m	The proportion of outcome variance at the cluster level explained by covariates in the model with the moderator.
q.mod	The number of cluster-level covariates in the model (except the treatment indicator, moderator, and the interaction term). The default value is 1.

c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
gammalim	The range for numerically solving the root of standardized moderation effect (gamma). Default is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Jlim	The range for numerically solving the root of the sample size requirement(J).
binary	Logical; The moderator is binary if TRUE, and continuous if FALSE. The default is TRUE.
mlim	The range for numerically solving the root of budget (m). The default is NULL, which mlim = Jlim times the costs for each site and its members.
rounded	Logical; round n and p that are from functions od.2 to integer and two decimal places, respectively if TRUE, otherwise no rounding; default value is TRUE.
Q	The proportion of binary moderator that coded as 1. Default is 0.50.
power	Statistical power.mod for a moderation effect.

Value

Required budget (and/or required level-2 sample size), statistical power, or MDMES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

Examples

```
myod <- od.2.mod(icc = .2, r12 = .5, r22 = .5,
                 c1 = 10, c1t = 100, c2 = 50, c2t = 500,
                 gamma = 0.2, d = 0.2)
mypower <- power.2.mod(expr = myod, m=myod$out$m, gamma = 0.2); mypower$out
mym <- power.2.mod(expr = myod, power.mod = .80, gamma = 0.2); mym$out
```

---

power.2m *Budget and/or sample size, power, MDES calculation for two-level MRTs detecting main effects*

Description

This function can calculate required budget for desired power, power or minimum detectable effect size (MDES) under fixed budget for two-level multisite randomized trials (MRTs). It also can perform conventional power analyses (e.g., required sample size, power, and MDES calculation).

Usage

```
power.2m(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  d = NULL,
  power = NULL,
  m = NULL,
  n = NULL,
  J = NULL,
  p = NULL,
  icc = NULL,
  r12 = NULL,
  r22m = NULL,
  q = 1,
  c1 = NULL,
  c2 = NULL,
  c1t = NULL,
  omega = NULL,
  dlim = NULL,
  powerlim = NULL,
  Jlim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	Returned objects from function od.2m or od.2m.mod; default is NULL; if expr is specified, parameter values of icc, r12, r22m, c1, c2, c1t, p, and n used or solved in function od.2m will be passed to current function; only the values of p and n that specified or solved in function od.2m can be overwritten if constraint is specified.
constraint	Specify the constrained values of p and/or n in list format to overwrite those from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
d	Effect size.
power	Statistical power.

m	Total budget.
n	The level-1 sample size per level-2 unit.
J	The number of sites.
p	The proportion of level-1 units to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
r12	The proportion of level-1 variance explained by covariates.
r22m	The proportion of variance of site-specific treatment effect explained by covariates.
q	The number of covariates at level 2.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit (site).
c1t	The cost of sampling one level-1 unit in treatment condition.
omega	The standardized variance of site-specific treatment effect.
dlim	The range for solving the root of effect size (d) numerically, default value is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Jlim	The range for searching the root of level-2 sample size (J) numerically, default is c(4, 10e10).
mlim	The range for searching the root of budget (m) numerically, default is the costs sampling Jlim level-2 units or c(4 * Jcost, 1e+10 * Jcost) with Jcost = (1 - p) * c1 * n + p * c1t * n + c2.
rounded	Logical; round the values of p, n/J/K that are from functions od.4 to two decimal places and integer, respectively if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (and/or required level-2 sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2022). Optimal sample allocation in multisite randomized trials. The Journal of Experimental Education, 90(3), 693-711. <https://doi.org/10.1080/00220973.2020.1830361

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.2m(icc = 0.2, omega = 0.02, r12 = 0.5, r22m = 0.5,
              c1 = 1, c2 = 10, c1t = 10,
              varlim = c(0, 0.005))
  myod1$out # n = 23.5, p = 0.24
```

```
# ------- Power analyses by default considering costs and budget -------
# Required budget and sample size
  mym.1 <- power.2m(expr = myod1, d = 0.2, q = 1, power = 0.8)
  mym.1$out  # m = 1882, J = 22.8
  # mym.1$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  mym.1 <- power.2m(d = 0.2, power = 0.8, q = 1,
                icc = 0.2, omega = 0.02, r12 = 0.5, r22m = 0.5,
                c1 = 1, c2 = 10, c1t = 10,
                n = 23.5, p = 0.24)
# Required budget and sample size with constrained p
  mym.2 <- power.2m(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5))
  mym.2$out  # m = 2424, J = 18
# Required budget and sample size with constrained p and n
  mym.3 <- power.2m(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5, n = 5))
  mym.3$out  # m = 2502, J = 66.7

# Power calculation
  mypower <- power.2m(expr = myod1, q = 1, d = 0.2, m = 1882)
  mypower$out  # power = 0.80
# Power calculation under constrained p (p = 0.5)
  mypower.1 <- power.2m(expr = myod1, q = 1, d = 0.2, m = 1882,
                constraint = list(p = 0.5))
  mypower.1$out  # power = 0.68

# MDES calculation
  mymdes <- power.2m(expr = myod1, q = 1, power = 0.80, m = 1882)
  mymdes$out  # d = 0.20


# ------- Conventional power analyses with cost.model = FALSE-------
# Required sample size
  myJ <- power.2m(cost.model = FALSE, expr = myod1, d = 0.2,
  q = 1, power = 0.8)
  myJ$out  # J = 22.8
  # myL$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  myJ <- power.2m(cost.model = FALSE, d = 0.2, power = 0.8, q = 1,
                icc = 0.2, omega = 0.02, r12 = 0.5, r22m = 0.5,
                c1 = 1, c2 = 10, c1t = 10,
                n = 23.5, p = 0.24)

# Power calculation
  mypower1 <- power.2m(cost.model = FALSE, expr = myod1, J = 22.8,
  d = 0.2, q = 1)
  mypower1$out  # power = 0.80

# MDES calculation
  mymdes1 <- power.2m(cost.model = FALSE, expr = myod1, J = 22.8,
  power = 0.8, q = 1)
```

```
mymdes1$out  # d = 0.20
```

