Package 'rSPDE'

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Type Package

Title Rational Approximations of Fractional Stochastic Partial Differential Equations

Version 2.5.2

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Description Functions that compute rational approximations of fractional elliptic stochastic partial differential equations. The package also contains functions for common statistical usage of these approximations. The main references for rSPDE are Bolin, Simas and Xiong (2023) <doi:10.1080/10618600.2023.2231051> for the covariance-based method and Bolin and Kirchner (2020) <doi:10.1080/10618600.2019.1665537> for the operator-based rational approximation. These can be generated by the citation function in R.

Depends R (>= 3.5.0), Matrix

Imports stats, methods, fmesher (>= 0.2.0), lifecycle, broom

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URL https://davidbolin.github.io/rSPDE/

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Suggests knitr, rmarkdown, INLA (>= 24.12.01), testthat, ggplot2, lattice, splancs, optimParallel, RSpectra, numDeriv, inlabru (>= 2.12.0), sn, viridis, doParallel, foreach, tidyr, dplyr, GeneralizedHyperbolic, gridExtra, MetricGraph (>= 1.4.1), sf

Additional_repositories https://inla.r-inla-download.org/R/testing

BugReports https://github.com/davidbolin/rSPDE/issues

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Contents

`rSPDE-package` *Rational approximations of fractional SPDEs.*

Description

`rSPDE` is used for approximating fractional elliptic SPDEs

$$L^{\beta}(\tau u(s)) = W,$$

where L is a differential operator and $\beta > 0$ is a general fractional power.

Details

The approximation is based on a rational approximation of the fractional operator, and allows for computationally efficient inference and simulation.

The main functions for computing rational approximation objects are:

`fractional.operators()` works for general rational operators

`matern.operators()` works for random fields with stationary Matern covariance functions

`spde.matern.operators()` works for random fields with defined as solutions to a possibly non-stationary Matern-type SPDE model.

`rspde.matern()` R-INLA implementation of the covariance-based rational approximation for random fields with stationary Matern covariance functions

Basic statistical operations such as likelihood evaluations (see `[rSPDE.loglike]`, `[rSPDE.matern.loglike]`) and kriging predictions (see `[predict.rSPDEobj]`, `[predict.CBrSPDEobj]`) using the rational approximations are also implemented.

For illustration purposes, the package contains a simple FEM implementation for models on R. For spatial models, the FEM implementation in the `R-INLA` package is recommended.

For a more detailed introduction to the package, see the rSPDE Vignettes.

Author(s)

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See Also

Useful links:

- https://davidbolin.github.io/rSPDE/
- Report bugs at https://github.com/davidbolin/rSPDE/issues

`augment.rspde_lme` *Augment data with information from a* `rspde_lme` *object*

Description

Augment accepts a model object and a dataset and adds information about each observation in the dataset. It includes predicted values in the `.fitted` column, residuals in the `.resid` column, and standard errors for the fitted values in a `.se.fit` column. It also contains the New columns always begin with a . prefix to avoid overwriting columns in the original dataset.

Usage

```
## S3 method for class 'rspde_lme'
augment(
  x,
  newdata = NULL,
  loc = NULL,
  mesh = FALSE,
  which_repl = NULL,
  se_fit = FALSE,
  conf_int = FALSE,
  pred_int = FALSE,
  level = 0.95,
  n_samples = 100,
  ...
)
```

Arguments

`x`	A `rspde_lme` object.
`newdata`	A `data.frame` or a `list` containing the covariates, the edge number and the distance on edge for the locations to obtain the prediction. If `NULL`, the fitted values will be given for the original locations where the model was fitted.
`loc`	Prediction locations. Can either be a `data.frame`, a `matrix` or a character vector, that contains the names of the columns of the coordinates of the locations. For models using `metric_graph` objects, plase use `edge_number` and `distance_on_edge` instead.
`mesh`	Obtain predictions for mesh nodes? The graph must have a mesh, and either `only_latent` is set to TRUE or the model does not have covariates.

`which_repl`	Which replicates to obtain the prediction. If `NULL` predictions will be obtained for all replicates. Default is `NULL`.
`se_fit`	Logical indicating whether or not a .se.fit column should be added to the augmented output. If TRUE, it only returns a non-NA value if type of prediction is 'link'.
`conf_int`	Logical indicating whether or not confidence intervals for the fitted variable should be built.
`pred_int`	Logical indicating whether or not prediction intervals for future observations should be built.
`level`	Level of confidence and prediction intervals if they are constructed.
`n_samples`	Number of samples when computing prediction intervals.
`...`	Additional arguments. Expert use only.

Value

A `tidyr::tibble()` with columns:

- `.fitted` Fitted or predicted value.
- `.fittedlwrconf` Lower bound of the confidence interval, if conf_int = TRUE
- `.fitteduprconf` Upper bound of the confidence interval, if conf_int = TRUE
- `.fittedlwrpred` Lower bound of the prediction interval, if pred_int = TRUE
- `.fitteduprpred` Upper bound of the prediction interval, if pred_int = TRUE
- `.fixed` Prediction of the fixed effects.
- `.random` Prediction of the random effects.
- `.resid` The ordinary residuals, that is, the difference between observed and fitted values.
- `.se_fit` Standard errors of fitted values, if se_fit = TRUE.

See Also

glance.rspde_lme

`bru_get_mapper.inla_rspde_anisotropic2d` *rSPDE anisotropic inlabru mapper*

Description

rSPDE anisotropic inlabru mapper

Usage

```
bru_get_mapper.inla_rspde_anisotropic2d(model, ...)
```

Arguments

model	An inla_rspde_anisotropic2d object for which to construct or extract a mapper
...	Arguments passed on to other methods

---

bru_get_mapper.inla_rspde_fintrinsic
rSPDE inlabru mapper

Description

rSPDE inlabru mapper

Usage

```
bru_get_mapper.inla_rspde_fintrinsic(model, ...)
```

Arguments

model	An inla_rspde_fintrinsic object for which to construct or extract a mapper
...	Arguments passed on to other methods

---

bru_get_mapper.inla_rspde_matern1d
rSPDE stationary inlabru mapper

Description

rSPDE stationary inlabru mapper

Usage

```
bru_get_mapper.inla_rspde_matern1d(model, ...)

ibm_n.bru_mapper_inla_rspde_matern1d(mapper, ...)

ibm_values.bru_mapper_inla_rspde_matern1d(mapper, ...)

ibm_jacobian.bru_mapper_inla_rspde_matern1d(mapper, input, ...)
```

Arguments

model	An inla_rspde_matern1d object for which to construct or extract a mapper
...	Arguments passed on to other methods
mapper	A bru_mapper_inla_rspde_matern1d object
input	The values for which to produce a mapping matrix

`bru_get_mapper.inla_rspde_spacetime`
rSPDE space time inlabru mapper

Description

rSPDE space time inlabru mapper

Usage

```
bru_get_mapper.inla_rspde_spacetime(model, ...)
```

Arguments

`model`	An `inla_rspde_spacetime` object for which to construct or extract a mapper
`...`	Arguments passed on to other methods

`bru_get_mapper.intrinsic_matern`
rSPDE inlabru mapper

Description

rSPDE inlabru mapper

Usage

```
bru_get_mapper.intrinsic_matern(model, ...)

ibm_n.bru_mapper_intrinsic_matern(mapper, ...)

ibm_values.bru_mapper_intrinsic_matern(mapper, ...)

ibm_jacobian.bru_mapper_intrinsic_matern(mapper, input, ...)
```

Arguments

`model`	An `intrinsic_matern` object for which to construct or extract a mapper
`...`	Arguments passed on to other methods
`mapper`	A `bru_mapper_intrinsic_matern` object
`input`	The values for which to produce a mapping matrix

construct.spde.matern.loglike *Constructor of Matern loglikelihood functions for non-stationary models.*

Description

This function evaluates the log-likelihood function for observations of a non-stationary Gaussian process defined as the solution to the SPDE

$$(\kappa(s) - \Delta)^{\beta}(\tau(s)u(s)) = W.$$

The observations are assumed to be generated as $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using a rational approximation of the fractional SPDE model.

Usage

```
construct.spde.matern.loglike(
  object,
  Y,
  A,
  sigma.e = NULL,
  mu = 0,
  nu = NULL,
  m = NULL,
  log_scale = TRUE,
  return_negative_likelihood = TRUE
)
```

Arguments

object	The rational SPDE approximation, computed using matern.operators()
Y	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
A	An observation matrix that links the measurement location to the finite element basis.
sigma.e	IF non-null, the standard deviation of the measurement noise will be kept fixed in the returned likelihood.
mu	Expectation vector of the latent field (default = 0).
nu	If non-null, the shape parameter will be kept fixed in the returned likelihood.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
log_scale	Should the parameters be evaluated in log-scale?
return_negative_likelihood	Return minus the likelihood to turn the maximization into a minimization?

Value

The log-likelihood function. The parameters of the returned function are given in the order theta, nu, sigma.e, whenever they are available.

See Also

matern.operators(), predict.CBrSPDEobj()

Examples

```
# this example illustrates how the function can be used for maximum
# likelihood estimation
# Sample a Gaussian Matern process on R using a rational approximation
set.seed(123)
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51
# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)
tau <- rep(0.5, n.x)
nu <- 0.8
alpha <- nu + 0.5
kappa <- rep(1, n.x)
# Matern parameterization
# compute rational approximation
op <- spde.matern.operators(
  loc_mesh = x,
  kappa = kappa, tau = tau, alpha = alpha,
  parameterization = "spde", d = 1
)
# Sample the model
u <- simulate(op, n.rep)
# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)
# define negative likelihood function for optimization using matern.loglike
mlik <- construct.spde.matern.loglike(op, Y, A)
#' #The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(1 / sqrt(var(c(Y))), sqrt(8), 0.9, 0.01))
# run estimation and display the results
theta <- optim(theta0, mlik)
print(data.frame(
  tau = c(tau[1], exp(theta$par[1])), kappa = c(kappa[1], exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
```

```
))


# SPDE parameterization
# compute rational approximation
op <- spde.matern.operators(
  kappa = kappa, tau = tau, alpha = alpha,
  loc_mesh = x, d = 1,
  parameterization = "spde"
)
# Sample the model
u <- simulate(op, n.rep)
# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)
# define negative likelihood function for optimization using matern.loglike
mlik <- construct.spde.matern.loglike(op, Y, A)
#' #The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(1 / sqrt(var(c(Y))), sqrt(8), 0.9, 0.01))
# run estimation and display the results
theta <- optim(theta0, mlik)
print(data.frame(
  tau = c(tau[1], exp(theta$par[1])), kappa = c(kappa[1], exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`covariance_mesh` *Covariance between mesh nodes*

Description

Generic for computing the covariance between mesh nodes for model objects.

Usage

```
covariance_mesh(object, ...)
```

Arguments

`object`	A model object.
`...`	Additional arguments passed to methods.

cov_function_mesh *Covariance between mesh nodes and locations*

Description

Generic for computing the covariance between mesh nodes and locations for model objects.

Usage

```
cov_function_mesh(object, ...)
```

Arguments

object	A model object.
...	Additional arguments passed to methods.

create_train_test_indices *Create train and test splits for cross-validation*

Description

Creates train and test splits for cross-validation by handling multiple data types and supporting k-fold, leave-one-out (LOO), and leave-percentage-out (LPO) methods. Handles missing values and maintains data structure across multiple datasets.

Usage

```
create_train_test_indices(
  data_list,
  cv_type = c("k-fold", "loo", "lpo"),
  k = 5,
  percentage = 20,
  number_folds = 10
)
```

Arguments

data_list	A list of datasets, one per likelihood. Each dataset can be a data.frame, SpatialPointsDataFrame, or metric_graph_data object
cv_type	Type of cross-validation: "k-fold", "loo", or "lpo". Default is "k-fold"
k	Number of folds for k-fold CV. Default is 5
percentage	Training data percentage for LPO CV (1-99). Default is 20
number_folds	Number of folds for LPO CV. Default is 10

Details

The function handles NA values by removing rows with any missing values before creating splits. For multiple datasets, indices are mapped back to their original positions in each dataset.

Value

A list where each element contains:

`train`	Indices for training data mapped to original datasets
`test`	Indices for test data mapped to original datasets

`cross_validation` *Perform cross-validation on a list of fitted models.*

Description

Obtain several scores for a list of fitted models according to a folding scheme.

Usage

```
cross_validation(
  models,
  model_names = NULL,
  scores = c("mae", "mse", "crps", "scrps", "dss"),
  cv_type = c("k-fold", "loo", "lpo"),
  weight_thr = NULL,
  k = 5,
  percentage = 20,
  number_folds = 10,
  n_samples = 1000,
  return_scores_folds = FALSE,
  orientation_results = c("negative", "positive"),
  include_best = TRUE,
  train_test_indexes = NULL,
  return_train_test = FALSE,
  return_post_samples = FALSE,
  return_true_test_values = FALSE,
  parallelize_RP = FALSE,
  n_cores_RP = parallel::detectCores() - 1,
  true_CV = TRUE,
  save_settings = FALSE,
  model_options_bru = list(),
  print = TRUE,
  fit_verbose = FALSE
)
```

Arguments

models	A fitted model obtained from calling the bru() function or a list of fitted models. All models in the list must have the same number of likelihoods and must be fitted to identical datasets.
model_names	A vector containing the names of the models to appear in the returned data.frame. If NULL, the names will be of the form Model 1, Model 2, and so on. By default, it will try to obtain the name from the models list.
scores	A vector containing the scores to be computed. The options are "mse", "crps", "scrps", "dss", "wcrps" and "swcrps". By default, all scores are computed.
cv_type	The type of the folding to be carried out. The options are k-fold for k-fold cross-validation, in which case the parameter k should be provided, loo, for leave-one-out and lpo for leave-percentage-out, in this case, the parameter percentage should be given, and also the number_folds with the number of folds to be done. The default is k-fold.
weight_thr	When computing "wcrps" or "swcrps", the threshold to be used to compute the weights. Must be supplied if any of these scores are requested. No default value is provided.
k	The number of folds to be used in k-fold cross-validation. Will only be used if cv_type is k-fold.
percentage	The percentage (from 1 to 99) of the data to be used to train the model. Will only be used if cv_type is lpo.
number_folds	Number of folds to be done if cv_type is lpo.
n_samples	Number of samples to compute the posterior statistics to be used to compute the scores.
return_scores_folds	If TRUE, the scores for each fold will also be returned.
orientation_results	character vector. The options are "negative" and "positive". If "negative", the smaller the scores the better. If "positive", the larger the scores the better.
include_best	Should a row indicating which model was the best for each score be included?
train_test_indexes	A list where each element corresponds to a fold. Each fold contains: • train: A list of training index vectors, one for each likelihood. • test: A list of test index vectors, one for each likelihood, with the same length as train. This list is typically obtained by setting the argument return_train_test to TRUE. When supplying train_test_indexes, the cv_type, k, percentage and number_folds arguments are ignored.
return_train_test	Logical. Should the training and test indexes be returned? If 'TRUE' the train and test indexes will the 'train_test' element of the returned list.
return_post_samples	If TRUE the posterior samples will be included in the returned list.
return_true_test_values	If TRUE the true test values will be included in the returned list.

parallelize_RP	Logical. Should the computation of CRPS and SCRPS (and for some cases, DSS) be parallelized?
n_cores_RP	Number of cores to be used if parallelize_rp is TRUE.
true_CV	Should a TRUE cross-validation be performed? If TRUE the models will be fitted on the training dataset. If FALSE, the parameters will be kept fixed at the ones obtained in the result object.
save_settings	Logical. If TRUE, the settings used in the cross-validation will also be returned.
model_options_bru	A list of options to be passed to inlabru.
print	Should partial results be printed throughout the computation?
fit_verbose	Should INLA's run during cross-validation be verbose?

Value

A data.frame with the fitted models and the corresponding scores.

---

folded.matern.covariance.1d

The 1d folded Matern covariance function

Description

folded.matern.covariance.1d evaluates the 1d folded Matern covariance function over an interval $[0, L]$.

Usage

```
folded.matern.covariance.1d(
  h,
  m,
  kappa,
  nu,
  sigma,
  L = 1,
  N = 10,
  boundary = c("neumann", "dirichlet", "periodic")
)
```

Arguments

h, m	Vectors of arguments of the covariance function.
kappa	Range parameter.
nu	Shape parameter.
sigma	Standard deviation.

L	The upper bound of the interval $[0, L]$. By default, L=1.
N	The truncation parameter.
boundary	The boundary condition. The possible conditions are "neumann" (default), "dirichlet" or "periodic".

Details

folded.matern.covariance.1d evaluates the 1d folded Matern covariance function over an interval $[0, L]$ under different boundary conditions. For periodic boundary conditions

$$C_{\mathcal{P}}(h, m) = \sum_{k=-\infty}^{\infty} (C(h - m + 2kL),$$

for Neumann boundary conditions

$$C_{\mathcal{N}}(h, m) = \sum_{k=-\infty}^{\infty} (C(h - m + 2kL) + C(h + m + 2kL)),$$

and for Dirichlet boundary conditions:

$$C_{\mathcal{D}}(h, m) = \sum_{k=-\infty}^{\infty} (C(h - m + 2kL) - C(h + m + 2kL)),$$

where $C(\cdot)$ is the Matern covariance function:

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^{\nu} K_{\nu}(\kappa h).$$

We consider the truncation:

$$C_{\mathcal{P},N}(h, m) = \sum_{k=-N}^{N} C(h-m+2kL), C_{\mathcal{N},N}(h, m) = \sum_{k=-\infty}^{\infty} (C(h-m+2kL)+C(h+m+2kL)),$$

and

$$C_{\mathcal{D},N}(h, m) = \sum_{k=-N}^{N} (C(h - m + 2kL) - C(h + m + 2kL)).$$

Value

A matrix with the corresponding covariance values.

Examples

```
x <- seq(from = 0, to = 1, length.out = 101)
plot(x, folded.matern.covariance.1d(rep(0.5, length(x)), x,
  kappa = 10, nu = 1 / 5, sigma = 1
),
type = "l", ylab = "C(h)", xlab = "h"
)
```

`folded.matern.covariance.2d` *The 2d folded Matern covariance function*

Description

`folded.matern.covariance.2d` evaluates the 2d folded Matern covariance function over an interval $[0, L] \times [0, L]$.

Usage

```
folded.matern.covariance.2d(
  h,
  m,
  kappa,
  nu,
  sigma,
  L = 1,
  N = 10,
  boundary = c("neumann", "dirichlet", "periodic", "R2")
)
```

Arguments

`h, m`	Vectors with two coordinates.
`kappa`	Range parameter.
`nu`	Shape parameter.
`sigma`	Standard deviation.
`L`	The upper bound of the square $[0, L] \times [0, L]$. By default, `L=1`.
`N`	The truncation parameter.
`boundary`	The boundary condition. The possible conditions are `"neumann"` (default), `"dirichlet"`, `"periodic"` or `"R2"`.

Details

`folded.matern.covariance.2d` evaluates the 1d folded Matern covariance function over an interval $[0, L] \times [0, L]$ under different boundary conditions. For periodic boundary conditions

$$C_{\mathcal{P}}((h_1, h_2), (m_1, m_2)) = \sum_{k_2=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} (C(\|(h_1 - m_1 + 2k_1L, h_2 - m_2 + 2k_2L)\|),$$

for Neumann boundary conditions

$$C_{\mathcal{N}}((h_1, h_2), (m_1, m_2)) = \sum_{k_2=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} (C(\|(h_1-m_1+2k_1L, h_2-m_2+2k_2L)\|)+C(\|(h_1-m_1+2k_1L, h_2+m_2+2k_2L)\|$$

and for Dirichlet boundary conditions:

$$C_{\mathcal{D}}((h_1,h_2),(m_1,m_2)) = \sum_{k_2=-\infty}^{\infty}\sum_{k_1=-\infty}^{\infty}(C(\|(h_1-m_1+2k_1L, h_2-m_2+2k_2L)\|)-C(\|(h_1-m_1+2k_1L, h_2+m_2+2k_2L)\|$$

where $C(\cdot)$ is the Matern covariance function:

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^{\nu}K_{\nu}(\kappa h).$$

We consider the truncation for k_1, k_2 from $-N$ to N.

Value

The correspoding covariance.

Examples

```
h <- c(0.5, 0.5)
m <- c(0.5, 0.5)
folded.matern.covariance.2d(h, m, kappa = 10, nu = 1 / 5, sigma = 1)
```

`fractional.operators` *Rational approximations of fractional operators*

Description

`fractional.operators` is used for computing an approximation, which can be used for inference and simulation, of the fractional SPDE

$$L^{\beta}(\tau u(s)) = W.$$

Here L is a differential operator, $\beta > 0$ is the fractional power, τ is a positive scalar or vector that scales the variance of the solution u, and W is white noise.

Usage

```
fractional.operators(L, beta, C, scale.factor, m = 1, tau = 1)
```

Arguments

`L`	A finite element discretization of the operator L.
`beta`	The positive fractional power.
`C`	The mass matrix of the finite element discretization.
`scale.factor`	A constant c is a lower bound for the the smallest eigenvalue of the non-discretized operator L.

m	The order of the rational approximation, which needs to be a positive integer. The default value is 1. Higer values gives a more accurate approximation, which are more computationally expensive to use for inference. Currently, the largest value of m that is implemented is 4.
tau	The constant or vector that scales the variance of the solution. The default value is 1.

Details

The approximation is based on a rational approximation of the fractional operator, resulting in an approximate model on the form

$$P_l u(s) = P_r W,$$

where $P_j = p_j(L)$ are non-fractional operators defined in terms of polynomials p_j for $j = l, r$. The order of p_r is given by m and the order of p_l is $m + m_\beta$ where m_β is the integer part of β if $\beta > 1$ and $m_\beta = 1$ otherwise.

The discrete approximation can be written as $u = P_r x$ where $x \sim N(0, Q^{-1})$ and $Q = P_l^T C^{-1} P_l$. Note that the matrices P_r and Q may be be ill-conditioned for $m > 1$. In this case, the methods in `operator.operations()` should be used for operations involving the matrices, since these methods are more numerically stable.

Value

`fractional.operators` returns an object of class "rSPDEobj". This object contains the following quantities:

Pl	The operator P_l.
Pr	The operator P_r.
C	The mass lumped mass matrix.
Ci	The inverse of C.
m	The order of the rational approximation.
beta	The fractional power.
type	String indicating the type of approximation.
Q	The matrix `t(Pl) %*% solve(C,Pl)`.
type	String indicating the type of approximation.
Pl.factors	List with elements that can be used to assemble P_l.
Pr.factors	List with elements that can be used to assemble P_r.

See Also

`matern.operators()`, `spde.matern.operators()`, `matern.operators()`

Examples

```
# Compute rational approximation of a Gaussian process with a
# Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
  (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op <- fractional.operators(
  L = fem$G + kappa^2 * fem$C, beta = (nu + 1 / 2) / 2,
  C = fem$C, scale.factor = kappa^2, tau = tau
)

v <- t(rSPDE.A1d(x, 0.5))
c.approx <- Sigma.mult(op, v)

# plot the result and compare with the true Matern covariance
plot(x, matern.covariance(abs(x - 0.5), kappa, nu, sigma),
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c.approx, col = 2)
```

`get.initial.values.rSPDE` *Initial values for log-likelihood optimization in rSPDE models with a latent stationary Gaussian Matern model*

Description

Auxiliar function to obtain domain-based initial values for log-likelihood optimization in rSPDE models with a latent stationary Gaussian Matern model

Usage

```
get.initial.values.rSPDE(
  mesh = NULL,
  mesh.range = NULL,
  graph.obj = NULL,
  n.spde = 1,
  dim = NULL,
  B.tau = NULL,
```

```
  B.kappa = NULL,
  B.sigma = NULL,
  B.range = NULL,
  nu = NULL,
  parameterization = c("matern", "spde"),
  include.nu = TRUE,
  log.scale = TRUE,
  nu.upper.bound = NULL
)
```

Arguments

mesh	An in INLA mesh
mesh.range	The range of the mesh.
graph.obj	A metric_graph object. To be used in case both mesh and mesh.range are NULL.
n.spde	The number of basis functions in the mesh model.
dim	The dimension of the domain.
B.tau	Matrix with specification of log-linear model for τ. Will be used if parameterization = 'spde'.
B.kappa	Matrix with specification of log-linear model for κ. Will be used if parameterization = 'spde'.
B.sigma	Matrix with specification of log-linear model for σ. Will be used if parameterization = 'matern'.
B.range	Matrix with specification of log-linear model for ρ, which is a range-like parameter (it is exactly the range parameter in the stationary case). Will be used if parameterization = 'matern'.
nu	The smoothness parameter.
parameterization	Which parameterization to use? matern uses range, std. deviation and nu (smoothness). spde uses kappa, tau and nu (smoothness). The default is matern.
include.nu	Should we also provide an initial guess for nu?
log.scale	Should the results be provided in log scale?
nu.upper.bound	Should an upper bound for nu be considered?

Value

A vector of the form (theta_1,theta_2,theta_3) or where theta_1 is the initial guess for tau, theta_2 is the initial guess for kappa and theta_3 is the initial guess for nu.

gg_df *Data frame for result objects from R-INLA fitted models to be used in ggplot2*

Description

Data frame for result objects from R-INLA fitted models to be used in ggplot2

Usage

```
gg_df(result, ...)
```

Arguments

result	a result object for which the data frame is desired
...	further arguments passed to or from other methods.

Value

A data frame containing the posterior densities.

gg_df.rspde_result *Data frame for rspde_result objects to be used in ggplot2*

Description

Returns a ggplot-friendly data-frame with the marginal posterior densities.

Usage

```
## S3 method for class 'rspde_result'
gg_df(
  result,
  parameter = result$params,
  transform = TRUE,
  restrict_x_axis = NULL,
  restrict_quantiles = NULL,
  ...
)
```

Arguments

result	An rspde_result object.
parameter	Vector. Which parameters to get the posterior density in the data.frame? The options are std.dev, range, tau, kappa and nu.
transform	Should the posterior density be given in the original scale?
restrict_x_axis	Variables to restrict the range of x axis based on quantiles.
restrict_quantiles	Named list of quantiles to restrict x axis. It should contain the name of the parameter along with a vector with two elements specifying the lower and upper quantiles. The names should be match the ones in result$params. For example, if we want to restrict nu to the 0.05 and 0.95 quantiles we do restrict_quantiles = c(0.05, 0.95).
...	currently not used.

Value

A data frame containing the posterior densities.

---

glance.rspde_lme *Glance at an* rspde_lme *object*

Description

Glance accepts a rspde_lme object and returns a tidyr::tibble() with exactly one row of model summaries. The summaries are the square root of the estimated variance of the measurement error, residual degrees of freedom, AIC, BIC, log-likelihood, the type of latent model used in the fit and the total number of observations.

Usage

```
## S3 method for class 'rspde_lme'
glance(x, ...)
```

Arguments

x	An rspde_lme object.
...	Currently not used.

Value

A tidyr::tibble() with exactly one row and columns:

- nobs Number of observations used.
- sigma the square root of the estimated residual variance
- logLik The log-likelihood of the model.
- AIC Akaike's Information Criterion for the model.
- BIC Bayesian Information Criterion for the model.
- deviance Deviance of the model.
- df.residual Residual degrees of freedom.
- model.type Type of latent model fitted.

See Also

augment.rspde_lme

---

graph_data_rspde *Data extraction from metric graphs for 'rSPDE' models*

Description

Extracts data from metric graphs to be used by 'INLA' and 'inlabru'.

Usage

```
graph_data_rspde(
  graph_rspde,
  name = "field",
  repl = NULL,
  repl_col = NULL,
  group = NULL,
  group_col = NULL,
  only_pred = FALSE,
  time = NULL,
  bru = FALSE,
  tibble = FALSE,
  drop_na = FALSE,
  drop_all_na = TRUE
)
```

Arguments

graph_rspde	An inla_metric_graph_spde or inla_rspde_spacetime object built with the rspde.metric_graph() or rspde.spacetime() function.
name	A character string with the base name of the effect.
repl	Which replicates? If there is no replicates, one can set repl to NULL. If one wants all replicates, then one sets to repl to .all.
repl_col	Which "column" of the data contains the replicate variable?
group	Which groups? If there is no groups, one can set group to NULL. If one wants all groups, then one sets to group to .all.
group_col	Which "column" of the data contains the group variable?
only_pred	Should only return the data.frame to the prediction data?
time	Column containing times for space time models. Not needed when using inlabru. Only for INLA implementation of space time model.
bru	Should the data be processed for inlabru?
tibble	Should the data be returned as a tidyr::tibble?
drop_na	Should the rows with at least one NA for one of the columns be removed? DEFAULT is FALSE. This option is turned to FALSE if only_pred is TRUE.
drop_all_na	Should the rows with all variables being NA be removed? DEFAULT is TRUE. This option is turned to FALSE if only_pred is TRUE.

Value

An 'INLA' and 'inlabru' friendly list with the data.

---

ibm_n.bru_mapper_inla_rspde_fintrinsic

rSPDE inlabru mapper

Description

rSPDE inlabru mapper

Usage

```
ibm_n.bru_mapper_inla_rspde_fintrinsic(mapper, ...)

ibm_values.bru_mapper_inla_rspde_fintrinsic(mapper, ...)

ibm_jacobian.bru_mapper_inla_rspde_fintrinsic(mapper, input, ...)

bru_get_mapper.inla_rspde(model, ...)

ibm_n.bru_mapper_inla_rspde(mapper, ...)
```

```
ibm_values.bru_mapper_inla_rspde(mapper, ...)

ibm_jacobian.bru_mapper_inla_rspde(mapper, input, ...)
```

Arguments

mapper	A bru_mapper_inla_rspde object
...	Arguments passed on to other methods
input	The values for which to produce a mapping matrix
model	An inla_rspde object for which to construct or extract a mapper

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE) &&
  requireNamespace("inlabru", quietly = TRUE)) {
  library(INLA)
  library(inlabru)

  set.seed(123)
  m <- 100
  loc_2d_mesh <- matrix(runif(m * 2), m, 2)
  mesh_2d <- inla.mesh.2d(
    loc = loc_2d_mesh,
    cutoff = 0.05,
    max.edge = c(0.1, 0.5)
  )
  sigma <- 1
  range <- 0.2
  nu <- 0.8
  kappa <- sqrt(8 * nu) / range
  op <- matern.operators(
    mesh = mesh_2d, nu = nu,
    range = range, sigma = sigma, m = 2,
    parameterization = "matern"
  )
  u <- simulate(op)
  A <- inla.spde.make.A(
    mesh = mesh_2d,
    loc = loc_2d_mesh
  )
  sigma.e <- 0.1
  y <- A %*% u + rnorm(m) * sigma.e
  y <- as.vector(y)

  data_df <- data.frame(
    y = y, x1 = loc_2d_mesh[, 1],
    x2 = loc_2d_mesh[, 2]
  )
```

```
  rspde_model <- rspde.matern(
    mesh = mesh_2d,
    nu_upper_bound = 2
  )

  cmp <- y ~ Intercept(1) +
    field(cbind(x1,x2), model = rspde_model)


  rspde_fit <- bru(cmp, data = data_df)
  summary(rspde_fit)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`intrinsic.matern.operators` *Covariance-based approximations of intrinsic fields*

Description

`intrinsic.matern.operators` is used for computing a covariance-based rational SPDE approximation of intrinsic fields on R^d defined through the SPDE

$$(-\Delta)^{\beta/2}(\kappa^2 - \Delta)^{\alpha/2}(\tau u) = \mathcal{W}$$

Usage

```
intrinsic.matern.operators(
  kappa,
  tau,
  alpha,
  beta = 1,
  G = NULL,
  C = NULL,
  d = NULL,
  mesh = NULL,
  graph = NULL,
  loc_mesh = NULL,
  m_alpha = 2,
  m_beta = 2,
  compute_higher_order = FALSE,
  return_block_list = FALSE,
  type_rational_approximation = c("brasil", "chebfun", "chebfunLB"),
  fem_mesh_matrices = NULL,
  scaling = NULL,
  opts = NULL
)
```

Arguments

kappa	range parameter
tau	precision parameter
alpha	Smoothness parameter
beta	Smoothness parameter
G	The stiffness matrix of a finite element discretization of the domain of interest.
C	The mass matrix of a finite element discretization of the domain of interest.
d	The dimension of the domain.
mesh	An inla mesh.
graph	An optional metric_graph object. Replaces d, C and G.
loc_mesh	locations for the mesh for d=1.
m_alpha	The order of the rational approximation for the Matérn part, which needs to be a positive integer. The default value is 2.
m_beta	The order of the rational approximation for the intrinsic part, which needs to be a positive integer. The default value is 2.
compute_higher_order	Logical. Should the higher order finite element matrices be computed?
return_block_list	Logical. For type = "covariance", should the block parts of the precision matrix be returned separately as a list?
type_rational_approximation	Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".
fem_mesh_matrices	A list containing FEM-related matrices. The list should contain elements c0, g1, g2, g3, etc.
scaling	scaling factor, see details.
opts	options for numerical calulcation of the scaling, see details.

Details

The covariance operator

$$\tau^{-2}(-\Delta)^{\beta}(\kappa^2 - \Delta)^{\alpha}$$

is approximated based on rational approximations of the two fractional components. The Laplacians are equipped with homogeneous Neumann boundary conditions. Unless supplied, the scaling is computed as the lowest positive eigenvalue of `sqrt(solve(c0))%*%g1%*%sqrt(solve(c0))`. opts provides a list of options for the numerical calculation of the scaling factor, which is done using `Rspectra::eigs_sym`. See the help of that function for details.

Value

`intrinsic.matern.operators` returns an object of class "intrinsicCBrSPDEobj". This object is a list including the following quantities:

`C`	The mass lumped mass matrix.
`Ci`	The inverse of `C`.
`GCi`	The stiffness matrix G times `Ci`
`Q`	The precision matrix.
`Q_list`	A list containing the blocks required to assemble the precision matrix.
`alpha`	The fractional power of the Matérn part of the operator.
`beta`	The fractional power of the intrinsic part of the operator.
`kappa`	Range parameter of the covariance function
`tau`	Scale parameter of the covariance function.
`m_alpha`	The order of the rational approximation for the Matérn part.
`m_beta`	The order of the rational approximation for the intrinsic part.
`m`	The total number of blocks in the precision matrix.
`n`	The number of mesh nodes.
`d`	The dimension of the domain.
`type_rational_approximation`	String indicating the type of rational approximation.
`higher_order`	Boolean indicating if higher order FEM-related matrices are computed.
`return_block_list`	Boolean indicating if the precision matrix is returned as a list with the blocks.
`fem_mesh_matrices`	A list containing the mass lumped mass matrix, the stiffness matrix and the higher-order FEM-related matrices.
`make_A`	Use `make_A()` to compute the projection matrix linking the field to observation locations.
`variogram`	A function to compute the variogram of the model at a specified node.
`A`	Matrix that sums the components in the approximation to the mesh nodes.
`scaling`	The scaling used in the intrinsic part of the model.

Examples