`power.2m.111` *Budget and/or sample size, power, MDES calculation for MRTs investigating mediation effects with individual-level mediators*

Description

This function can calculate required budget for desired power and power under a fixed budget for multisite-randomized trials (MRTs) with individual mediators probing mediation effects. It also can perform conventional power analyses (e.g., required sample size and power calculation).

Usage

```
power.2m.111(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  a = NULL,
  b = NULL,
  power = NULL,
  m = NULL,
  test = NULL,
  n = NULL,
  p = NULL,
  c1 = NULL,
  c1t = NULL,
  c2 = NULL,
  r12 = 0,
  r22m = 0,
  r12m = 0,
  icc.m = NULL,
  omega = NULL,
  icc = NULL,
  J = NULL,
  q = 0,
  q.a = 0,
  q.b = 0,
  max.iter = 300,
  powerlim = NULL,
  Jlim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	returned object from function od.2m.111; default value is NULL; if expr is specified, parameter values of a, b, c1, c1t, and p used or solved in function od.2m.111 will be passed to the current function; only the values of p and n that specified or solved in function od.2m.111 can be overwritten if constraint is specified.
constraint	specify the constrained value of p and/or n in a list format to overwrite that/those from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
a	The treatment effect on the mediator.
b	The within treatment correlation between the outcome and the mediator.
power	Statistical power.
m	Total budget.
test	The type of test will be used to detect mediation effects. Default is the joint significance test (i.e., test = "joint"). Another choice is the Sobel test by specifying the argument as test = "sobel".
n	The level-1 sample size per level-2 unit.
p	The proportion of level-1 units to be assigned to treatment.
c1	The cost of sampling one level-1 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2	The cost of sampling one level-2 unit (site).
r12	The proportion of level-1 variance explained by covariates.
r22m	The proportion of variance of site-specific treatment effect explained by covariates.
r12m	The proportion of within treatment mediator variance at the level one explained by covariates.
icc.m	The intraclass correlation coefficient for the mediator.
omega	The standardized variance of site-specific treatment effect.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
J	The number of sites.
q	The number of covariates at level 2.
q.a	The number of covariates at the individual level of the mediator model (except the treatment indicator).

q.b	The number of covariates in the outcome model (except the treatment indicator and the mediator).
max.iter	Maximal number of function evaluations when used as the stopping criterion. Default is 200.
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Jlim	The range for searching the root of level-2 sample size (J) numerically, default is c(4, 10e10).
mlim	the range for searching the root of budget (m) numerically, default value is the costs sampling nlim units.
rounded	Logical; round the values of p, n/J/K that are from functions od.4 to two decimal places and integer, respectively if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (or required sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

---

power.2m.mod *Statistical power, sample size (and/or budget), minimum detectable moderator effect size calculation for two-level multisite randomized trials (MRTs) detecting moderation effects*

Description

This function can calculate power, required sample size/budget for desired power, or minimum detectable moderation effect size (MDMES) under a fixed budget in two-level MRTs. It also can perform conventional power analyses (e.g., required sample size, power, and MDMES calculation).

Usage

```
power.2m.mod(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  omega = NULL,
  gamma = NULL,
  power.mod = NULL,
  m = NULL,
  n = NULL,
  J = NULL,
  p = NULL,
```

```
  icc = NULL,
  r12 = NULL,
  r22m = NULL,
  q.mod = 1,
  c1 = NULL,
  c2 = NULL,
  c1t = NULL,
  gammalim = c(0, 5),
  powerlim = c(1e-10, 1 - 1e-10),
  Jlim = c(2.5, 1e+10),
  mod.level = 2,
  binary = TRUE,
  mlim = NULL,
  rounded = TRUE,
  Q = 0.5
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	Returned objects from function od.2m.mod; default is NULL; if expr is specified, parameter values of icc, r12, r22m, c1, c2, c1t, p, and n used or solved in function od.2m.mod will be passed to the current function; only the values of p and n that specified or solved in function od.2m.mod can be overwritten if constraint is specified.
constraint	Specify the constrained values of p and/or n in list format to overwrite those from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
omega	The treatment-by-site variance of the outcome.
gamma	The standardized moderated treatment effect.
power.mod	Statistical power for the moderation effect. The default is .80.
m	Total budget, default is the total costs of sampling 60 sites.
n	The level-1 sample size per level-2 unit.
J	The number of sites.
p	The proportion of level-1 units within each level 2 unit to be assigned to treatment.
icc	The unconditional intraclass correlation coefficient (ICC) in population or in each treatment condition.
r12	The proportion of level-1 variance explained by covariates.