```
if (requireNamespace("RSpectra", quietly = TRUE)) {
  x <- seq(from = 0, to = 10, length.out = 201)
  beta <- 1
  alpha <- 1
  kappa <- 1
  op <- intrinsic.matern.operators(
    kappa = kappa, tau = 1, alpha = alpha,
    beta = beta, loc_mesh = x, d = 1
  )
```

```
  # Compute and plot the variogram of the model
  Sigma <- op$A[,-1] %*% solve(op$Q[-1,-1], t(op$A[,-1]))
  One <- rep(1, times = ncol(Sigma))
  D <- diag(Sigma)
  Gamma <- 0.5 * (One %*% t(D) + D %*% t(One) - 2 * Sigma)
  k <- 100
  plot(x, Gamma[k, ], type = "l")
  lines(x,
    variogram.intrinsic.spde(x[k], x, kappa, alpha, beta, L = 10, d = 1),
    col = 2, lty = 2
  )
}
```

`intrinsic.operators` *Covariance-based approximations of intrinsic fields*

Description

`intrinsic.matern.operators` is used for computing a covariance-based rational SPDE approximation of intrinsic fields on R^d defined through the SPDE

$$(-\Delta)^{\beta/2}(\tau u) = \mathcal{W}$$

Usage

```
intrinsic.operators(
  tau = NULL,
  beta = NULL,
  G = NULL,
  C = NULL,
  d = NULL,
  mesh = NULL,
  graph = NULL,
  loc_mesh = NULL,
  m = 1,
  compute_higher_order = FALSE,
  return_block_list = FALSE,
  type_rational_approximation = c("brasil", "chebfun", "chebfunLB"),
  fem_mesh_matrices = NULL,
  scaling = NULL,
  opts = NULL
)
```

Arguments

`tau`	precision parameter
`beta`	Smoothness parameter
`G`	The stiffness matrix of a finite element discretization of the domain of interest.

C	The mass matrix of a finite element discretization of the domain of interest.
d	The dimension of the domain.
mesh	An inla mesh.
graph	An optional metric_graph object. Replaces d, C and G.
loc_mesh	locations for the mesh for d=1.
m	The order of the rational approximation for the intrinsic part, which needs to be a positive integer. The default value is 2.
compute_higher_order	Logical. Should the higher order finite element matrices be computed?
return_block_list	Logical. For type = "covariance", should the block parts of the precision matrix be returned separately as a list?
type_rational_approximation	Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".
fem_mesh_matrices	A list containing FEM-related matrices. The list should contain elements c0, g1, g2, g3, etc.
scaling	scaling factor, see details.
opts	options for numerical calulcation of the scaling, see details.

Details

The covariance operator

$$\tau^{-2}(-\Delta)^{\beta}$$

is approximated based on a rational approximation. The Laplacian is equipped with homogeneous Neumann boundary conditions and a zero-mean constraint is additionally imposed to obtained a non-intrinsic model. The scaling is computed as the lowest positive eigenvalue of sqrt(solve(c0))%*%g1sqrt(solve(c0)). opts provides a list of options for the numerical calculation of the scaling factor, which is done using Rspectra::eigs_sym. See the help of that function for details.

Value

intrinsic.operators returns an object of class "intrinsicCBrSPDEobj".

Examples

```
if (requireNamespace("RSpectra", quietly = TRUE)) {
  x <- seq(from = 0, to = 10, length.out = 201)
  beta <- 1
  alpha <- 1
  op <- intrinsic.operators(tau = 1, beta = beta, loc_mesh = x, d = 1)
  # Compute and plot the variogram of the model
  Sigma <- op$A[,-1] %*% solve(op$Q[-1,-1], t(op$A[,-1]))
  One <- rep(1, times = ncol(Sigma))
  D <- diag(Sigma)
  Gamma <- 0.5 * (One %*% t(D) + D %*% t(One) - 2 * Sigma)
```

```
  k <- 100
  plot(x, Gamma[k, ], type = "l")
  lines(x,
    variogram.intrinsic.spde(x[k], x, kappa = 0, alpha = 0,
    beta = beta, L = 10, d = 1),
    col = 2, lty = 2
  )
}
```

---

make_A *Projection matrix for model objects*

Description

Generic for computing the projection matrix that links observation locations to model mesh nodes.

Usage

```
make_A(object, ...)
```

Arguments

object	A model object.
...	Additional arguments passed to methods.

---

matern.covariance *The Matern covariance function*

Description

matern.covariance evaluates the Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h).$$

Usage

```
matern.covariance(h, kappa, nu, sigma)
```

Arguments

h	Distances to evaluate the covariance function at.
kappa	Range parameter.
nu	Shape parameter.
sigma	Standard deviation.

Value

A vector with the values C(h).

Examples

```
x <- seq(from = 0, to = 1, length.out = 101)
plot(x, matern.covariance(abs(x - 0.5), kappa = 10, nu = 1 / 5, sigma = 1),
  type = "l", ylab = "C(h)", xlab = "h"
)
```

`matern.operators` *Rational approximations of stationary Gaussian Matern random fields*

Description

`matern.operators` is used for computing a rational SPDE approximation of a stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h)$$

Usage

```
matern.operators(
  kappa = NULL,
  tau = NULL,
  alpha = NULL,
  sigma = NULL,
  range = NULL,
  nu = NULL,
  G = NULL,
  C = NULL,
  d = NULL,
  mesh = NULL,
  graph = NULL,
  range_mesh = NULL,
  loc_mesh = NULL,
  m = 1,
  type = c("covariance", "operator"),
  parameterization = c("spde", "matern"),
  compute_higher_order = FALSE,
  return_block_list = FALSE,
  type_rational_approximation = c("brasil", "chebfun", "chebfunLB"),
  compute_logdet = FALSE
)
```

Arguments

kappa	Parameter kappa of the SPDE representation. If NULL, the range parameter will be used. If the range is also NULL, a starting value based on the mesh will be supplied.
tau	Parameter tau of the SPDE representation. If both sigma and tau are NULL, a starting value based on the mesh will be supplied.
alpha	Parameter alpha of the SPDE representation. If alpha is NULL, a starting value will be supplied.
sigma	Standard deviation of the covariance function. Used if parameterization is matern. If NULL, tau will be used. If tau is also NULL, a starting value based on the mesh will be supplied.
range	Range parameter of the covariance function. Used if parameterization is matern. If range is NULL, a starting value based on the mesh will be supplied.
nu	Shape parameter of the covariance function. Used if parameterization is matern. If NULL, a starting value will be supplied.
G	The stiffness matrix of a finite element discretization of the domain of interest. Does not need to be given if either mesh or graph is supplied.
C	The mass matrix of a finite element discretization of the domain of interest. Does not need to be given if either mesh or graph is supplied.
d	The dimension of the domain. Does not need to be given if either mesh or graph is provided.
mesh	An optional fmesher mesh. Replaces d, C and G.
graph	An optional metric_graph object. Replaces d, C and G.
range_mesh	The range of the mesh. Will be used to provide starting values for the parameters. Will be used if mesh and graph are NULL, and if one of the parameters (kappa or tau for spde parameterization, or sigma or range for matern parameterization) are not provided.
loc_mesh	The mesh locations used to construct the matrices C and G. This option should be provided if one wants to use the rspde_lme() function and will not provide neither graph nor mesh. Only works for 1d data. Does not work for metric graphs. For metric graphs you should supply the graph using the graph argument.
m	The order of the rational approximation, which needs to be a positive integer. The default value is 1.
type	The type of the rational approximation. The options are "covariance" and "operator". The default is "covariance".
parameterization	Which parameterization to use? matern uses range, std. deviation and nu (smoothness). spde uses kappa, tau and alpha. The default is spde.
compute_higher_order	Logical. Should the higher order finite element matrices be computed?
return_block_list	Logical. For type = "covariance", should the block parts of the precision matrix be returned separately as a list?

`type_rational_approximation`
: Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

`compute_logdet`
: Should log determinants be computed while building the model? (For covariance-based models)

Details

If `type` is "covariance", we use the covariance-based rational approximation of the fractional operator. In the SPDE approach, we model u as the solution of the following SPDE:

$$L^{\alpha/2}(\tau u) = \mathcal{W},$$

where $L = -\Delta + \kappa^2 I$ and $\mathcal{W}$ is the standard Gaussian white noise. The covariance operator of u is given by $L^{-\alpha}$. Now, let L_h be a finite-element approximation of L. We can use a rational approximation of order m on $L_h^{-\alpha}$ to obtain the following approximation:

$$L_{h,m}^{-\alpha} = L_h^{-m_\alpha} p(L_h^{-1}) q(L_h^{-1})^{-1},$$

where $m_\alpha = \lfloor \alpha \rfloor$, p and q are polynomials arising from such rational approximation. From this approximation we construct an approximate precision matrix for u.

If `type` is "operator", the approximation is based on a rational approximation of the fractional operator $(\kappa^2 - \Delta)^\beta$, where $\beta = (\nu + d/2)/2$. This results in an approximate model of the form

$$P_l u(s) = P_r W,$$

where $P_j = p_j(L)$ are non-fractional operators defined in terms of polynomials p_j for $j = l, r$. The order of p_r is given by `m` and the order of p_l is $m + m_\beta$ where m_β is the integer part of β if $\beta > 1$ and $m_\beta = 1$ otherwise.

The discrete approximation can be written as $u = P_r x$ where $x \sim N(0, Q^{-1})$ and $Q = P_l^T C^{-1} P_l$. Note that the matrices P_r and Q may be be ill-conditioned for $m > 1$. In this case, the methods in `operator.operations()` should be used for operations involving the matrices, since these methods are more numerically stable.

Value

If `type` is "covariance", then `matern.operators` returns an object of class "CBrSPDEobj". This object is a list containing the following quantities:

`C`
: The mass lumped mass matrix.

`Ci`
: The inverse of `C`.

`GCi`
: The stiffness matrix G times `Ci`

`Gk`
: The stiffness matrix G along with the higher-order FEM-related matrices G2, G3, etc.

`fem_mesh_matrices`
: A list containing the mass lumped mass matrix, the stiffness matrix and the higher-order FEM-related matrices.

`m`
: The order of the rational approximation.

alpha	The fractional power of the precision operator.
type	String indicating the type of approximation.
d	The dimension of the domain.
nu	Shape parameter of the covariance function.
kappa	Range parameter of the covariance function
tau	Scale parameter of the covariance function.
sigma	Standard deviation of the covariance function.
type	String indicating the type of approximation.

If type is "operator", then matern.operators returns an object of class "rSPDEobj". This object contains the quantities listed in the output of fractional.operators(), the G matrix, the dimension of the domain, as well as the parameters of the covariance function.

See Also

fractional.operators(), spde.matern.operators(), matern.operators()

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
nobs <- 101
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
  parameterization = "matern"
)

v <- t(rSPDE.A1d(x, 0.5))
# Compute the precision matrix
Q <- op_cov$Q
# A matrix here is the identity matrix
A <- Diagonal(nobs)
# We need to concatenate 3 A's since we are doing a covariance-based rational
# approximation of order 2
Abar <- cbind(A, A, A)
w <- rbind(v, v, v)
# The approximate covariance function:
c_cov.approx <- (Abar) %*% solve(Q, w)
c.true <- folded.matern.covariance.1d(
```

```
  rep(0.5, length(x)),
  abs(x), kappa, nu, sigma
)

# plot the result and compare with the true Matern covariance
plot(x, c.true,
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c_cov.approx, col = 2)


# Compute the operator-based rational approximation of a Gaussian
# process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op <- matern.operators(
  range = range, sigma = sigma, nu = nu,
  loc_mesh = x, d = 1,
  type = "operator",
  parameterization = "matern"
)

v <- t(rSPDE.A1d(x, 0.5))
c.approx <- Sigma.mult(op, v)
c.true <- folded.matern.covariance.1d(
  rep(0.5, length(x)),
  abs(x), kappa, nu, sigma
)

# plot the result and compare with the true Matern covariance
plot(x, c.true,
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximation"
)
lines(x, c.approx, col = 2)
```

`matern.rational` *Rational approximation of the Matern fields on intervals and metric graphs*

Description

The function is used for computing an approximation, which can be used for inference and simulation, of the fractional SPDE

$$(\kappa^2 - \Delta)^{\alpha/2}(\tau u(s)) = W$$

on intervals or metric graphs. Here W is Gaussian white noise, κ controls the range, $\alpha = \nu + 1/2$ with $\nu > 0$ controls the smoothness and τ is related to the marginal variances through

$$\sigma^2 = \frac{\Gamma(\nu)}{\tau^2 \Gamma(\alpha) 2\sqrt{\pi}\kappa^{2\nu}}.$$

Usage

```
matern.rational(
  graph = NULL,
  loc = NULL,
  bc = c("free", "Neumann", "Dirichlet"),
  kappa = NULL,
  range = NULL,
  nu = NULL,
  sigma = NULL,
  tau = NULL,
  alpha = NULL,
  m = 2,
  parameterization = c("matern", "spde"),
  type_rational_approximation = "brasil",
  type_interp = "spline"
)
```

Arguments

graph	Metric graph object. The default is NULL, which means that a stationary Matern model on the line is created.
loc	Locations where to evaluate the model.
bc	Specifies the boundary conditions. The default is "free" which gives stationary Matern models on intervals. Other options are "Neumann" or "Dirichlet".
kappa	Range parameter
range	practical correlation range
nu	Smoothness parameter
sigma	Standard deviation
tau	Precision parameter
alpha	Smoothness parameter
m	The order of the approximation
parameterization	Which parameterization to use? `matern` uses range, std. deviation and nu (smoothness). `spde` uses kappa, tau and alpha. The default is `matern`.

type_rational_approximation
: Method used to compute the coefficients of the rational approximation.

type_interp
: Interpolation method for the rational coefficients.

Value

A model object for the the approximation

Examples

```
s <- seq(from = 0, to = 1, length.out = 101)
kappa <- 20
sigma <- 2
nu <- 0.8
r <- sqrt(8*nu)/kappa #range parameter
op_cov <- matern.rational(loc = s, nu = nu, range = r, sigma = sigma, m = 2,
parameterization = "matern")
cov.true <- matern.covariance(abs(s-s[1]), kappa = kappa, sigma = sigma, nu = nu)
cov.approx <- op_cov$covariance(ind = 1)

plot(s, cov.true)
lines(s, cov.approx, col = 2)
```

---

matern.rational.cov *Rational approximation of the Matern covariance*

Description

Computes a rational approximation of the Matern covariance function on intervals.

Usage

```
matern.rational.cov(
  h,
  order,
  kappa,
  nu,
  sigma,
  type_rational = "brasil",
  type_interp = "linear"
)
```

Arguments

h
: Distances to compute the covariance for

order
: The order of the approximation

kappa
: Range parameter

nu
: Smoothness parameter

sigma	Standard deviation
type_rational	Method used to compute the coefficients of the rational approximation.
type_interp	Interpolation method for the rational coefficients.

Value

The covariance matrix of the approximation

Examples

```
h <- seq(from = 0, to = 1, length.out = 100)
cov.true <- matern.covariance(h, kappa = 10, sigma = 1, nu = 0.8)
cov.approx <- matern.rational.cov(h, kappa = 10, sigma = 1, nu = 0.8, order = 2)

plot(h, cov.true)
lines(h, cov.approx, col = 2)
```

---

matern2d.operators *Rational approximations of stationary anisotropic Gaussian Matern random fields*

Description

matern2d.operators is used for computing a rational SPDE approximation of a stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\sqrt{h^T H^{-1} h})^\nu K_\nu(\sqrt{h^T H^{-1} h})$$

, based on a SPDE representation of the form

$$(I - \nabla \cdot (H\nabla))^{(\nu+1)/2} u = c\sigma W$$

, where $c>0$ is a constant. The matrix H is defined as

$$\begin{bmatrix} h_x^2 & h_x h_y h_{xy} \\ h_x h_y h_{xy} & h_y^2 \end{bmatrix}$$

Usage

```
matern2d.operators(
  hx = NULL,
  hy = NULL,
  hxy = NULL,
  nu = NULL,
  sigma = NULL,
  mesh = NULL,
  fem = NULL,
```

```
  m = 1,
  type_rational_approximation = c("brasil", "chebfun", "chebfunLB"),
  return_fem_matrices = FALSE
)
```

Arguments

hx	Parameter in the H matrix.
hy	Parameter in the H matrix.
hxy	Parameter in the H matrix.
nu	Smoothness parameter.
sigma	standard deviation parameter.
mesh	An fmesher mesh.
fem	Optional precomputed FEM matrices.
m	The order of the rational approximation, which needs to be a positive integer. The default value is 1.
type_rational_approximation	Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".
return_fem_matrices	Should the FEM matrices be returned?

Value

An object of type CBrSPDEobj2d

See Also

fractional.operators(), spde.matern.operators(), matern.operators()

Examples

```
library(fmesher)
n_loc <- 2000
loc_2d_mesh <- matrix(runif(n_loc * 2), n_loc, 2)
mesh_2d <- fm_mesh_2d(loc = loc_2d_mesh, cutoff = 0.03, max.edge = c(0.1, 0.5))
op <- matern2d.operators(mesh = mesh_2d)
```

`operator.operations` *Operations with the Pr and Pl operators*

Description

Functions for multiplying and solving with the P_r and P_l operators as well as the latent precision matrix $Q = P_l C^{-1} P_l$ and covariance matrix $\Sigma = P_r Q^{-1} P_r^T$. These operations are done without first assembling P_r, P_l in order to avoid numerical problems caused by ill-conditioned matrices.