r22m	The proportion of variance of site-specific treatment effect explained by covariates.
q.mod	The number of predictors at the moderator level in the moderation model.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit (site).
c1t	The cost of sampling one level-1 unit in treatment condition.
gammalim	The range for numerically solving the root of standardized moderation effect (gamma). Default is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Jlim	The range for solving the root of level-2 sample size (J) numerically. Change the default values to a larger range (e.g., starting with a smaller value) if f() values at end points are not of opposite sign. For example, use Jlim = c(1.5, 1e10).
mod.level	The level of the moderator is at. The moderator is at level 1 if mod.level is 1, and at level 2 if mod.level is 2. The default is mod.level = 1.
binary	Logical; The moderator is binary if TRUE, and continuous if FALSE. The default is TRUE.
mlim	The range for numerically solving the root of budget (m). The default is NULL, which mlim = Jlim times the costs for each site and its members.
rounded	Logical; round the values of p, n/J/K that are from functions od.4 to two decimal places and integer, respectively if TRUE, otherwise no rounding; default value is TRUE.
Q	The proportion of units in one group for the binary moderator. Default is 0.5.
power	Statistical power.mod for a moderation effect.

Value

Required budget (and/or required level-2 sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

Examples

```
myod <- od.2m.mod(icc = .2, r12 = .5, r22m = .5,
                  c1 = 10, c1t = 100, c2 = 50,
                  omega = .01, gamma = 0.1)
mypower <- power.2m.mod(expr = myod, m=myod$out$m, gamma = 0.1); mypower$out
mym <- power.2m.mod(expr = myod, power.mod = .80, gamma = 0.1); mym$out
myod <- od.2m.only.mod(icc = .2, r12 = .5, r22m = .5,
                       c1 = 10, c1t = 100, c2 = 50, omega = .01)
mypower <- power.2m.mod(expr = myod, power.mod = .8, gamma = 0.1)
mypower$out
```

power.3 *Budget and/or sample size, power, MDES calculation for three-level CRTs detecting main effects*

Description

This function can calculate required budget for desired power, power or minimum detectable effect size (MDES) under fixed budget for three-level cluster randomized trials (CRTs). It also can perform conventional power analyses (e.g., required sample size, power, and MDES calculation).

Usage

```
power.3(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  d = NULL,
  power = NULL,
  m = NULL,
  n = NULL,
  J = NULL,
  K = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32 = NULL,
  q = NULL,
  c1 = NULL,
  c2 = NULL,
  c3 = NULL,
  c1t = NULL,
  c2t = NULL,
  c3t = NULL,
  dlim = NULL,
  powerlim = NULL,
  Klim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise

	conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	Returned objects from function od.3; default is NULL; if expr is specified, parameter values of icc2, icc3, r12, r22, r32, c1, c2, c3, c1t, c2t, c3t, p, n, and J used or solved in function od.3 will be passed to the current function; only the values of p, n, and/or J that specified or solved in function od.3 can be overwritten if constraint is specified.
constraint	Specify the constrained values of p, n, and/or J in list format to overwrite those from expr; default is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
d	Effect size.
power	Statistical power.
m	Total budget.
n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
K	The total level-3 sample size.
p	The proportion of level-3 clusters/units assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32	The proportion of level-3 variance explained by covariates.
q	The number of covariates at level 3.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c3	The cost of sampling one level-3 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
c3t	The cost of sampling one level-3 unit in treatment condition.
dlim	The range for solving the root of effect size (d) numerically, default value is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Klim	The range for searching the root of level-3 sample size (K) numerically, default value is c(4, 1e+10).
mlim	The range for searching the root of budget (m) numerically, default value is the costs sampling Klim level-3 units across treatment conditions or c(4 * Kcost, 1e+10 * Kcost) with Kcost = ((1 - p) * (c1 * n * J + c2 * J + c3) + p * (c1t * n * J + c2t * J + c3t)).
rounded	Logical; round the values of p, n/J that are from functions od.3 to two decimal places and integer, respectively if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (and/or required level-3 sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2020). Optimal sample allocation under unequal costs in cluster-randomized trials. Journal of Educational and Behavioral Statistics, 45(4): 446–474. <https://doi.org/10.3102/1076998620912418>

Examples

```
# Unconstrained optimal design
  myod1 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250)
  myod1$out # output  # n = 7.9, J = 3.2, p = 0.28

# ------- Power analyses by default considering costs and budget -------
# Required budget and sample size
  mym.1 <- power.3(expr = myod1, d = 0.2, q = 1, power = 0.8)
  mym.1$out  # m = 16032, K = 97.3
  #mym.1$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  mym.1 <- power.3(d = 0.2, power = 0.8, q = 1,
                 icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5,
                 c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
                 n = 8, J = 3, p = 0.28)
# Required budget and sample size with constrained p
  mym.2 <- power.3(expr = myod1, d = 0.2, q = 1, power = 0.8,
                 constraint = list(p = 0.5))
  mym.2$out  # m = 19239, K = 78.8
# Required budget and sample size with constrained p and J
  mym.3 <- power.3(expr = myod1, d = 0.2, q = 1, power = 0.8,
                 constraint = list(p = 0.5, J = 20))
  mym.3$out  # m = 39774, K = 46.9