Usage

```
Pr.mult(obj, v, transpose = FALSE)

Pr.solve(obj, v, transpose = FALSE)

Pl.mult(obj, v, transpose = FALSE)

Pl.solve(obj, v, transpose = FALSE)

Q.mult(obj, v)

Q.solve(obj, v)

Qsqrt.mult(obj, v, transpose = FALSE)

Qsqrt.solve(obj, v, transpose = FALSE)

Sigma.mult(obj, v)

Sigma.solve(obj, v)
```

Arguments

`obj`	rSPDE object
`v`	vector to apply the operation to
`transpose`	set to TRUE if the operation should be performed with the transposed object

Details

`Pl.mult`, `Pr.mult`, and `Q.mult` multiplies the vector with the respective object. Changing `mult` to `solve` in the function names multiplies the vector with the inverse of the object. `Qsqrt.mult` and `Qsqrt.solve` performs the operations with the square-root type object $Q_r = C^{-1/2} P_l$ defined so that $Q = Q_r^T Q_r$.

Value

A vector with the values of the operation

precision *Get the precision matrix of CBrSPDEobj objects*

Description

Function to get the precision matrix of a CBrSPDEobj object

Usage

```
precision(object, ...)

## S3 method for class 'CBrSPDEobj'
precision(
  object,
  nu = NULL,
  kappa = NULL,
  sigma = NULL,
  range = NULL,
  tau = NULL,
  m = NULL,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern.operators()
...	Currently not used.
nu	If non-null, update the shape parameter of the covariance function.
kappa	If non-null, update the range parameter of the covariance function.
sigma	If non-null, update the standard deviation of the covariance function.
range	If non-null, update the range parameter of the covariance function.
tau	If non-null, update the parameter tau.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.

Value

The precision matrix.

See Also

simulate.CBrSPDEobj(), matern.operators()

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8
range <- 0.2

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
  (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
  parameterization = "matern"
)

# Get the precision matrix:
prec_matrix <- precision(op_cov)
```

---

precision.CBrSPDEobj2d

Get the precision matrix of CBrSPDEobj2d objects

Description

Function to get the precision matrix of a CBrSPDEobj2d object

Usage

```
## S3 method for class 'CBrSPDEobj2d'
precision(
  object,
  nu = NULL,
  hx = NULL,
  hy = NULL,
  hxy = NULL,
  sigma = NULL,
  m = NULL,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern2d.operators()
nu	If non-null, update the shape parameter of the covariance function.
hx	If non-null, update the hx parameter.
hy	If non-null, update the hy parameter.
hxy	If non-null, update the hxy parameter.
sigma	If non-null, update the standard deviation of the covariance function.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
...	Currently not used.

Value

The precision matrix.

See Also

simulate.CBrSPDEobj2d(), matern2d.operators()

Examples

```
library(fmesher)
n_loc <- 2000
loc_2d_mesh <- matrix(runif(n_loc * 2), n_loc, 2)
mesh_2d <- fm_mesh_2d(loc = loc_2d_mesh, cutoff = 0.03, max.edge = c(0.1, 0.5))
op <- matern2d.operators(mesh = mesh_2d)
Q <- precision(op)
```

---

precision.inla_rspde *Get the precision matrix of* inla_rspde *objects*

Description

Function to get the precision matrix of an inla_rspde object created with the rspde.matern() function.

Usage

```
## S3 method for class 'inla_rspde'
precision(object, theta = NULL, ...)
```

Arguments

object	The inla_rspde object obtained with the rspde.matern() function.
theta	If null, the starting values for theta will be used. Otherwise, it must be suplied as a vector. For stationary models, we have theta = c(log(tau), log(kappa), nu). For nonstationary models, we have theta = c(theta_1, theta_2, ..., theta_n, nu).
...	Currently not used.

Value

The precision matrix.

See Also

precision.CBrSPDEobj(), matern.operators()

---

precision.intrinsicCBrSPDEobj

Get the precision matrix of intrinsicCBrSPDEobj objects

Description

Function to get the precision matrix of a intrinsicCBrSPDEobj object

Usage

```
## S3 method for class 'intrinsicCBrSPDEobj'
precision(
  object,
  kappa = NULL,
  tau = NULL,
  alpha = NULL,
  beta = NULL,
  ld = FALSE,
  ...
)
```

Arguments

object	The model object computed using intrinsic.matern.operators()
kappa	If non-null, update the range parameter.
tau	If non-null, update the precision parameter.
alpha	If non-null, update the alpha parameter.
beta	If non-null, update the beta parameter.
ld	If TRUE, return the log determinant of the precision matrix instead of the precision matrix. By default FALSE.
...	Currently not used.

Value

The precision matrix.

See Also

simulate.intrinsicCBrSPDEobj(), intrinsic.matern.operators()

Examples

```
if (requireNamespace("RSpectra", quietly = TRUE)) {
  x <- seq(from = 0, to = 10, length.out = 201)
  beta <- 1
  alpha <- 1
  kappa <- 1
  op <- intrinsic.matern.operators(
    kappa = kappa, tau = 1, alpha = alpha,
    beta = beta, loc_mesh = x, d = 1
  )
Q <- precision(op)
}
```

---

precision.rSPDEobj1d *Get the precision matrix of rSPDEobj1d objects*

Description

Function to get the precision matrix of a rSPDEobj1d object

Usage

```
## S3 method for class 'rSPDEobj1d'
precision(
  object,
  loc = NULL,
  nu = NULL,
  kappa = NULL,
  sigma = NULL,
  range = NULL,
  tau = NULL,
  m = NULL,
  ordering = c("field", "location"),
  ldl = FALSE,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern.rational()
loc	If non-null, update the locations where to evaluate the model.
nu	If non-null, update the shape parameter of the covariance function.
kappa	If non-null, update the range parameter of the covariance function.
sigma	If non-null, update the standard deviation of the covariance function.
range	If non-null, update the range parameter of the covariance function.
tau	If non-null, update the parameter tau.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
ordering	Return the matrices ordered by field or by location?
ldl	Directly build the LDL factorization of the precision matrix?
...	Currently not used.

Value

A list containing the precision matrix Q of the process and its derivatives if they exist, and a matrix A that extracts the elements corresponding to the process. If ldl=TRUE, the LDL factorization is returned instead of Q. If the locations are not ordered, the precision matrix is given for the ordered locations, but the A matrix returns to the original order.

See Also

simulate.rSPDEobj1d(), matern.rational()

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
sigma <- 1
nu <- 0.8
range <- 0.2

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)

op_cov <- matern.rational(
  loc = x, nu = nu,
  range = range, sigma = sigma, m = 2,
  parameterization = "matern"
)

# Get the precision matrix:
prec_matrix <- precision(op_cov)
```

precision.spacetimeobj *Get the precision matrix of spacetimeobj objects*

Description

Function to get the precision matrix of a spacetimeobj object

Usage

```
## S3 method for class 'spacetimeobj'
precision(object, kappa = NULL, sigma = NULL, gamma = NULL, rho = NULL, ...)
```

Arguments

object	The model object computed using spacetime.operators()
kappa	If non-null, update the range parameter of the covariance function.
sigma	If non-null, update the standard deviation of the covariance function.
gamma	If non-null, update the temporal range parameter of the covariance function.
rho	If non-null, update the drift parameter of the covariance function.
...	Currently not used.

Value

The precision matrix.

See Also

simulate.spacetimeobj(), spacetime.operators()

Examples

```
s <- seq(from = 0, to = 20, length.out = 101)
t <- seq(from = 0, to = 20, length.out = 31)

op_cov <- spacetime.operators(space_loc = s, time_loc = t,
                              kappa = 5, sigma = 10, alpha = 1,
                              beta = 2, rho = 1, gamma = 0.05)
prec_matrix <- precision(op_cov)
```

`predict.CBrSPDEobj` *Prediction of a fractional SPDE using the covariance-based rational SPDE approximation*

Description

The function is used for computing kriging predictions based on data $Y_i = u(s_i) + \epsilon_i$, where ϵ is mean-zero Gaussian measurement noise and $u(s)$ is defined by a fractional SPDE $(\kappa^2 I - \Delta)^{\alpha/2}(\tau u(s)) = W$, where W is Gaussian white noise and $\alpha = \nu + d/2$, where d is the dimension of the domain.

Usage

```
## S3 method for class 'CBrSPDEobj'
predict(
  object,
  A,
  Aprd,
  Y,
  sigma.e,
  mu = 0,
  compute.variances = FALSE,
  posterior_samples = FALSE,
  n_samples = 100,
  only_latent = FALSE,
  compute_var_method = c("direct", "loop"),
  ...
)
```

Arguments

`object`	The covariance-based rational SPDE approximation, computed using `matern.operators()`
`A`	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
`Aprd`	A matrix linking the prediction locations to the basis of the FEM approximation of the latent model.
`Y`	A vector with the observed data, can also be a matrix where the columns are observations of independent replicates of u.
`sigma.e`	The standard deviation of the Gaussian measurement noise. Put to zero if the model does not have measurement noise.
`mu`	Expectation vector of the latent field (default = 0).
`compute.variances`	Set to also TRUE to compute the kriging variances.
`posterior_samples`	If `TRUE`, posterior samples will be returned.

n_samples	Number of samples to be returned. Will only be used if sampling is TRUE.
only_latent	Should the posterior samples be only given to the laten model?
compute_var_method	Method to compute the variances. Options are: "direct", "loop", and "selected_inv". The "direct" method computes the variance directly, which is faster but can be memory intensive. The "loop" method computes the variance by looping over the elements, which is more memory efficient but slower.
...	further arguments passed to or from other methods.

Value

A list with elements

mean	The kriging predictor (the posterior mean of u|Y).
variance	The posterior variances (if computed).

Examples

```
set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
sigma.e <- 0.3
range <- 0.2

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
  (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))

# Compute the covariance-based rational approximation
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
  parameterization = "matern"
)

# Sample the model
u <- simulate(op_cov)

# Create some data
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
Y <- as.vector(A %*% u + sigma.e * rnorm(10))

# compute kriging predictions at the FEM grid
A.krig <- rSPDE.A1d(x, x)
u.krig <- predict(op_cov,
```

```
  A = A, Aprd = A.krig, Y = Y, sigma.e = sigma.e,
  compute.variances = TRUE
)

plot(obs.loc, Y,
  ylab = "u(x)", xlab = "x", main = "Data and prediction",
  ylim = c(
    min(u.krig$mean - 2 * sqrt(u.krig$variance)),
    max(u.krig$mean + 2 * sqrt(u.krig$variance))
  )
)
lines(x, u.krig$mean)
lines(x, u.krig$mean + 2 * sqrt(u.krig$variance), col = 2)
lines(x, u.krig$mean - 2 * sqrt(u.krig$variance), col = 2)
```

`predict.CBrSPDEobj2d` *Prediction of an anisotropic Whittle-Matern field*

Description

The function is used for computing kriging predictions based on data $Y_i = u(s_i) + \epsilon_i$, where ϵ is mean-zero Gaussian measurement noise and $u(s)$ is defined by a SPDE as described in `matern2d.operators()`.

Usage

```
## S3 method for class 'CBrSPDEobj2d'
predict(
  object,
  A,
  Aprd,
  Y,
  sigma.e,
  mu = 0,
  compute.variances = FALSE,
  posterior_samples = FALSE,
  n_samples = 100,
  only_latent = FALSE,
  ...
)
```

Arguments

`object`	The covariance-based rational SPDE approximation, computed using `matern2d.operators()`
`A`	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
`Aprd`	A matrix linking the prediction locations to the basis of the FEM approximation of the latent model.

Y	A vector with the observed data, can also be a matrix where the columns are observations of independent replicates of u.
sigma.e	The standard deviation of the Gaussian measurement noise. Put to zero if the model does not have measurement noise.
mu	Expectation vector of the latent field (default = 0).
compute.variances	Set to also TRUE to compute the kriging variances.
posterior_samples	If TRUE, posterior samples will be returned.
n_samples	Number of samples to be returned. Will only be used if sampling is TRUE.
only_latent	Should the posterior samples be only given to the laten model?
...	further arguments passed to or from other methods.

Value

A list with elements

mean	The kriging predictor (the posterior mean of u|Y).
variance	The posterior variances (if computed).

Examples

```
library(fmesher)
n_loc <- 2000
loc_2d_mesh <- matrix(runif(n_loc * 2), n_loc, 2)
mesh_2d <- fm_mesh_2d(loc = loc_2d_mesh, cutoff = 0.01, max.edge = c(0.1, 0.5))
op <- matern2d.operators(hx = 0.08, hy = 0.08, hxy = 0.5, nu = 0.5,
sigma = 1, mesh = mesh_2d)
u <- simulate(op)
n.obs <- 2000
obs.loc <- cbind(runif(n.obs),runif(n.obs))
A <- fm_basis(mesh_2d,obs.loc)
sigma.e <- 0.1
Y <- as.vector(A%*%u + sigma.e*rnorm(n.obs))
A <- make_A(op, obs.loc)
proj <- fm_evaluator(mesh_2d, dims = c(100, 100),
            xlim = c(0,1), ylim = c(0,1))
Aprd <- make_A(op, proj$lattice$loc)
u.krig <- predict(op, A = A, Aprd = Aprd, Y = Y, sigma.e = sigma.e)
```

---

predict.inla_rspde_matern1d

Predict method for 'inlabru' stationary Matern 1d models

Description

Auxiliar function to obtain predictions of the stationary Matern 1d models using 'inlabru'.

Usage

```
## S3 method for class 'inla_rspde_matern1d'
predict(
  object,
  cmp,
  bru_fit,
  newdata = NULL,
  formula = NULL,
  n.samples = 100,
  seed = 0L,
  probs = c(0.025, 0.5, 0.975),
  return_original_order = TRUE,
  num.threads = NULL,
  include = NULL,
  exclude = NULL,
  drop = FALSE,
  tolerance = 1e-04,
  ...
)
```

Arguments

object	An inla_rspde_matern1d object built with the rspde.matern1d() function.
cmp	The 'inlabru' component used to fit the model.
bru_fit	A fitted model using 'inlabru' or 'INLA'.
newdata	A data.frame of covariates needed for the prediction.
formula	A formula where the right hand side defines an R expression to evaluate for each generated sample. If NULL, the latent and hyperparameter states are returned as named list elements. See Details for more information.
n.samples	Integer setting the number of samples to draw in order to calculate the posterior statistics. The default is rather low but provides a quick approximate result.
seed	Random number generator seed passed on to inla.posterior.sample()
probs	A numeric vector of probabilities with values in the standard unit interval to be passed to stats::quantile
return_original_order	Should the predictions be returned in the original order?
num.threads	Specification of desired number of threads for parallel computations. Default NULL, leaves it up to 'INLA'. When seed != 0, overridden to "1:1"
include	Character vector of component labels that are needed by the predictor expression; Default: NULL (include all components that are not explicitly excluded)
exclude	Character vector of component labels that are not used by the predictor expression. The exclusion list is applied to the list as determined by the include parameter; Default: NULL (do not remove any components from the inclusion list)

drop	logical; If keep=FALSE, data is a SpatialDataFrame, and the prediciton summary has the same number of rows as data, then the output is a SpatialDataFrame object. Default FALSE.
tolerance	Tolerance for merging locations.
...	Additional arguments passed on to `inla.posterior.sample()`.

Value

A list with predictions.

`predict.intrinsicCBrSPDEobj` *Prediction of an intrinsic Whittle-Matern model*

Description

The function is used for computing kriging predictions based on data $Y_i = u(s_i) + \epsilon_i$, where ϵ is mean-zero Gaussian measurement noise and $u(s)$ is defined by an intrinsic SPDE as described in `intrinsic.matern.operators()`.

Usage

```
## S3 method for class 'intrinsicCBrSPDEobj'
predict(
  object,
  A,
  Aprd,
  Y,
  sigma.e,
  mu = 0,
  compute.variances = FALSE,
  posterior_samples = FALSE,
  n_samples = 100,
  only_latent = FALSE,
  mean_correction = FALSE,
  ind_mean = 1,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using `intrinsic.matern.operators()`
A	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
Aprd	A matrix linking the prediction locations to the basis of the FEM approximation of the latent model.

Y	A vector with the observed data, can also be a matrix where the columns are observations of independent replicates of u.
sigma.e	The standard deviation of the Gaussian measurement noise. Put to zero if the model does not have measurement noise.
mu	Expectation vector of the latent field (default = 0).
compute.variances	Set to also TRUE to compute the kriging variances.
posterior_samples	If TRUE, posterior samples will be returned.
n_samples	Number of samples to be returned. Will only be used if sampling is TRUE.
only_latent	Should the posterior samples be only given to the laten model?
mean_correction	Should mean correction be used for extreme value models?
ind_mean	Index of the mesh node to condition on for the mean correction.
...	further arguments passed to or from other methods.

Value

A list with elements

mean	The kriging predictor (the posterior mean of u|Y).
variance	The posterior variances (if computed).