# Power calculation
  mypower <- power.3(expr = myod1, q = 1, d = 0.2, m = 16032)
  mypower$out  # power = 0.80
# Power calculation under constrained p (p = 0.5)
  mypower.1 <- power.3(expr = myod1, q = 1, d = 0.2, m = 16032,
                 constraint = list(p = 0.5))
  mypower.1$out  # power = 0.72

# MDES calculation
  mymdes <- power.3(expr = myod1, q = 1, power = 0.80, m = 16032)
  mymdes$out  # d = 0.20


# ------- Conventional power analyses with cost.model = FALSE-------
# Required sample size
  myK <- power.3(cost.model = FALSE, expr = myod1, d = 0.2, q = 1, power = 0.8)
```

```
  myK$out  # K = 97.3
  #myK$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  myK <- power.3(cost.model = FALSE, d = 0.2, power = 0.8, q = 1,
                 icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5,
                 n = 8, J = 3, p = 0.28)

# Power calculation
  mypower1 <- power.3(cost.model = FALSE, expr = myod1, K = 97, d = 0.2, q = 1)
  mypower1$out  # power = 0.80

# MDES calculation
  mymdes1 <- power.3(cost.model = FALSE, expr = myod1, K = 97, power = 0.8, q = 1)
  mymdes1$out  # d = 0.20
```

`power.3m` *Budget and/or sample size, power, MDES calculation for three-level MRTs detecting main effects*

Description

This function can calculate required budget for desired power, power or minimum detectable effect size (MDES) under fixed budget for three-level multisite randomized trials (MRTs). It also can perform conventional power analyses (e.g., required sample size, power, and MDES calculation).

Usage

```
power.3m(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  d = NULL,
  power = NULL,
  m = NULL,
  n = NULL,
  J = NULL,
  K = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32m = NULL,
  q = NULL,
  c1 = NULL,
```

```
  c2 = NULL,
  c3 = NULL,
  c1t = NULL,
  c2t = NULL,
  omega = NULL,
  dlim = NULL,
  powerlim = NULL,
  Klim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	Returned objects from function od.3m; default is NULL; if expr is specified, parameter values of icc2, icc3, r12, r22, r32m, c1, c2, c3, c1t, c2t, p, n, and J used or solved in function od.3m will be passed to current function; only the values of p, n, and/or J that specified or solved in function od.3m can be overwritten if constraint is specified.
constraint	Specify the constrained values of p, n, and/or J, in list format to overwrite those from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
d	Effect size.
power	Statistical power.
m	Total budget.
n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
K	The level-3 sample size per level-4 unit.
p	The proportion of level-2 units to be assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32m	The proportion of variance of site-specific treatment effect explained by covariates.
q	The number of covariates at level 3.
c1	The cost of sampling one level-1 unit in control condition.

c2	The cost of sampling one level-2 unit in control condition.
c3	The cost of sampling one level-3 unit (site).
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
omega	The standardized variance of site-specific treatment effect.
dlim	The range for solving the root of effect size (d) numerically, default value is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Klim	The range for searching the root of level-3 sample size (K) numerically, default value is c(4, 1e+10).
mlim	The range for searching the root of budget (m) numerically, default is the costs sampling Klim level-3 units or c(4 * Kcost, 1e+10 * Kcost) with Kcost = ((1 - p) * (c1 * n * J + c2 * J) + p * (c1t * n * J + c2t * J) + c3.
rounded	Logical; round the values of p, n/J/K that are from functions od.4 to two decimal places and integer, respectively if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (and/or required level-3 sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

References

Shen, Z., & Kelcey, B. (2022). Optimal sampling ratios in three-level multisite experiments. Journal of Research on Educational Effectiveness, 15(1), 130-150. <https://doi.org/10.1080/19345747.2021.1953200>

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.3m(icc2 = 0.2, icc3 = 0.1, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32m = 0.5,
              c1 = 1, c2 = 5,
              c1t = 1, c2t = 200, c3 = 200,
              varlim = c(0, 0.005))
  myod1$out # n = 13.1, J = 15.3, p = 0.23

# ------- Power analyses by default considering costs and budget -------
# Required budget and sample size
  mym.1 <- power.3m(expr = myod1, d = 0.2, q = 1, power = 0.8)
  mym.1$out  # m = 15491, K = 13.6
  # mym.1$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  mym.1 <- power.3m(d = 0.2, power = 0.8, q = 1,
                 icc2 = 0.2, icc3 = 0.1, omega = 0.02,
                  r12 = 0.5, r22 = 0.5, r32m = 0.5,
```