Examples

```
if (requireNamespace("RSpectra", quietly = TRUE)) {
  x <- seq(from = 0, to = 10, length.out = 201)
  beta <- 1
  alpha <- 1
  kappa <- 1
  op <- intrinsic.matern.operators(
    kappa = kappa, tau = 1, alpha = alpha,
    beta = beta, loc_mesh = x, d = 1
  )
# Create some data
u <-  simulate(op)
sigma.e <- 0.1
obs.loc <- runif(n = 20, min = 0, max = 10)
A <- rSPDE.A1d(x, obs.loc)
Y <- as.vector(A %*% u + sigma.e * rnorm(20))

# compute kriging predictions at the FEM grid
A.krig <- rSPDE.A1d(x, x)
u.krig <- predict(op,
  A = A, Aprd = A.krig, Y = Y, sigma.e = sigma.e,
  compute.variances = TRUE
)
```

```
plot(obs.loc, Y,
  ylab = "u(x)", xlab = "x", main = "Data and prediction",
  ylim = c(
    min(u.krig$mean - 2 * sqrt(u.krig$variance)),
    max(u.krig$mean + 2 * sqrt(u.krig$variance))
  )
)
lines(x, u.krig$mean)
lines(x, u.krig$mean + 2 * sqrt(u.krig$variance), col = 2)
lines(x, u.krig$mean - 2 * sqrt(u.krig$variance), col = 2)
}
```

`predict.rSPDEobj` *Prediction of a fractional SPDE using a rational SPDE approximation*

Description

The function is used for computing kriging predictions based on data $Y_i = u(s_i) + \epsilon_i$, where ϵ is mean-zero Gaussian measurement noise and $u(s)$ is defined by a fractional SPDE $L^\beta u(s) = W$, where W is Gaussian white noise.

Usage

```
## S3 method for class 'rSPDEobj'
predict(
  object,
  A,
  Aprd,
  Y,
  sigma.e,
  compute.variances = FALSE,
  posterior_samples = FALSE,
  n_samples = 100,
  only_latent = FALSE,
  ...
)
```

Arguments

`object`	The rational SPDE approximation, computed using `fractional.operators()`, `matern.operators()`, or `spde.matern.operators()`.
`A`	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
`Aprd`	A matrix linking the prediction locations to the basis of the FEM approximation of the latent model.
`Y`	A vector with the observed data, can also be a matrix where the columns are observations of independent replicates of u.

sigma.e	The standard deviation of the Gaussian measurement noise. Put to zero if the model does not have measurement noise.
compute.variances	Set to also TRUE to compute the kriging variances.
posterior_samples	If TRUE, posterior samples will be returned.
n_samples	Number of samples to be returned. Will only be used if sampling is TRUE.
only_latent	Should the posterior samples be only given to the latent model?
...	further arguments passed to or from other methods.

Value

A list with elements

mean	The kriging predictor (the posterior mean of u|Y).
variance	The posterior variances (if computed).
samples	A matrix containing the samples if sampling is TRUE.

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
sigma.e <- 0.3
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation
op <- matern.operators(
  range = range, sigma = sigma,
  nu = nu, loc_mesh = x, d = 1,
  parameterization = "matern"
)

# Sample the model
u <- simulate(op)

# Create some data
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
Y <- as.vector(A %*% u + sigma.e * rnorm(10))

# compute kriging predictions at the FEM grid
A.krig <- rSPDE.A1d(x, x)
u.krig <- predict(op,
  A = A, Aprd = A.krig, Y = Y, sigma.e = sigma.e,
```

```
  compute.variances = TRUE
)

plot(obs.loc, Y,
  ylab = "u(x)", xlab = "x", main = "Data and prediction",
  ylim = c(
    min(u.krig$mean - 2 * sqrt(u.krig$variance)),
    max(u.krig$mean + 2 * sqrt(u.krig$variance))
  )
)
lines(x, u.krig$mean)
lines(x, u.krig$mean + 2 * sqrt(u.krig$variance), col = 2)
lines(x, u.krig$mean - 2 * sqrt(u.krig$variance), col = 2)
```

`predict.rspde_lme` *Prediction of a mixed effects regression model on a metric graph.*

Description

Prediction of a mixed effects regression model on a metric graph.

Usage

```
## S3 method for class 'rspde_lme'
predict(
  object,
  newdata = NULL,
  loc = NULL,
  time = NULL,
  mesh = FALSE,
  which_repl = NULL,
  compute_variances = FALSE,
  posterior_samples = FALSE,
  n_samples = 100,
  sample_latent = FALSE,
  return_as_list = FALSE,
  return_original_order = TRUE,
  ...,
  data = lifecycle::deprecated()
)
```

Arguments

`object`	The fitted object with the `rspde_lme()` function
`newdata`	A `data.frame` or a `list` containing the covariates, the edge number and the distance on edge for the locations to obtain the prediction.

loc	Prediction locations. Can either be a data.frame, a matrix or a character vector, that contains the names of the columns of the coordinates of the locations. For models using metric_graph objects, plase use edge_number and distance_on_edge instead.
time	Prediction times for spatio-temporal models.
mesh	Obtain predictions for mesh nodes? The graph must have a mesh, and either only_latent is set to TRUE or the model does not have covariates.
which_repl	Which replicates to use? If NULL all replicates will be used.
compute_variances	Set to also TRUE to compute the kriging variances.
posterior_samples	If TRUE, posterior samples will be returned.
n_samples	Number of samples to be returned. Will only be used if sampling is TRUE.
sample_latent	Do posterior samples only for the random effects?
return_as_list	Should the means of the predictions and the posterior samples be returned as a list, with each replicate being an element?
return_original_order	Should the results be return in the original (input) order or in the order inside the graph?
...	Additional arguments. Expert use only.
data	**[Deprecated]** Use newdata instead.

Value

A list with elements mean, which contains the means of the predictions, fe_mean, which is the prediction for the fixed effects, re_mean, which is the prediction for the random effects, variance (if compute_variance is TRUE), which contains the variances of the predictions, samples (if posterior_samples is TRUE), which contains the posterior samples.

---

predict.spacetimeobj *Prediction of a space-time SPDE*

Description

The function is used for computing kriging predictions based on data $Y_i = u(s_i, t_i) + \epsilon_i$, where ϵ is mean-zero Gaussian measurement noise and $u(s, t)$ is defined by a spatio-temporal SPDE as described in spacetime.operators().

Usage

```
## S3 method for class 'spacetimeobj'
predict(
  object,
  A,
  Aprd,
  Y,
  sigma.e,
  mu = 0,
  compute.variances = FALSE,
  posterior_samples = FALSE,
  n_samples = 100,
  only_latent = FALSE,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using spacetime.operators()
A	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
Aprd	A matrix linking the prediction locations to the basis of the FEM approximation of the latent model.
Y	A vector with the observed data, can also be a matrix where the columns are observations of independent replicates of u.
sigma.e	The standard deviation of the Gaussian measurement noise. Put to zero if the model does not have measurement noise.
mu	Expectation vector of the latent field (default = 0).
compute.variances	Set to also TRUE to compute the kriging variances.
posterior_samples	If TRUE, posterior samples will be returned.
n_samples	Number of samples to be returned. Will only be used if sampling is TRUE.
only_latent	Should the posterior samples be only given to the laten model?
...	further arguments passed to or from other methods.

Value

A list with elements

mean	The kriging predictor (the posterior mean of u|Y).
variance	The posterior variances (if computed).

Examples

```
s <- seq(from = 0, to = 20, length.out = 101)
t <- seq(from = 0, to = 20, length.out = 31)

op_cov <- spacetime.operators(space_loc = s, time_loc = t,
                              kappa = 5, sigma = 10, alpha = 1,
                              beta = 2, rho = 1, gamma = 0.05)
# generate data
sigma.e <- 0.01
n.obs <- 500
obs.loc <- data.frame(x = max(s)*runif(n.obs),
                      t = max(t)*runif(n.obs))
A <- rSPDE.Ast(space_loc = s, time_loc = t, obs.s = obs.loc$x, obs.t = obs.loc$t)
Aprd <- Diagonal(dim(A)[2])
x <- simulate(op_cov, nsim = 1)
Y <- A%*%x + sigma.e*rnorm(n.obs)
u.krig <- predict(op_cov, A, Aprd, Y, sigma.e)
```

---

rational.order *Get the order of rational approximation.*

Description

Get the order of rational approximation.

Usage

```
rational.order(object)
```

Arguments

object	A CBrSPDEobj object or an inla_rspde object.

Value

The order of rational approximation.

---

rational.order<- *Changing the order of the rational approximation*

Description

Changing the order of the rational approximation

Usage

```
rational.order(x) <- value
```

Arguments

x	A CBrSPDE or an rpsde.inla object
value	The order of rational approximation.

Value

An object of the same class with the new order of rational approximation.

---

rational.type *Get type of rational approximation.*

Description

Get type of rational approximation.

Usage

```
rational.type(object)
```

Arguments

object	A CBrSPDEobj object or an inla_rspde object.

Value

The type of rational approximation.

---

rational.type<- *Changing the type of the rational approximation*

Description

Changing the type of the rational approximation

Usage

```
rational.type(x) <- value
```

Arguments

x	A CBrSPDE or an rpsde.inla object
value	The type of rational approximation. The current options are "chebfun", "brasil" and "chebfunLB"

Value

An object of the same class with the new rational approximation.

`require.nowarnings` *Warnings free loading of add-on packages*

Description

Turn off all warnings for require(), to allow clean completion of examples that require unavailable Suggested packages.

Usage

```
require.nowarnings(package, lib.loc = NULL, character.only = FALSE)
```

Arguments

`package`	The name of a package, given as a character string.
`lib.loc`	a character vector describing the location of R library trees to search through, or `NULL`. The default value of `NULL` corresponds to all libraries currently known to `.libPaths()`. Non-existent library trees are silently ignored.
`character.only`	a logical indicating whether `package` can be assumed to be a character string.

Details

`require(package)` acts the same as `require(package, quietly = TRUE)` but with warnings turned off. In particular, no warning or error is given if the package is unavailable. Most cases should use `requireNamespace(package, quietly = TRUE)` instead, which doesn't produce warnings.

Value

`require.nowarnings` returns (invisibly) `TRUE` if it succeeds, otherwise `FALSE`

See Also

`require()`

Examples

```
## This should produce no output:
if (require.nowarnings(nonexistent)) {
  message("Package loaded successfully")
}
```

rSPDE.A1d *Observation matrix for finite element discretization on R*

Description

A finite element discretization on R can be written as $u(s) = \sum_i^n u_i \varphi_i(s)$ where $\varphi_i(s)$ is a piecewise linear "hat function" centered at location x_i. This function computes an $m \times n$ matrix A that links the basis function in the expansion to specified locations $s = (s_1, \ldots, s_m)$ in the domain through $A_i j = \varphi_j(s_i)$.

Usage

```
rSPDE.A1d(x, loc)
```

Arguments

x	The locations of the nodes in the FEM discretization.
loc	The locations $(s_1, \ldots, s_m)$

Value

The sparse matrix A.

Author(s)

David Bolin <davidbolin@gmail.com>

See Also

rSPDE.fem1d()

Examples

```
# create mass and stiffness matrices for a FEM discretization on [0,1]
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# create the observation matrix for some locations in the domain
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
```

`rspde.anistropic2d` *Rational approximations of stationary anisotropic Gaussian Matern random fields*

Description

`rspde.anistropic2d` computes a Finite Element Method (FEM) approximation of a Gaussian random field defined as the solution to the stochastic partial differential equation (SPDE):

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\sqrt{h^T H^{-1} h})^{\nu} K_{\nu}(\sqrt{h^T H^{-1} h})$$

, based on a SPDE representation of the form

$$(I - \nabla \cdot (H\nabla))^{(\nu+1)/2} u = c\sigma W$$

, where $c>0$ is a constant. The matrix H is defined as

$$\begin{bmatrix} h_x^2 & h_x h_y h_{xy} \\ h_x h_y h_{xy} & h_y^2 \end{bmatrix}$$

Usage

```
rspde.anistropic2d(
  mesh,
  nu = NULL,
  nu.upper.bound = 2,
  rspde.order = 1,
  prior.hx = NULL,
  prior.hy = NULL,
  prior.hxy = NULL,
  prior.sigma = NULL,
  prior.precision = NULL,
  prior.nu = NULL,
  prior.nu.dist = "lognormal",
  nu.prec.inc = 0.01,
  type.rational.approx = "brasil",
  shared_lib = "detect",
  debug = FALSE,
  ...
)
```

Arguments

`mesh`	Spatial mesh for the FEM approximation.
`nu`	If nu is set to a parameter, nu will be kept fixed and will not be estimated. If nu is `NULL`, it will be estimated.
`nu.upper.bound`	Upper bound for the smoothness parameter ν. If `NULL`, it will be set to 2.

`rspde.order` The order of the covariance-based rational SPDE approach. The default order is 1.

`prior.hx` A list specifying the prior for the parameter h_x in the matrix H. This list may contain two elements: `mean` and/or `precision`, both of which must be numeric scalars. The precision refers to the prior on $\log(h_x)$. If `NULL`, default values will be used. The `mean` value is also used as starting value for hx.

`prior.hy` A list specifying the prior for the parameter h_y in the matrix H. This list may contain two elements: `mean` and/or `precision`, both of which must be numeric scalars. The precision refers to the prior on $\log(h_x)$. If `NULL`, default values will be used. The `mean` value is also used as starting value for hy.

`prior.hxy` A list specifying the prior for the parameter h_x in the matrix H. This list may contain two elements: `mean` and/or `precision`, both of which must be numeric scalars. The precision refers to the prior on $\log((h_{xy}+1)/(1-h_{xy}))$. If `NULL`, default values will be used. The `mean` value is also used as starting value for hxy.

`prior.sigma` A list specifying the prior for the variance parameter σ. This list may contain two elements: `mean` and/or `precision`, both of which must be numeric scalars. The precision refers to the prior on $\log(\sigma)$. If `NULL`, default values will be used. The `mean` value is also used as starting value for sigma.

`prior.precision`
A precision matrix for $\log(h_x), \log(h_y), \log((h_{xy}+1)/(1-h_{xy})), \log(\sigma)$. This matrix replaces the precision element from `prior.kappa`, `prior.sigma`, `prior.gamma`, and `prior.rho` respectively. For dimension 1 `prior.precision` must be a 4x4 matrix. For dimension 2, ρ is a vector of length 2, so in this case `prior.precision` must be a 5x5 matrix. If `NULL`, a diagonal precision matrix with default values will be used.

`prior.nu` a list containing the elements `mean` and `prec` for beta distribution, or `loglocation` and `logscale` for a truncated lognormal distribution. `loglocation` stands for the location parameter of the truncated lognormal distribution in the log scale. `prec` stands for the precision of a beta distribution. `logscale` stands for the scale of the truncated lognormal distribution on the log scale. Check details below.

`prior.nu.dist` The distribution of the smoothness parameter. The current options are "beta" or "lognormal". The default is "lognormal".

`nu.prec.inc` Amount to increase the precision in the beta prior distribution. Check details below.

`type.rational.approx`
Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

`shared_lib` String specifying which shared library to use for the Cgeneric implementation. Options are "detect", "INLA", or "rSPDE". You may also specify the direct path to a .so (or .dll) file.

`debug` Logical value indicating whether to enable INLA debug mode.

`...` Additional arguments passed internally for configuration purposes.

Value

An object of class `inla_rspde_spacetime` representing the FEM approximation of the space-time Gaussian random field.

`rSPDE.Ast` *Observation matrix for space-time models*

Description

Observation matrix for space-time models

Usage

```
rSPDE.Ast(
  mesh_space = NULL,
  space_loc = NULL,
  mesh_time = NULL,
  time_loc = NULL,
  graph = NULL,
  obs.s = NULL,
  obs.t = NULL,
  ind.field = NULL
)
```

Arguments

`mesh_space`	mesh object for models on 1d or 2d domains
`space_loc`	mesh locations for models on 1d domains
`mesh_time`	mesh object for time discretization
`time_loc`	mesh locations for time discretization
`graph`	MetricGraph object for models on metric graphs
`obs.s`	spatial locations of observations
`obs.t`	time points for observations
`ind.field`	indices for field (if Dirichlet conditions are used)

Value

Observation matrix linking observation locations to mesh nodes

Examples

```
s <- seq(from = 0, to = 20, length.out = 11)
t <- seq(from = 0, to = 20, length.out = 5)
n.obs <- 10
obs.loc <- data.frame(x = max(s)*runif(n.obs),
t = max(t)*runif(n.obs))
A <- rSPDE.Ast(space_loc = s,time_loc = t,
               obs.s = obs.loc$x, obs.t = obs.loc$t)
```

---

rSPDE.construct.matern.loglike

Constructor of Matern loglikelihood functions.

Description

This function returns a log-likelihood function for a Gaussian process with a Matern covariance function, that is observed under Gaussian measurement noise: $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using the a rational approximation of the fractional SPDE model corresponding to the Gaussian process.

Usage

```
rSPDE.construct.matern.loglike(
  object,
  Y,
  A,
  sigma.e = NULL,
  mu = 0,
  nu = NULL,
  tau = NULL,
  kappa = NULL,
  sigma = NULL,
  range = NULL,
  parameterization = c("spde", "matern"),
  m = NULL,
  log_scale = TRUE,
  return_negative_likelihood = TRUE
)
```

Arguments

object	The rational SPDE approximation, computed using matern.operators()
Y	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
A	An observation matrix that links the measurement location to the finite element basis.

sigma.e	IF non-null, the standard deviation of the measurement noise will be kept fixed in the returned likelihood.
mu	Expectation vector of the latent field (default = 0).
nu	If non-null, the shape parameter will be kept fixed in the returned likelihood.
tau	If non-null, the tau parameter will be kept fixed in the returned likelihood. (Replaces sigma)
kappa	If non-null, the range parameter will be kept fixed in the returned likelihood.
sigma	If non-null, the standard deviation will be kept fixed in the returned likelihood.
range	If non-null, the range parameter will be kept fixed in the returned likelihood. (Replaces kappa)
parameterization	If spde, then one will use the parameters tau and kappa. If matern, then one will use the parameters sigma and range.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
log_scale	Should the parameters be evaluated in log-scale?
return_negative_likelihood	Return minus the likelihood to turn the maximization into a minimization?