```
                 c1 = 1, c2 = 5,
                 c1t = 1, c2t = 200, c3 = 200,
                 n = 13, J = 15, p = 0.23)
# Required budget and sample size with constrained p
  mym.2 <- power.3m(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5))
  mym.2$out  # m = 21072, K = 10.9
# Required budget and sample size with constrained p and n
  mym.3 <- power.3m(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5, n = 20))
  mym.3$out  # m = 21252, K = 10.4

# Power calculation
  mypower <- power.3m(expr = myod1, q = 1, d = 0.2, m = 15491)
  mypower$out  # power = 0.80
# Power calculation under constrained p (p = 0.5)
  mypower.1 <- power.3m(expr = myod1, q = 1, d = 0.2, m = 15491,
                constraint = list(p = 0.5))
  mypower.1$out  # power = 0.62

# MDES calculation
  mymdes <- power.3m(expr = myod1, q = 1, power = 0.80, m = 15491)
  mymdes$out  # d = 0.20


# ------- Conventional power analyses with cost.model = FALSE-------
# Required sample size
  myK <- power.3m(cost.model = FALSE, expr = myod1, d = 0.2, q = 1, power = 0.8)
  myK$out  # K = 13.6
  # myK$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  myK <- power.3m(cost.model = FALSE, d = 0.2, power = 0.8, q = 1,
                 icc2 = 0.2, icc3 = 0.1, omega = 0.02,
                 r12 = 0.5, r22 = 0.5, r32m = 0.5,
                 c1 = 1, c2 = 5,
                 c1t = 1, c2t = 200, c3 = 200,
                 n = 13, J = 15, p = 0.23)

# Power calculation
  mypower1 <- power.3m(cost.model = FALSE, expr = myod1, K = 13.6, d = 0.2, q = 1)
  mypower1$out  # power = 0.80

# MDES calculation
  mymdes1 <- power.3m(cost.model = FALSE, expr = myod1, K = 13.6, power = 0.8, q = 1)
  mymdes1$out  # d = 0.20
```

`power.4` *Budget and/or sample size, power, MDES calculation for four-level CRTs detecting main effects*

Description

This function can calculate required budget for desired power, power or minimum detectable effect size (MDES) under fixed budget for four-level cluster randomized trials (CRTs). It also can perform conventional power analyses (e.g., required sample size, power, and MDES calculation).

Usage

```
power.4(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  d = NULL,
  power = NULL,
  m = NULL,
  n = NULL,
  J = NULL,
  K = NULL,
  L = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  icc4 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32 = NULL,
  r42 = NULL,
  q = NULL,
  c1 = NULL,
  c2 = NULL,
  c3 = NULL,
  c4 = NULL,
  c1t = NULL,
  c2t = NULL,
  c3t = NULL,
  c4t = NULL,
  dlim = NULL,
  powerlim = NULL,
  Llim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

cost.model	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise

	conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
expr	Returned objects from function od.4; default value is NULL; if expr is specified, parameter values of icc2, icc3, icc4, r12, r22, r32, r42, c1, c2, c3, c4, c1t, c2t, c3t, c4t, p, n, J, and K used or solved in function od.4 will be passed to current function; only the values of p, n, J, and/or K that specified or solved in function od.4 can be overwritten if constraint is specified.
constraint	Specify the constrained values of p, n, J, and/or K in list format to overwrite those from expr; default value is NULL.
sig.level	Significance level or type I error rate, default value is 0.05.
two.tailed	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
d	Effect size.
power	Statistical power.
m	Total budget.
n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
K	The level-3 sample size per level-4 unit.
L	The total level-4 sample size.
p	The proportion of level-4 clusters/units to be assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
icc4	The unconditional intraclass correlation coefficient (ICC) at level 4.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32	The proportion of level-3 variance explained by covariates.
r42	The proportion of level-4 variance explained by covariates.
q	The number of covariates at level 4.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c3	The cost of sampling one level-3 unit in control condition.
c4	The cost of sampling one level-4 unit in control condition.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
c3t	The cost of sampling one level-3 unit in treatment condition.
c4t	The cost of sampling one level-4 unit in treatment condition.
dlim	The range for solving the root of effect size (d) numerically, default value is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).

Llim	The range for solving the root of level-4 sample size (L) numerically, default value is c(4, 1e+10).
mlim	The range for searching the root of budget (m) numerically, default value is the costs sampling Llim level-4 units across treatment conditions or c(4 * Lcost, 1e+10 * Lcost) with Lcost = ((1 - p) * (c1 * n * J * K + c2 * J * K + c3 * K + c4) + p * (c1t * n * J * K + c2t * J * K + c3t * K + c4t)).
rounded	Logical; round the values of p, n/J/K that are from functions od.4 to two decimal places and integer, respectively if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (and/or required level-4 sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