Value

The log-likelihood function. The parameters of the returned function are given in the order sigma, kappa, nu, sigma.e, whenever they are available.

See Also

matern.operators(), predict.CBrSPDEobj()

Examples

```
# this example illustrates how the function can be used for maximum
# likelihood estimation

set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
nu <- 0.8
sigma <- 1
sigma.e <- 0.1
n.rep <- 10
n.obs <- 200
n.x <- 51
range <- 0.2
# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
# Compute the covariance-based rational approximation
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
```

```
  parameterization = "matern"
)
# Sample the model
u <- simulate(op_cov, n.rep)
# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)

# Define the negative likelihood function for optimization
# using CBrSPDE.matern.loglike
# Matern parameterization
loglike <- rSPDE.construct.matern.loglike(op_cov, Y, A, parameterization = "matern")

# The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- c(
  get.initial.values.rSPDE(mesh.range = 1, dim = 1),
  log(0.1 * sd(as.vector(Y)))
)
# run estimation and display the results
theta <- optim(theta0, loglike,
  method = "L-BFGS-B"
)
print(data.frame(
  sigma = c(sigma, exp(theta$par[1])), range = c(range, exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`rSPDE.fem1d` *Finite element calculations for problems on R*

Description

This function computes mass and stiffness matrices for a FEM approximation on R, assuming Neumann boundary conditions. These matrices are needed when discretizing the operators in rational approximations.

Usage

```
rSPDE.fem1d(x)
```

Arguments

`x`	Locations of the nodes in the FEM approximation.

Value

The function returns a list with the following elements

G	The stiffness matrix with elements $(\nabla\phi_i, \nabla\phi_j)$.
C	The mass matrix with elements (ϕ_i, ϕ_j).
Cd	Mass lumped mass matrix.
B	Matrix with elements $(\nabla\phi_i, \phi_j)$.

Author(s)

David Bolin <davidbolin@gmail.com>

See Also

rSPDE.A1d()

Examples

```
# create mass and stiffness matrices for a FEM discretization on [0,1]
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)
```

---

rSPDE.fem2d *Finite element calculations for problems in 2D*

Description

This function computes mass and stiffness matrices for a mesh in 2D, assuming Neumann boundary conditions.

Usage

```
rSPDE.fem2d(FV, P)
```

Arguments

FV	Matrix where each row defines a triangle
P	Locations of the nodes in the mesh.

Value

The function returns a list with the following elements

G	The stiffness matrix with elements $(\nabla\phi_i, \nabla\phi_j)$.
C	The mass matrix with elements (ϕ_i, ϕ_j).
Cd	The mass lumped matrix with diagonal elements $(\phi_i, 1)$.
Hxx	Matrix with elements $(\partial_x\phi_i, \partial_x\phi_j)$.
Hyy	Matrix with elements $(\partial_y\phi_i, \partial_y\phi_j)$.
Hxy	Matrix with elements $(\partial_x\phi_i, \partial_y\phi_j)$.
Hyx	Matrix with elements $(\partial_y\phi_i, \partial_x\phi_j)$.
Bx	Matrix with elements $(\partial_x\phi_i, \phi_j)$.
By	Matrix with elements $(\partial_y\phi_i, \phi_j)$.

Author(s)

David Bolin <davidbolin@gmail.com>

See Also

rSPDE.fem1d()

Examples

```
P <- rbind(c(0, 0), c(1, 0), c(1, 1), c(0, 1))
FV <- rbind(c(1, 2, 3), c(2, 3, 4))
fem <- rSPDE.fem2d(FV, P)
```

---

rspde.intrinsic *Rational approximations of fractional intrinsic fields*

Description

rspde.intrinsic computes a Finite Element Method (FEM) approximation of a Gaussian random field defined as the solution to the stochastic partial differential equation (SPDE):

$$(-\Delta)^{(\nu+d/2)/2}\tau u = W$$

.

Usage

```
rspde.intrinsic(
  mesh,
  nu = NULL,
  nu.upper.bound = 2,
  mean.correction = FALSE,
  rspde.order = 1,
  prior.tau = NULL,
  prior.nu = NULL,
  prior.nu.dist = "lognormal",
  nu.prec.inc = 0.01,
  diagonal = 1e-05,
  type.rational.approx = "brasil",
  shared_lib = "detect",
  debug = FALSE,
  cache = TRUE,
  scaling = NULL,
  opts = NULL,
  ...
)
```

Arguments

mesh	Spatial mesh for the FEM approximation.
nu	If nu is set to a parameter, nu will be kept fixed and will not be estimated. If nu is NULL, it will be estimated.
nu.upper.bound	Upper bound for the smoothness parameter ν. If NULL, it will be set to 2.
mean.correction	Add mean correction for extreme value models?
rspde.order	The order of the covariance-based rational SPDE approach. The default order is 1.
prior.tau	A list specifying the prior for the variance parameter τ. This list may contain two elements: mean and/or precision, both of which must be numeric scalars.
prior.nu	a list containing the elements mean and prec for beta distribution, or loglocation and logscale for a truncated lognormal distribution. loglocation stands for the location parameter of the truncated lognormal distribution in the log scale. prec stands for the precision of a beta distribution. logscale stands for the scale of the truncated lognormal distribution on the log scale. Check details below.
prior.nu.dist	The distribution of the smoothness parameter. The current options are "beta" or "lognormal". The default is "lognormal".
nu.prec.inc	Amount to increase the precision in the beta prior distribution. Check details below.
diagonal	Number added to diagonal of Q for increased stability.

`type.rational.approx`
: Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

`shared_lib`
: String specifying which shared library to use for the Cgeneric implementation. Options are "detect", "INLA", or "rSPDE". You may also specify the direct path to a .so (or .dll) file.

`debug`
: Logical value indicating whether to enable INLA debug mode.

`cache`
: Use caching internally in the estimation?

`scaling`
: A positive numeric value of length 1 for scaling the model. If NULL (default), it will be computed using RSpectra::eigs. Must be positive if provided.

`opts`
: A list of options passed to RSpectra::eigs function. See RSpectra documentation for available options.

`...`
: Additional arguments passed internally for configuration purposes.

Value

An object of class `inla_rspde_intrinsic` representing the FEM approximation of the intrinsic Gaussian random field.

`rSPDE.loglike` *Object-based log-likelihood function for latent Gaussian fractional SPDE model*

Description

This function evaluates the log-likelihood function for a fractional SPDE model $L^\beta u(s) = W$ that is observed under Gaussian measurement noise: $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables and $x(s) = \mu(s) + u(s)$, where $\mu(s)$ is the expectation vector of the latent field.

Usage

```
rSPDE.loglike(obj, Y, A, sigma.e, mu = 0)
```

Arguments

`obj`
: The rational SPDE approximation, computed using `fractional.operators()`, `matern.operators()`, or `spde.matern.operators()`.

`Y`
: The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.

`A`
: An observation matrix that links the measurement location to the finite element basis.

`sigma.e`
: The standard deviation of the measurement noise.

`mu`
: Expectation vector of the latent field (default = 0).

Value

The log-likelihood value.

Note

This example below shows how the function can be used to evaluate the likelihood of a latent Matern model.

See Also

spde.matern.loglike()

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
sigma.e <- 0.3
range <- 0.2

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation
op <- matern.operators(
  range = range, sigma = sigma, nu = nu,
  loc_mesh = x, d = 1,
  type = "operator", parameterization = "matern"
)

# Sample the model
u <- simulate(op)

# Create some data
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
Y <- as.vector(A %*% u + sigma.e * rnorm(10))

# compute log-likelihood of the data
lik1 <- rSPDE.loglike(op, Y, A, sigma.e)
cat(lik1)
```

---

rspde.make.A *Observation/prediction matrices for rSPDE models.*

Description

Constructs observation/prediction weight matrices for rSPDE models based on `inla.mesh` or `inla.mesh.1d` objects.

Usage

```
rspde.make.A(
  mesh = NULL,
  loc = NULL,
  A = NULL,
  dim = NULL,
  rspde.order = 1,
  nu = NULL,
  index = NULL,
  group = NULL,
  repl = 1L,
  n.group = NULL,
  n.repl = NULL
)
```

Arguments

`mesh`	An `inla.mesh`, an `inla.mesh.1d` object or a `metric_graph` object.
`loc`	Locations, needed if an INLA mesh is provided
`A`	The A matrix from the standard SPDE approach, such as the matrix returned by `inla.spde.make.A`. Should only be provided if `mesh` is not provided.
`dim`	the dimension. Should only be provided if an `mesh` is not provided.
`rspde.order`	The order of the covariance-based rational SPDE approach.
`nu`	If `NULL`, then the model will assume that nu will be estimated. If nu is fixed, you should provide the value of nu.
`index`	For each observation/prediction value, an index into loc. Default is seq_len(nrow(A.loc)).
`group`	For each observation/prediction value, an index into the group model.
`repl`	For each observation/prediction value, the replicate index.
`n.group`	The size of the group model.
`n.repl`	The total number of replicates.

Value

The A matrix for rSPDE models.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)) {
  library(INLA)
```

```
  set.seed(123)
  loc <- matrix(runif(100 * 2) * 100, 100, 2)
  mesh <- inla.mesh.2d(
    loc = loc,
    cutoff = 50,
    max.edge = c(50, 500)
  )
  A <- rspde.make.A(mesh, loc = loc, rspde.order = 3)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`rspde.make.index` *rSPDE model index vector generation*

Description

Generates a list of named index vectors for an rSPDE model.

Usage

```
rspde.make.index(
  name,
  n.spde = NULL,
  n.group = 1,
  n.repl = 1,
  mesh = NULL,
  rspde.order = 1,
  nu = NULL,
  dim = NULL
)
```

Arguments

`name`	A character string with the base name of the effect.
`n.spde`	The number of basis functions in the mesh model.
`n.group`	The size of the group model.
`n.repl`	The total number of replicates.
`mesh`	An `inla.mesh`, an `inla.mesh.1d` object or a `metric_graph` object.
`rspde.order`	The order of the rational approximation
`nu`	If `NULL`, then the model will assume that nu will be estimated. If nu is fixed, you should provide the value of nu.
`dim`	the dimension of the domain. Should only be provided if `mesh` is not provided.

Value

A list of named index vectors.

name	Indices into the vector of latent variables
name.group	'group' indices
name.repl	Indices for replicates

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)) {
  library(INLA)

  set.seed(123)

  m <- 100
  loc_2d_mesh <- matrix(runif(m * 2), m, 2)
  mesh_2d <- inla.mesh.2d(
    loc = loc_2d_mesh,
    cutoff = 0.05,
    max.edge = c(0.1, 0.5)
  )
  sigma <- 1
  range <- 0.2
  nu <- 0.8
  kappa <- sqrt(8 * nu) / range
  op <- matern.operators(
    mesh = mesh_2d, nu = nu,
    range = range, sigma = sigma, m = 2,
    parameterization = "matern"
  )
  u <- simulate(op)
  A <- inla.spde.make.A(
    mesh = mesh_2d,
    loc = loc_2d_mesh
  )
  sigma.e <- 0.1
  y <- A %*% u + rnorm(m) * sigma.e
  Abar <- rspde.make.A(mesh = mesh_2d, loc = loc_2d_mesh)
  mesh.index <- rspde.make.index(name = "field", mesh = mesh_2d)
  st.dat <- inla.stack(
    data = list(y = as.vector(y)),
    A = Abar,
    effects = mesh.index
  )
  rspde_model <- rspde.matern(
    mesh = mesh_2d,
    nu.upper.bound = 2
  )
  f <- y ~ -1 + f(field, model = rspde_model)
  rspde_fit <- inla(f,
```

```
    data = inla.stack.data(st.dat),
    family = "gaussian",
    control.predictor =
      list(A = inla.stack.A(st.dat))
  )
  result <- rspde.result(rspde_fit, "field", rspde_model)
  summary(result)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`rspde.matern` *Matern rSPDE model object for INLA*

Description

Creates an INLA object for a stationary Matern model with general smoothness parameter.

Usage

```
rspde.matern(
  mesh,
  nu.upper.bound = NULL,
  rspde.order = 1,
  nu = NULL,
  B.sigma = matrix(c(0, 1, 0), 1, 3),
  B.range = matrix(c(0, 0, 1), 1, 3),
  parameterization = c("spde", "matern", "matern2"),
  B.tau = matrix(c(0, 1, 0), 1, 3),
  B.kappa = matrix(c(0, 0, 1), 1, 3),
  start.nu = NULL,
  start.theta = NULL,
  prior.nu = NULL,
  theta.prior.mean = NULL,
  theta.prior.prec = 0.1,
  prior.std.dev.nominal = 1,
  prior.range.nominal = NULL,
  prior.kappa.mean = NULL,
  prior.tau.mean = NULL,
  start.lstd.dev = NULL,
  start.lrange = NULL,
  start.ltau = NULL,
  start.lkappa = NULL,
  prior.theta.param = c("theta", "spde"),
  prior.nu.dist = c("beta", "lognormal"),
  nu.prec.inc = 1,
  type.rational.approx = c("brasil", "chebfun", "chebfunLB"),
```

```
  debug = FALSE,
  shared_lib = "detect",
  ...
)
```

Arguments

mesh	The mesh to build the model. It can be an inla.mesh or an inla.mesh.1d object. Otherwise, should be a list containing elements d, the dimension, C, the mass matrix, and G, the stiffness matrix.
nu.upper.bound	Upper bound for the smoothness parameter. If NULL, it will be set to 2.
rspde.order	The order of the covariance-based rational SPDE approach. The default order is 1.
nu	If nu is set to a parameter, nu will be kept fixed and will not be estimated. If nu is NULL, it will be estimated.
B.sigma	Matrix with specification of log-linear model for σ (for 'matern' parameterization) or for σ^2 (for 'matern2' parameterization). Will be used if parameterization = 'matern' or parameterization = 'matern2'.
B.range	Matrix with specification of log-linear model for ρ, which is a range-like parameter (it is exactly the range parameter in the stationary case). Will be used if parameterization = 'matern' or parameterization = 'matern2'.
parameterization	Which parameterization to use? matern uses range, std. deviation and nu (smoothness). spde uses kappa, tau and nu (smoothness). matern2 uses range-like (1/kappa), variance and nu (smoothness). The default is spde.
B.tau	Matrix with specification of log-linear model for τ. Will be used if parameterization = 'spde'.
B.kappa	Matrix with specification of log-linear model for κ. Will be used if parameterization = 'spde'.
start.nu	Starting value for nu.
start.theta	Starting values for the model parameters. In the stationary case, if parameterization='matern', then theta[1] is the std.dev and theta[2] is the range parameter. If parameterization = 'spde', then theta[1] is tau and theta[2] is kappa.
prior.nu	a list containing the elements mean and prec for beta distribution, or loglocation and logscale for a truncated lognormal distribution. loglocation stands for the location parameter of the truncated lognormal distribution in the log scale. prec stands for the precision of a beta distribution. logscale stands for the scale of the truncated lognormal distribution on the log scale. Check details below.
theta.prior.mean	A vector for the mean priors of theta.
theta.prior.prec	A precision matrix for the prior of theta.
prior.std.dev.nominal	Prior std. deviation to be used for the priors and for the starting values.

`prior.range.nominal`
: Prior range to be used for the priors and for the starting values.

`prior.kappa.mean`
: Prior kappa to be used for the priors and for the starting values.

`prior.tau.mean` Prior tau to be used for the priors and for the starting values.

`start.lstd.dev` Starting value for log of std. deviation. Will not be used if start.ltau is non-null. Will be only used in the stationary case and if `parameterization = 'matern'`.

`start.lrange` Starting value for log of range. Will not be used if start.lkappa is non-null. Will be only used in the stationary case and if `parameterization = 'matern'`.

`start.ltau` Starting value for log of tau. Will be only used in the stationary case and if `parameterization = 'spde'`.

`start.lkappa` Starting value for log of kappa. Will be only used in the stationary case and if `parameterization = 'spde'`.

`prior.theta.param`
: Should the lognormal prior be on `theta` or on the SPDE parameters (`tau` and `kappa` on the stationary case)?

`prior.nu.dist` The distribution of the smoothness parameter. The current options are "beta" or "lognormal". The default is "lognormal".

`nu.prec.inc` Amount to increase the precision in the beta prior distribution. Check details below.

`type.rational.approx`
: Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

`debug` INLA debug argument

`shared_lib` Which shared lib to use for the cgeneric implementation? If "detect", it will check if the shared lib exists locally, in which case it will use it. Otherwise it will use INLA's shared library. If "INLA", it will use the shared lib from INLA's installation. If 'rSPDE', then it will use the local installation (does not work if your installation is from CRAN). Otherwise, you can directly supply the path of the .so (or .dll) file.

`...` Only being used internally.

`prior.kappa` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale.

`prior.tau` a list containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale.

`prior.range` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale. Will not be used if prior.kappa is non-null.

`prior.std.dev` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale. Will not be used if prior.tau is non-null.

Value

An INLA model.

`rspde.matern.intrinsic` *Intrinsic Matern rSPDE model object for INLA*

Description

Creates an INLA object for a stationary intrinsic Matern model. Currently, alpha is fixed to 2 and beta is fixed to 1.

Usage

```
rspde.intrinsic.matern(
  mesh,
  alpha = 2,
  mean.correction = FALSE,
  prior.lkappa.mean = NULL,
  prior.ltau.mean = 1,
  prior.lkappa.prec = 0.1,
  prior.ltau.prec = 0.1,
  start.ltau = NULL,
  start.lkappa = NULL,
  true.scaling = TRUE,
  diagonal = 0,
  debug = FALSE,
  shared_lib = "detect",
  ...
)
```

Arguments

`mesh`	The mesh to build the model. It can be an `inla.mesh` or an `inla.mesh.1d` object. Otherwise, should be a list containing elements d, the dimension, C, the mass matrix, and G, the stiffness matrix.
`alpha`	Smoothness parameter, need to be 1 or 2.
`mean.correction`	Add mean correction for extreme value models?
`prior.lkappa.mean`	Prior on log kappa to be used for the priors and for the starting values.
`prior.ltau.mean`	Prior on log tau to be used for the priors and for the starting values.
`prior.lkappa.prec`	Precision to be used on the prior on log kappa to be used for the priors and for the starting values.
`prior.ltau.prec`	Precision to be used on the prior on log tau to be used for the priors and for the starting values.

start.ltau	Starting value for log of tau.
start.lkappa	Starting value for log of kappa.
true.scaling	Compute the true normalizing constant manually? Default TRUE. The alternative is to set this to FALSE and set the diagonal argument to some small positive value. In the latter case, the model is approximated by a non-intrinsic model with a precision matrix that has the diagonal value added to the diagonal.
diagonal	Value of diagonal correction for INLA stability. Default 0.
debug	INLA debug argument
shared_lib	Which shared lib to use for the cgeneric implementation? If "detect", it will check if the shared lib exists locally, in which case it will use it. Otherwise it will use INLA's shared library. If "INLA", it will use the shared lib from INLA's installation. If 'rSPDE', then it will use the local installation (does not work if your installation is from CRAN). Otherwise, you can directly supply the path of the .so (or .dll) file.
...	Only being used internally.

Value

An INLA model.

`rSPDE.matern.loglike` *Object-based log-likelihood function for latent Gaussian fractional SPDE model using the rational approximations*

Description

This function evaluates the log-likelihood function for a Gaussian process with a Matern covariance function, that is observed under Gaussian measurement noise: $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using the a rational approximation of the fractional SPDE model corresponding to the Gaussian process.

Usage

```
rSPDE.matern.loglike(
  object,
  Y,
  A,
  sigma.e,
  mu = 0,
  nu = NULL,
  kappa = NULL,
  sigma = NULL,
  range = NULL,
  tau = NULL,
  m = NULL
)
```

Arguments

object	The rational SPDE approximation, computed using matern.operators()
Y	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
A	An observation matrix that links the measurement location to the finite element basis.
sigma.e	The standard deviation of the measurement noise.
mu	Expectation vector of the latent field (default = 0).
nu	If non-null, update the shape parameter of the covariance function.
kappa	If non-null, update the range parameter of the covariance function.
sigma	If non-null, update the standard deviation of the covariance function.
range	If non-null, update the range parameter of the covariance function.
tau	If non-null, update the parameter tau.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.

Value

The log-likelihood value.

See Also

matern.operators(), predict.CBrSPDEobj()

Examples

```
# this example illustrates how the function can be used for maximum likelihood estimation

set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
nu <- 0.8
kappa <- 5
sigma <- 1
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51
range <- 0.2

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)

tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
  (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))

# Compute the covariance-based rational approximation
```

```
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
  parameterization = "matern"
)

# Sample the model
u <- simulate(op_cov, n.rep)

# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)

# Define the negative likelihood function for optimization
# using CBrSPDE.matern.loglike

# Notice that we are also using sigma instead of tau, so it can be compared
# to matern.loglike()
mlik_cov <- function(theta, Y, A, op_cov) {
  kappa <- exp(theta[1])
  sigma <- exp(theta[2])
  nu <- exp(theta[3])
  return(-rSPDE.matern.loglike(
    object = op_cov, Y = Y,
    A = A, kappa = kappa, sigma = sigma,
    nu = nu, sigma.e = exp(theta[4])
  ))
}

# The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(sqrt(8), 1 / sqrt(var(c(Y))), 0.9, 0.01))

# run estimation and display the results
theta <- optim(theta0, mlik_cov,
  Y = Y, A = A, op_cov = op_cov,
  method = "L-BFGS-B"
)

print(data.frame(
  range = c(range, exp(theta$par[1])), sigma = c(sigma, exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`rspde.matern.precision` *Precision matrix of the covariance-based rational approximation of stationary Gaussian Matern random fields*

Description

`rspde.matern.precision` is used for computing the precision matrix of the covariance-based rational SPDE approximation of a stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2(\nu-1)\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h)$$

Usage

```
rspde.matern.precision(
  kappa,
  nu,
  tau = NULL,
  sigma = NULL,
  rspde.order,
  dim,
  fem_mesh_matrices,
  only_fractional = FALSE,
  return_block_list = FALSE,
  type_rational_approx = "brasil"
)
```

Arguments

`kappa`	Range parameter of the covariance function.
`nu`	Shape parameter of the covariance function.
`tau`	Scale parameter of the covariance function. If sigma is not provided, tau should be provided.
`sigma`	Standard deviation of the covariance function. If tau is not provided, sigma should be provided.
`rspde.order`	The order of the rational approximation
`dim`	The dimension of the domain
`fem_mesh_matrices`	A list containing the FEM-related matrices. The list should contain elements c0, g1, g2, g3, etc.
`only_fractional`	Logical. Should only the fractional-order part of the precision matrix be returned?

return_block_list
: Logical. For type = "covariance", should the block parts of the precision matrix be returned separately as a list?

type_rational_approx
: Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

Value

The precision matrix

Examples

```
set.seed(123)
nobs <- 101
x <- seq(from = 0, to = 1, length.out = nobs)
fem <- rSPDE.fem1d(x)
kappa <- 40
sigma <- 1
d <- 1
nu <- 2.6
tau <- sqrt(gamma(nu) / (kappa^(2 * nu) * (4 * pi)^(d / 2) *
  gamma(nu + d / 2)))
range <- sqrt(8 * nu) / kappa
op_cov <- matern.operators(
  loc_mesh = x, nu = nu, range = range, sigma = sigma,
  d = 1, m = 2, compute_higher_order = TRUE,
  parameterization = "matern"
)
v <- t(rSPDE.A1d(x, 0.5))
c.true <- matern.covariance(abs(x - 0.5), kappa, nu, sigma)
Q <- rspde.matern.precision(
  kappa = kappa, nu = nu, tau = tau, rspde.order = 2, d = 1,
  fem_mesh_matrices = op_cov$fem_mesh_matrices
)
A <- Diagonal(nobs)
Abar <- cbind(A, A, A)
w <- rbind(v, v, v)
c.approx_cov <- (Abar) %*% solve(Q, w)

# plot the result and compare with the true Matern covariance
plot(x, matern.covariance(abs(x - 0.5), kappa, nu, sigma),
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c.approx_cov, col = 2)
```

`rspde.matern.precision.integer` *Precision matrix of stationary Gaussian Matern random fields with integer covariance exponent*

Description

`rspde.matern.precision.integer.opt` is used for computing the precision matrix of stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2(\nu-1)\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h)$$

, where $\alpha = \nu + d/2$ is a natural number.

Usage

```
rspde.matern.precision.integer(
  kappa,
  nu,
  tau = NULL,
  sigma = NULL,
  dim,
  fem_mesh_matrices
)
```

Arguments

`kappa`	Range parameter of the covariance function.
`nu`	Shape parameter of the covariance function.
`tau`	Scale parameter of the covariance function.
`sigma`	Standard deviation of the covariance function. If tau is not provided, sigma should be provided.
`dim`	The dimension of the domain
`fem_mesh_matrices`	A list containing the FEM-related matrices. The list should contain elements c0, g1, g2, g3, etc.

Value

The precision matrix

Examples

```
set.seed(123)
nobs <- 101
x <- seq(from = 0, to = 1, length.out = nobs)
fem <- rSPDE.fem1d(x)
kappa <- 40
sigma <- 1
d <- 1
nu <- 0.5
tau <- sqrt(gamma(nu) / (kappa^(2 * nu) *
  (4 * pi)^(d / 2) * gamma(nu + d / 2)))
range <- sqrt(8 * nu) / kappa
op_cov <- matern.operators(
  loc_mesh = x, nu = nu, range = range, sigma = sigma,
  d = 1, m = 2, parameterization = "matern"
)
v <- t(rSPDE.A1d(x, 0.5))
c.true <- matern.covariance(abs(x - 0.5), kappa, nu, sigma)
Q <- rspde.matern.precision.integer(
  kappa = kappa, nu = nu, tau = tau, d = 1,
  fem_mesh_matrices = op_cov$fem_mesh_matrices
)
A <- Diagonal(nobs)
c.approx_cov <- A %*% solve(Q, v)

# plot the result and compare with the true Matern covariance
plot(x, matern.covariance(abs(x - 0.5), kappa, nu, sigma),
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c.approx_cov, col = 2)
```

`rspde.matern.precision.integer.opt` *Optimized precision matrix of stationary Gaussian Matern random fields with integer covariance exponent*

Description

`rspde.matern.precision.integer.opt` is used for computing the optimized version of the precision matrix of stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h),$$

where $\alpha = \nu + d/2$ is a natural number.

Usage

```
rspde.matern.precision.integer.opt(
  kappa,
  nu,
  tau,
  d,
  fem_matrices,
  graph = NULL
)
```

Arguments

kappa	Range parameter of the covariance function.
nu	Shape parameter of the covariance function.
tau	Scale parameter of the covariance function.
d	The dimension of the domain
fem_matrices	A list containing the FEM-related matrices. The list should contain elements C, G, G_2, G_3, etc.
graph	The sparsity graph of the matrices. If NULL, only a vector of the elements will be returned, if non-NULL, a sparse matrix will be returned.

Value

The precision matrix

---

rspde.matern.precision.opt

Optimized precision matrix of the covariance-based rational approximation

---

Description

`rspde.matern.precision` is used for computing the optimized version of the precision matrix of the covariance-based rational SPDE approximation of a stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h).$$

Usage

```
rspde.matern.precision.opt(
  kappa,
  nu,
  tau,
  rspde.order,
```

```
  dim,
  fem_matrices,
  graph = NULL,
  sharp,
  type_rational_approx
)
```

Arguments

kappa	Range parameter of the covariance function.
nu	Shape parameter of the covariance function.
tau	Scale parameter of the covariance function.
rspde.order	The order of the rational approximation
dim	The dimension of the domain
fem_matrices	A list containing the FEM-related matrices. The list should contain elements C, G, G_2, G_3, etc.
graph	The sparsity graph of the matrices. If NULL, only a vector of the elements will be returned, if non-NULL, a sparse matrix will be returned.
sharp	The sparsity graph should have the correct sparsity (costs more to perform a sparsity analysis) or an upper bound for the sparsity?
type_rational_approx	Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

Value

The precision matrix

---

rspde.matern1d *Matern rSPDE model object for INLA*

Description

Creates an INLA object for a stationary Matern model with general smoothness parameter.

Usage

```
rspde.matern1d(
  loc,
  nu.upper.bound = NULL,
  rspde.order = 1,
  nu = NULL,
  parameterization = c("spde", "matern", "matern2"),
  start.nu = NULL,
  start.theta = NULL,
```

```
  prior.nu = NULL,
  theta.prior.mean = NULL,
  theta.prior.prec = 0.1,
  prior.std.dev.nominal = 1,
  prior.range.nominal = NULL,
  prior.kappa.mean = NULL,
  prior.tau.mean = NULL,
  start.lstd.dev = NULL,
  start.lrange = NULL,
  start.ltau = NULL,
  start.lkappa = NULL,
  prior.theta.param = c("theta", "spde"),
  prior.nu.dist = c("beta", "lognormal"),
  nu.prec.inc = 1,
  type.rational.approx = c("brasil", "chebfun", "chebfunLB"),
  debug = FALSE,
  shared_lib = "detect",
  ...
)
```

Arguments

loc	A vector of spatial locations.
nu.upper.bound	Upper bound for the smoothness parameter. If NULL, it will be set to 2.
rspde.order	The order of the covariance-based rational SPDE approach. The default order is 1.
nu	If nu is set to a parameter, nu will be kept fixed and will not be estimated. If nu is NULL, it will be estimated.
parameterization	Which parameterization to use? matern uses range, std. deviation and nu (smoothness). spde uses kappa, tau and nu (smoothness). matern2 uses range-like (1/kappa), variance and nu (smoothness). The default is spde.
start.nu	Starting value for nu.
start.theta	Starting values for the model parameters. In the stationary case, if parameterization='matern', then theta[1] is the std.dev and theta[2] is the range parameter. If parameterization = 'spde', then theta[1] is tau and theta[2] is kappa.
prior.nu	a list containing the elements mean and prec for beta distribution, or loglocation and logscale for a truncated lognormal distribution. loglocation stands for the location parameter of the truncated lognormal distribution in the log scale. prec stands for the precision of a beta distribution. logscale stands for the scale of the truncated lognormal distribution on the log scale. Check details below.
theta.prior.mean	A vector for the mean priors of theta.
theta.prior.prec	A precision matrix for the prior of theta.

`prior.std.dev.nominal`
: Prior std. deviation to be used for the priors and for the starting values.

`prior.range.nominal`
: Prior range to be used for the priors and for the starting values.

`prior.kappa.mean`
: Prior kappa to be used for the priors and for the starting values.

`prior.tau.mean` Prior tau to be used for the priors and for the starting values.

`start.lstd.dev` Starting value for log of std. deviation. Will not be used if start.ltau is non-null. Will be only used in the stationary case and if `parameterization = 'matern'`.

`start.lrange` Starting value for log of range. Will not be used if start.lkappa is non-null. Will be only used in the stationary case and if `parameterization = 'matern'`.

`start.ltau` Starting value for log of tau. Will be only used in the stationary case and if `parameterization = 'spde'`.

`start.lkappa` Starting value for log of kappa. Will be only used in the stationary case and if `parameterization = 'spde'`.

`prior.theta.param`
: Should the lognormal prior be on `theta` or on the SPDE parameters (`tau` and `kappa` on the stationary case)?

`prior.nu.dist` The distribution of the smoothness parameter. The current options are "beta" or "lognormal". The default is "lognormal".

`nu.prec.inc` Amount to increase the precision in the beta prior distribution.

`type.rational.approx`
: Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

`debug` INLA debug argument

`shared_lib` Which shared lib to use for the cgeneric implementation? If "detect", it will check if the shared lib exists locally, in which case it will use it. Otherwise it will use INLA's shared library. If "INLA", it will use the shared lib from INLA's installation. If 'rSPDE', then it will use the local installation (does not work if your installation is from CRAN). Otherwise, you can directly supply the path of the .so (or .dll) file.

`...` Only being used internally.

`prior.kappa` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale.

`prior.tau` a list containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale.

`prior.range` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale. Will not be used if prior.kappa is non-null.

`prior.std.dev` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale. Will not be used if prior.tau is non-null.

Value

An INLA model.

rspde.mesh.project *Calculate a lattice projection to/from an* inla.mesh *for rSPDE objects*

Description

Calculate a lattice projection to/from an inla.mesh for rSPDE objects

Usage

```
rspde.mesh.project(...)

rspde.mesh.projector(
  mesh,
  nu = NULL,
  rspde.order = 1,
  loc = NULL,
  lattice = NULL,
  xlim = NULL,
  ylim = NULL,
  dims = c(100, 100),
  projection = NULL,
  ...
)

## S3 method for class 'inla.mesh'
rspde.mesh.project(
  mesh,
  loc = NULL,
  field = NULL,
  rspde.order = 1,
  nu = NULL,
  ...
)

## S3 method for class 'rspde.mesh.projector'
rspde.mesh.project(projector, field, ...)

## S3 method for class 'inla.mesh.1d'
rspde.mesh.project(mesh, loc, field = NULL, rspde.order = 1, nu = NULL, ...)
```

Arguments

...	Additional parameters.
mesh	An inla.mesh or inla.mesh.1d object.
nu	The smoothness parameter. If NULL, it will be assumed that nu was estimated.
rspde.order	The order of the rational approximation.

loc	Projection locations. Can be a matrix or a SpatialPoints or a SpatialPointsDataFrame object.
lattice	An inla.mesh.lattice object.
xlim	X-axis limits for a lattice. For R2 meshes, defaults to covering the domain.
ylim	Y-axis limits for a lattice. For R2 meshes, defaults to covering the domain.
dims	Lattice dimensions.
projection	One of c("default", "longlat", "longsinlat", "mollweide").
field	Basis function weights, one per mesh basis function, describing the function to be evaluated at the projection locations.
projector	A rspde.mesh.projector object.

Details

This function is built upon the inla.mesh.project and inla.mesh.projector functions from INLA.

Value

A list with projection information for rspde.mesh.project. For rspde.mesh.projector(mesh, ...), a rspde.mesh.projector object. For rspde.mesh.project(projector, field, ...), a field projected from the mesh onto the locations given by the projector object.

---

rspde.metric_graph *Matern rSPDE model object for metric graphs in INLA*

Description

Creates an INLA object for a stationary Matern model on a metric graph with general smoothness parameter.

Usage