Examples

```
# Unconstrained optimal design
  myod1 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500)
  myod1$out # output # n = 7.1, J = 3.2, K = 4.2, p = 0.23

# ------- Power analyses by default considering costs and budget -------
# Required budget and sample size
  mym.1 <- power.4(expr = myod1, d = 0.2, q = 1, power = 0.8)
  mym.1$out  # m = 71161, L = 57.1
  #mym.1$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  mym.1 <- power.4(d = 0.2, power = 0.8, q = 1,
                icc2 = 0.2, icc3 = 0.1, icc4 = 0.05,
                r12 = 0.5, r22 = 0.5, r32 = 0.5, r42 = 0.5,
                c1 = 1, c2 = 5, c3 = 25, c4 = 125,
                c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
                n = 7, J = 3, K = 4, p = 0.23)
# Required budget and sample size with constrained p (p = 0.5)
  mym.2 <- power.4(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5))
  mym.2$out  # m = 93508, L = 41.1
# Required budget and sample size with constrained p and K
  mym.3 <- power.4(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5, K = 20))
  mym.3$out  # m = 157365, L = 25.7

# Power calculation
  mypower <- power.4(expr = myod1, q = 1, d = 0.2, m = 71161)
  mypower$out  # power = 0.80
# Power calculation under constrained p (p = 0.5)
  mypower.1 <- power.4(expr = myod1, q = 1, d = 0.2, m = 71161,
```

```
                constraint = list(p = 0.5))
  mypower.1$out  # power = 0.68

# MDES calculation
  mymdes <- power.4(expr = myod1, q = 1, power = 0.80, m = 71161)
  mymdes$out  # d = 0.20


# ------- Conventional power analyses with cost.model = FALSE-------
# Required sample size
  myL <- power.4(cost.model = FALSE, expr = myod1, d = 0.2, q = 1, power = 0.8)
  myL$out  # L = 57.1
#myL$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  myL <- power.4(cost.model = FALSE, d = 0.2, power = 0.8, q = 1,
                 icc2 = 0.2, icc3 = 0.1, icc4 = 0.05,
                 r12 = 0.5, r22 = 0.5, r32 = 0.5, r42 = 0.5,
                 n = 7, J = 3, K = 4, p = 0.23)

# Power calculation
  mypower1 <- power.4(cost.model = FALSE, expr = myod1, L = 57, d = 0.2, q = 1)
  mypower1$out  # power = 0.80

# MDES calculation
  mymdes1 <- power.4(cost.model = FALSE, expr = myod1, L = 57, power = 0.8, q = 1)
  mymdes1$out  # d = 0.20
```

`power.4m` *Budget and/or sample size, power, MDES calculation for four-level MRTs detecting main effects*

Description

This function can calculate required budget for desired power, power or minimum detectable effect size (MDES) under fixed budget for four-level multisite randomized trials (MRTs). It also can perform conventional power analyses (e.g., required sample size, power, and MDES calculation).

Usage

```
power.4m(
  cost.model = TRUE,
  expr = NULL,
  constraint = NULL,
  sig.level = 0.05,
  two.tailed = TRUE,
  d = NULL,
  power = NULL,
  m = NULL,
```

```
  n = NULL,
  J = NULL,
  K = NULL,
  L = NULL,
  p = NULL,
  icc2 = NULL,
  icc3 = NULL,
  icc4 = NULL,
  r12 = NULL,
  r22 = NULL,
  r32 = NULL,
  r42m = NULL,
  q = NULL,
  c1 = NULL,
  c2 = NULL,
  c3 = NULL,
  c4 = NULL,
  c1t = NULL,
  c2t = NULL,
  c3t = NULL,
  omega = NULL,
  dlim = NULL,
  powerlim = NULL,
  Llim = NULL,
  mlim = NULL,
  rounded = TRUE
)
```

Arguments

`cost.model`	Logical; power analyses accommodating costs and budget (e.g., required budget for desired power, power/MDES under fixed budget) if TRUE, otherwise conventional power analyses (e.g., required sample size, power, or MDES calculation); default value is TRUE.
`expr`	Returned objects from function `od.4m`; default is NULL; if `expr` is specified, parameter values of `icc2`, `icc3`, `icc4`, `r12`, `r22`, `r32`, `r42m`, `c1`, `c2`, `c3`, `c4`, `c1t`, `c2t`, `c3t`, `p`, `n`, `J`, and `K` used or solved in function `od.4m` will be passed to current function; only the values of `p`, `n`, `J`, and/or `K` that specified or solved in function `od.4m` can be overwritten if `constraint` is specified.
`constraint`	The constrained values of `p`, `n`, `J`, and/or `K` in list format to overwrite those from `expr`; default value is NULL.
`sig.level`	Significance level or type I error rate, default value is 0.05.
`two.tailed`	Logical; two-tailed tests if TRUE, otherwise one-tailed tests; default value is TRUE.
`d`	Effect size.
`power`	Statistical power.
`m`	Total budget.

n	The level-1 sample size per level-2 unit.
J	The level-2 sample size per level-3 unit.
K	The level-3 sample size per level-4 unit.
L	The total level-4 sample size.
p	The proportion of level-3 units to be assigned to treatment.
icc2	The unconditional intraclass correlation coefficient (ICC) at level 2.
icc3	The unconditional intraclass correlation coefficient (ICC) at level 3.
icc4	The unconditional intraclass correlation coefficient (ICC) at level 4.
r12	The proportion of level-1 variance explained by covariates.
r22	The proportion of level-2 variance explained by covariates.
r32	The proportion of level-3 variance explained by covariates.
r42m	The proportion of variance of site-specific treatment effect explained by covariates.
q	The number of covariates at level 4.
c1	The cost of sampling one level-1 unit in control condition.
c2	The cost of sampling one level-2 unit in control condition.
c3	The cost of sampling one level-3 unit in control condition.
c4	The cost of sampling one level-4 unit (cite.
c1t	The cost of sampling one level-1 unit in treatment condition.
c2t	The cost of sampling one level-2 unit in treatment condition.
c3t	The cost of sampling one level-3 unit in treatment condition.
omega	The standardized variance of site-specific treatment effect.
dlim	The range for solving the root of effect size (d) numerically, default value is c(0, 5).
powerlim	The range for solving the root of power (power) numerically, default value is c(1e-10, 1 - 1e-10).
Llim	The range for solving the root of level-4 sample size (L) numerically, default value is c(4, 1e+10).
mlim	The range for searching the root of budget (m) numerically, default is the costs sampling Llim level-4 units or c(4 * Lcost, 1e+10 * Lcost) with Lcost = ((1 - p) * (c1 * n * J * K + c2 * J * K + c3 * K) + p * (c1t * n * J * K + c2t * J * K + c3t * K) + c4.
rounded	Logical; round the values of p, n/J/K that are from functions od.4 to two decimal places and integer, respectively if TRUE, otherwise no rounding; default value is TRUE.

Value

Required budget (and/or required level-4 sample size), statistical power, or MDES depending on the specification of parameters. The function also returns the function name, design type, and parameters used in the calculation.

Examples

```
# Unconstrained optimal design #---------
  myod1 <- od.4m(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
              r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
              c1 = 1, c2 = 5, c3 = 25,
              c1t = 1, c2t = 50, c3t = 250, c4 = 500,
              varlim = c(0, 0.005))
  myod1$out # n = 8.3, J = 3.2, K = 4.9, p = 0.36

# ------- Power analyses by default considering costs and budget -------
# Required budget and sample size
  mym.1 <- power.4m(expr = myod1, d = 0.2, q = 1, power = 0.8)
  mym.1$out  # m = 30201, L = 20.6
  # mym.1$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  mym.1 <- power.4m(d = 0.2, power = 0.8, q = 1,
                icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
                r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
                c1 = 1, c2 = 5, c3 = 25,
                c1t = 1, c2t = 50, c3t = 250, c4 = 500,
                n = 8, J = 3, K = 5, p = 0.36)
# Required budget and sample size with constrained p
  mym.2 <- power.4m(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5))
  mym.2$out  # m = 33183, L = 19.3
# Required budget and sample size with constrained p and n
  mym.3 <- power.4m(expr = myod1, d = 0.2, q = 1, power = 0.8,
                constraint = list(p = 0.5, n = 20))
  mym.3$out  # m = 34262, L = 18.0

# Power calculation
  mypower <- power.4m(expr = myod1, q = 1, d = 0.2, m = 30201)
  mypower$out  # power = 0.80
# Power calculation under constrained p (p = 0.5)
  mypower.1 <- power.4m(expr = myod1, q = 1, d = 0.2, m = 30201,
                constraint = list(p = 0.5))
  mypower.1$out  # power = 0.76

# MDES calculation
  mymdes <- power.4m(expr = myod1, q = 1, power = 0.80, m = 30201)
  mymdes$out  # d = 0.20


# ------- Conventional power analyses with cost.model = FALSE-------
# Required sample size
  myL <- power.4m(cost.model = FALSE, expr = myod1, d = 0.2, q = 1, power = 0.8)
  myL$out  # L = 20.6
  # myL$par  # parameters and their values used for the function
# Or, equivalently, specify every argument in the function
  myL <- power.4m(cost.model = FALSE, d = 0.2, power = 0.8, q = 1,
                 icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, omega = 0.02,
                r12 = 0.5, r22 = 0.5, r32 = 0.5, r42m = 0.5,
```

```
                  c1 = 1, c2 = 5, c3 = 25,
                  c1t = 1, c2t = 50, c3t = 250, c4 = 500,
                  n = 8, J = 3, K = 5, p = 0.36)

# Power calculation
  mypower1 <- power.4m(cost.model = FALSE, expr = myod1, L = 20.6, d = 0.2, q = 1)
  mypower1$out  # power = 0.80

# MDES calculation
  mymdes1 <- power.4m(cost.model = FALSE, expr = myod1, L = 20.6, power = 0.8, q = 1)
  mymdes1$out  # d = 0.20
```

---

re *Relative efficiency (RE) calculation*

Description

Calculate the relative efficiency (RE) between two designs, it returns same results as those from function rpe.

Usage

```
re(od, subod, rounded = TRUE, verbose = TRUE)
```

Arguments

od	Returned object of first design (e.g., unconstrained optimal design) from function od.1, od.2, od.3, od.4, od.2m, od.3m, or od.4m.
subod	Returned object of second design (e.g., constrained optimal design) from function od.1, od.2, od.3, od.4, od.2m, od.3m, or od.4m.
rounded	Logical; round the values of p, n/J/K that are from functions to two decimal places and integer, respectively if TRUE, no rounding if FALSE; default is TRUE.
verbose	Logical; print the value of relative efficiency if TRUE, otherwise not; default is TRUE.

Value

Relative efficiency value.

References

(1) Shen, Z., & Kelcey, B. (2020). Optimal sample allocation under unequal costs in cluster-randomized trials. Journal of Educational and Behavioral Statistics, 45(4): 446–474. <https://doi.org/10.3102/107699862091
(2) Shen, Z., & Kelcey, B. (in press). Optimal sample allocation in multisite randomized trials. The Journal of Experimental Education. <https://doi.org/10.1080/00220973.2020.1830361> (3) Shen, Z., & Kelcey, B. (in press). Optimal sampling ratios in three-level multisite experiments. Journal of Research on Educational Effectiveness.

Examples

```
# Unconstrained optimal design of 2-level CRT #----------
  myod1 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              varlim = c(0.01, 0.02))
# Constrained optimal design with n = 20
  myod2 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              n = 20, varlim = c(0.005, 0.025))
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.88
# Constrained optimal design with p = 0.5
  myod2 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
             p = 0.5, varlim = c(0.005, 0.025))
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.90

# Unconstrained optimal design of 3-level CRT #----------
  myod1 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5,
             c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
             varlim = c(0.005, 0.025))
# Constrained optimal design with J = 20
  myod2 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5, J = 20,
             c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
             varlim = c(0, 0.025))
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.53

# Unconstrained optimal design of 4-level CRT #---------
  myod1 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, r12 = 0.5,
              r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
# Constrained optimal design with p = 0.5
  myod2 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, r12 = 0.5, p = 0.5,
              r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
# Relative efficiency (RE)
  myre <- re(od = myod1, subod= myod2)
  myre$re # RE = 0.78
```

---

rpe *Relative precision and efficiency (RPE) calculation*

Description

Calculate the relative precision and efficiency (RPE) between two designs, it returns same results as those from function re.

Usage

```
rpe(od, subod, rounded = TRUE, verbose = TRUE)
```

Arguments

od	Returned object of first design (e.g., unconstrained optimal design) from function od.1, od.2, od.3, od.4, od.2m, od.3m, or od.4m.
subod	Returned object of second design (e.g., constrained optimal design) from function od.1, od.2, od.3, od.4, od.2m, od.3m, or od.4m.
rounded	Logical; round the values of p, n/J/K that are from functions to two decimal places and integer, respectively if TRUE, no rounding if FALSE; default is TRUE.
verbose	Logical; print the value of relative precision and efficiency if TRUE, otherwise not; default is TRUE.

Value

Relative precision and efficiency value.

References

(1) Shen, Z., & Kelcey, B. (2020). Optimal sample allocation under unequal costs in cluster-randomized trials. Journal of Educational and Behavioral Statistics, 45(4): 446–474. <https://doi.org/10.3102/107699862091
(2) Shen, Z., & Kelcey, B. (in press). Optimal sample allocation in multisite randomized trials. The Journal of Experimental Education. <https://doi.org/10.1080/00220973.2020.1830361> (3) Shen, Z., & Kelcey, B. (in press). Optimal sampling ratios in three-level multisite experiments. Journal of Research on Educational Effectiveness.

Examples

```
# Unconstrained optimal design of 2-level CRT #----------
  myod1 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              varlim = c(0.01, 0.02))
# Constrained optimal design with n = 20
  myod2 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
              n = 20, varlim = c(0.005, 0.025))
# Relative precision and efficiency (RPE)
  myrpe <- rpe(od = myod1, subod= myod2)
  myrpe$rpe # RPE = 0.88
# Constrained optimal design with p = 0.5
  myod2 <- od.2(icc = 0.2, r12 = 0.5, r22 = 0.5, c1 = 1, c2 = 5, c1t = 1, c2t = 50,
             p = 0.5, varlim = c(0.005, 0.025))
# Relative precision and efficiency (RPE)
  myrpe <- rpe(od = myod1, subod= myod2)
```

```
  myrpe$rpe # RPE = 0.90

# Unconstrained optimal design of 3-level CRT #----------
  myod1 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5,
             c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
             varlim = c(0.005, 0.025))
# Constrained optimal design with J = 20
  myod2 <- od.3(icc2 = 0.2, icc3 = 0.1, r12 = 0.5, r22 = 0.5, r32 = 0.5, J = 20,
             c1 = 1, c2 = 5, c3 = 25, c1t = 1, c2t = 50, c3t = 250,
             varlim = c(0, 0.025))
# Relative precision and efficiency (RPE)
  myrpe <- rpe(od = myod1, subod= myod2)
  myrpe$rpe # RPE = 0.53

# Unconstrained optimal design of 4-level CRT #---------
  myod1 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, r12 = 0.5,
              r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
# Constrained optimal design with p = 0.5
  myod2 <- od.4(icc2 = 0.2, icc3 = 0.1, icc4 = 0.05, r12 = 0.5, p = 0.5,
              r22 = 0.5, r32 = 0.5, r42 = 0.5,
              c1 = 1, c2 = 5, c3 = 25, c4 = 125,
              c1t = 1, c2t = 50, c3t = 250, c4t = 2500,
              varlim = c(0, 0.01))
# Relative precision and efficiency (RPE)
  myrpe <- rpe(od = myod1, subod= myod2)
  myrpe$rpe # RPE = 0.78
```

Index