```
rspde.metric_graph(
  graph_obj,
  h = NULL,
  nu.upper.bound = 2,
  rspde.order = 1,
  nu = NULL,
  debug = FALSE,
  B.sigma = matrix(c(0, 1, 0), 1, 3),
  B.range = matrix(c(0, 0, 1), 1, 3),
  parameterization = c("matern", "spde"),
  B.tau = matrix(c(0, 1, 0), 1, 3),
  B.kappa = matrix(c(0, 0, 1), 1, 3),
  start.nu = NULL,
  start.theta = NULL,
```

```
  prior.nu = NULL,
  theta.prior.mean = NULL,
  theta.prior.prec = 0.1,
  prior.std.dev.nominal = 1,
  prior.range.nominal = NULL,
  prior.kappa.mean = NULL,
  prior.tau.mean = NULL,
  start.lstd.dev = NULL,
  start.lrange = NULL,
  start.ltau = NULL,
  start.lkappa = NULL,
  prior.theta.param = c("theta", "spde"),
  prior.nu.dist = c("lognormal", "beta"),
  nu.prec.inc = 1,
  type.rational.approx = c("brasil", "chebfun", "chebfunLB"),
  shared_lib = "INLA"
)
```

Arguments

graph_obj	The graph object to build the model. Needs to be of class metric_graph. It should have a built mesh. If the mesh is not built, one will be built using h=0.01 as default.
h	The width of the mesh in case the mesh was not built.
nu.upper.bound	Upper bound for the smoothness parameter.
rspde.order	The order of the covariance-based rational SPDE approach.
nu	If nu is set to a parameter, nu will be kept fixed and will not be estimated. If nu is NULL, it will be estimated.
debug	INLA debug argument
B.sigma	Matrix with specification of log-linear model for σ. Will be used if parameterization = 'matern'.
B.range	Matrix with specification of log-linear model for ρ, which is a range-like parameter (it is exactly the range parameter in the stationary case). Will be used if parameterization = 'matern'.
parameterization	Which parameterization to use? matern uses range, std. deviation and nu (smoothness). spde uses kappa, tau and nu (smoothness). The default is matern.
B.tau	Matrix with specification of log-linear model for τ. Will be used if parameterization = 'spde'.
B.kappa	Matrix with specification of log-linear model for κ. Will be used if parameterization = 'spde'.
start.nu	Starting value for nu.
start.theta	Starting values for the model parameters. In the stationary case, if parameterization='matern', then theta[1] is the std.dev and theta[2] is the range parameter. If parameterization = 'spde', then theta[1] is tau and theta[2] is kappa.

prior.nu a list containing the elements mean and prec for beta distribution, or loglocation and logscale for a truncated lognormal distribution. loglocation stands for the location parameter of the truncated lognormal distribution in the log scale. prec stands for the precision of a beta distribution. logscale stands for the scale of the truncated lognormal distribution on the log scale. Check details below.

theta.prior.mean
A vector for the mean priors of theta.

theta.prior.prec
A precision matrix for the prior of theta.

prior.std.dev.nominal
Prior std. deviation to be used for the priors and for the starting values.

prior.range.nominal
Prior range to be used for the priors and for the starting values.

prior.kappa.mean
Prior kappa to be used for the priors and for the starting values.

prior.tau.mean Prior tau to be used for the priors and for the starting values.

start.lstd.dev Starting value for log of std. deviation. Will not be used if start.ltau is non-null. Will be only used in the stationary case and if parameterization = 'matern'.

start.lrange Starting value for log of range. Will not be used if start.lkappa is non-null. Will be only used in the stationary case and if parameterization = 'matern'.

start.ltau Starting value for log of tau. Will be only used in the stationary case and if parameterization = 'spde'.

start.lkappa Starting value for log of kappa. Will be only used in the stationary case and if parameterization = 'spde'.

prior.theta.param
Should the lognormal prior be on theta or on the SPDE parameters (tau and kappa on the stationary case)?

prior.nu.dist The distribution of the smoothness parameter. The current options are "beta" or "lognormal". The default is "beta".

nu.prec.inc Amount to increase the precision in the beta prior distribution. Check details below.

type.rational.approx
Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

shared_lib Which shared lib to use for the cgeneric implementation? If "INLA", it will use the shared lib from INLA's installation. If 'rSPDE', then it will use the local installation (does not work if your installation is from CRAN). Otherwise, you can directly supply the path of the .so (or .dll) file.

prior.kappa a list containing the elements meanlog and sdlog, that is, the mean and standard deviation on the log scale.

prior.tau a list containing the elements meanlog and sdlog, that is, the mean and standard deviation on the log scale.

prior.range a list containing the elements meanlog and sdlog, that is, the mean and standard deviation on the log scale. Will not be used if prior.kappa is non-null.

`prior.std.dev` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale. Will not be used if prior.tau is non-null.

Value

An INLA model.

`rspde.result` *rSPDE result extraction from INLA estimation results*

Description

Extract field and parameter values and distributions for an rspde effect from an inla result object.

Usage

```
rspde.result(
  inla,
  name,
  rspde,
  compute.summary = TRUE,
  parameterization = "detect",
  n_samples = 5000,
  n_density = 1024
)
```

Arguments

`inla`	An `inla` object obtained from a call to `inla()`.
`name`	A character string with the name of the rSPDE effect in the inla formula.
`rspde`	The `inla_rspde` object used for the effect in the inla formula.
`compute.summary`	Should the summary be computed?
`parameterization`	If 'detect', the parameterization from the model will be used. Otherwise, the options are 'spde', 'matern' and 'matern2'.
`n_samples`	The number of samples to be used if parameterization is different from the one used to fit the model.
`n_density`	The number of equally spaced points to estimate the density.

Value

If the model was fitted with `matern` parameterization (the default), it returns a list containing:

`marginals.range`
: Marginal densities for the range parameter

`marginals.log.range`
: Marginal densities for log(range)

`marginals.std.dev`
: Marginal densities for std. deviation

`marginals.log.std.dev`
: Marginal densities for log(std. deviation)

`marginals.values`
: Marginal densities for the field values

`summary.log.range`
: Summary statistics for log(range)

`summary.log.std.dev`
: Summary statistics for log(std. deviation)

`summary.values` Summary statistics for the field values

If `compute.summary` is `TRUE`, then the list will also contain

`summary.kappa` Summary statistics for kappa

`summary.tau` Summary statistics for tau

If the model was fitted with the spde parameterization, it returns a list containing:

`marginals.kappa`
: Marginal densities for kappa

`marginals.log.kappa`
: Marginal densities for log(kappa)

`marginals.log.tau`
: Marginal densities for log(tau)

`marginals.tau` Marginal densities for tau

`marginals.values`
: Marginal densities for the field values

`summary.log.kappa`
: Summary statistics for log(kappa)

`summary.log.tau`
: Summary statistics for log(tau)

`summary.values` Summary statistics for the field values

If `compute.summary` is `TRUE`, then the list will also contain

`summary.kappa` Summary statistics for kappa

`summary.tau` Summary statistics for tau

For both cases, if nu was estimated, then the list will also contain

`marginals.nu` Marginal densities for nu

If nu was estimated and a beta prior was used, then the list will also contain

`marginals.logit.nu`
Marginal densities for logit(nu)

`summary.logit.nu`
Marginal densities for logit(nu)

If nu was estimated and a truncated lognormal prior was used, then the list will also contain

`marginals.log.nu`
Marginal densities for log(nu)

`summary.log.nu` Marginal densities for log(nu)

If nu was estimated and `compute.summary` is `TRUE`, then the list will also contain

`summary.nu` Summary statistics for nu

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)) {
  library(INLA)

  set.seed(123)

  m <- 100
  loc_2d_mesh <- matrix(runif(m * 2), m, 2)
  mesh_2d <- inla.mesh.2d(
    loc = loc_2d_mesh,
    cutoff = 0.05,
    max.edge = c(0.1, 0.5)
  )
  sigma <- 1
  range <- 0.2
  nu <- 0.8
  kappa <- sqrt(8 * nu) / range
  op <- matern.operators(
    mesh = mesh_2d, nu = nu,
    range = range, sigma = sigma, m = 2,
    parameterization = "matern"
  )
  u <- simulate(op)
  A <- inla.spde.make.A(
    mesh = mesh_2d,
    loc = loc_2d_mesh
  )
  sigma.e <- 0.1
  y <- A %*% u + rnorm(m) * sigma.e
  Abar <- rspde.make.A(mesh = mesh_2d, loc = loc_2d_mesh)
  mesh.index <- rspde.make.index(name = "field", mesh = mesh_2d)
```

```
  st.dat <- inla.stack(
    data = list(y = as.vector(y)),
    A = Abar,
    effects = mesh.index
  )
  rspde_model <- rspde.matern(
    mesh = mesh_2d,
    nu.upper.bound = 2
  )
  f <- y ~ -1 + f(field, model = rspde_model)
  rspde_fit <- inla(f,
    data = inla.stack.data(st.dat),
    family = "gaussian",
    control.predictor =
      list(A = inla.stack.A(st.dat))
  )
  result <- rspde.result(rspde_fit, "field", rspde_model)
  summary(result)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`rspde.spacetime` *Space-Time Random Fields via SPDE Approximation*

Description

`rspde.spacetime` computes a Finite Element Method (FEM) approximation of a Gaussian random field defined as the solution to the stochastic partial differential equation (SPDE):

$$du + \gamma(\kappa^2 + \kappa^{d/2}\rho \cdot \nabla - \Delta)^{\alpha} u = \sigma dW_C$$

where C is a Whittle-Matérn covariance operator with smoothness parameter β and range parameter κ. This function is designed to handle space-time random fields using either 1D spatial models or higher-dimensional FEM-based approaches.

Usage

```
rspde.spacetime(
  mesh_space = NULL,
  mesh_time = NULL,
  space_loc = NULL,
  time_loc = NULL,
  drift = TRUE,
  alpha,
  beta,
  prior.kappa = NULL,
  prior.sigma = NULL,
```

```
  prior.rho = NULL,
  prior.gamma = NULL,
  prior.precision = NULL,
  graph_dirichlet = TRUE,
  bounded_rho = TRUE,
  bound_rho = NULL,
  shared_lib = "detect",
  debug = FALSE,
  ...
)
```

Arguments

mesh_space	Spatial mesh for the FEM approximation, or a metric_graph object for handling models on metric graphs.
mesh_time	Temporal mesh for the FEM approximation.
space_loc	A vector of spatial locations for mesh nodes in 1D spatial models. This should be provided when mesh_space is not specified.
time_loc	A vector of temporal locations for mesh nodes. This should be provided when mesh_time is not specified.
drift	Logical value indicating whether the drift term should be included. If FALSE, the drift coefficient ρ is set to zero.
alpha	Integer smoothness parameter α.
beta	Integer smoothness parameter β.
prior.kappa	A list specifying the prior for the range parameter κ. This list may contain two elements: mean and/or precision, both of which must be numeric scalars (numeric vectors of length 1). The precision refers to the prior on $\log(\kappa)$. If NULL, default values will be used. The mean value is also used as starting value for kappa.
prior.sigma	A list specifying the prior for the variance parameter σ. This list may contain two elements: mean and/or precision, both of which must be numeric scalars. The precision refers to the prior on $\log(\sigma)$. If NULL, default values will be used. The mean value is also used as starting value for sigma.
prior.rho	A list specifying the prior for the drift coefficient ρ. This list may contain two elements: mean and/or precision, both of which must be numeric scalars if dimension is one, and numeric vectors of length 2 if dimension is 2. The precision applies directly to ρ without log transformation. If NULL, default values will be used. Will not be used if drift = FALSE. The mean value is also used as starting value for rho.
prior.gamma	A list specifying the prior for the weight γ in the SPDE operator. This list may contain two elements: mean and/or precision, both of which must be numeric scalars. The precision refers to the prior on $\log(\gamma)$. If NULL, default values will be used. The mean value is also used as starting value for gamma.
prior.precision	A precision matrix for $\log(\kappa), \log(\sigma), \log(\gamma), \rho$. This matrix replaces the precision element from prior.kappa, prior.sigma, prior.gamma, and prior.rho

respectively. For dimension 1 `prior.precision` must be a 4x4 matrix. For dimension 2, ρ is a vector of length 2, so in this case `prior.precision` must be a 5x5 matrix. If `NULL`, a diagonal precision matrix with default values will be used.

`graph_dirichlet`
: For models on metric graphs, use Dirichlet vertex conditions at vertices of degree 1?

`bounded_rho`
: Logical. Should `rho` be bounded to ensure the existence, uniqueness, and well-posedness of the solution? Defaults to `TRUE`. Note that this bounding is not a strict condition; there may exist values of rho beyond the upper bound that still satisfy these properties. When `bounded_rho = TRUE`, the `rspde_lme` models enforce bounded `rho` for consistency. If the estimated value of `rho` approaches the upper bound too closely, we recommend refitting the model with `bounded_rho = FALSE`. However, this should be done with caution, as it may lead to instability in some cases, though it can also result in a better model fit. The actual bound used for `rho` can be accessed from the `bound_rho` element of the returned object.

`bound_rho`
: A positive number specifying the bound for `rho`. If `NULL`, the default bound will be used.

`shared_lib`
: String specifying which shared library to use for the Cgeneric implementation. Options are "detect", "INLA", or "rSPDE". You may also specify the direct path to a .so (or .dll) file.

`debug`
: Logical value indicating whether to enable INLA debug mode.

`...`
: Additional arguments passed internally for configuration purposes.

Value

An object of class `inla_rspde_spacetime` representing the FEM approximation of the space-time Gaussian random field.

`rspde_lme` *rSPDE linear mixed effects models*

Description

Fitting linear mixed effects model with latent Whittle-Matern models.

Usage

```
rspde_lme(
  formula,
  loc,
  loc_time = NULL,
  data,
  model = NULL,
  repl = NULL,
```

```
  which_repl = NULL,
  optim_method = "L-BFGS-B",
  possible_methods = c("L-BFGS-B", "Nelder-Mead"),
  use_data_from_graph = TRUE,
  rspde_order = NULL,
  mean_correction = FALSE,
  previous_fit = NULL,
  fix_coeff = FALSE,
  model_options = list(),
  smoothness_upper_bound = 4,
  parallel = FALSE,
  n_cores = parallel::detectCores() - 1,
  optim_controls = list(),
  improve_hessian = FALSE,
  hessian_args = list(),
  alpha = lifecycle::deprecated(),
  nu = lifecycle::deprecated(),
  beta = lifecycle::deprecated(),
  starting_values_latent = lifecycle::deprecated(),
  start_sigma_e = lifecycle::deprecated(),
  start_alpha = lifecycle::deprecated(),
  start_nu = lifecycle::deprecated(),
  start_beta = lifecycle::deprecated(),
  nu_upper_bound = lifecycle::deprecated()
)
```

Arguments

formula	Formula object describing the relation between the response variables and the fixed effects. If the response variable is a matrix, each column of the matrix will be treated as a replicate.
loc	A vector with the names of the columns in data that contain the observation locations, or a matrix or a data.frame containing the observation locations. If the model is of class metric_graph, the locations must be either a matrix or a data.frame with two columns, or a character vector with the names of the two columns. The first column being the number of the edge, and the second column being the normalized position on the edge. If the model is a 2d model, loc must be either a matrix or data.frame with two columns or a character vector with the name of the two columns that contain the location, the first entry corresponding to the x entry and the second corresponding to the y entry.
loc_time	For spatio-temporal models, the name of the column in data that is the time variable, or a matrix or vector containing the observation time points.
data	A data.frame containing the data to be used.
model	Object generated by matern.operators(), spde.matern.operators() or spacetime.operators(). If NULL, simple linear regression will be performed.
repl	Vector indicating the replicate of each observation. If NULL it will assume there is only one replicate. If the model is generated from graphs from metric_graph

class and use_data_from_graph is TRUE, repl needs to be the name of the column inside the metric graph data that contains the replicate. If NULL it will assume there is only one replicate.

which_repl Which replicates to use? If NULL all replicates will be used.

optim_method The method to be used with optim function.

possible_methods
The optimization methods to try if the model fitting fails.

use_data_from_graph
Logical. Only for models generated from graphs from metric_graph class. In this case, should the data, the locations and the replicates be obtained from the graph object?

rspde_order The order of the rational approximation to be used while fitting the model. If not given, the order from the model object will be used. Not used for spatio-temporal models.

mean_correction
If TRUE, use mean correction for intrinsic models. Default FALSE.

previous_fit An object of class rspde_lme. Use the fitted coefficients as starting values.

fix_coeff If using a previous fit, should all coefficients be fixed at the starting values?

model_options A list containing additional options to be used in the model. This will take preference over the values extracted from previous_fit. Currently, it is possible to fix parameters during the estimation or change the starting values of the parameters. The general structure of the elements of the list is fix_parname and start_parname, where parname stands for the name of the parameter. If fix_parname is not NULL, then the model will be fitted with the parname being fixed at the value that was passed. If start_parname is not NULL, the model will be fitted using the value passed as starting value for parname. For 'rSPDE' and 'rSPDE1d' models, the possible elements of the list are fix_sigma_e, start_sigma_e, fix_nu, start_nu, fix_sigma, start_sigma, fix_range, start_range. Alternatively, one can also use the elements fix_sigma_e, start_sigma_e, fix_nu, start_nu, fix_tau, start_tau, fix_kappa, start_kappa. For 'spacetime' models, the possible elements of the list are fix_sigma_e, start_sigma_e, fix_kappa, start_kappa, fix_sigma, start_sigma, fix_gamma, start_gamma, fix_rho, start_rho. For dimension 2, the second coordinate of rho has name rho2 and must be passed as start_rho2 and fix_rho2. For 'rSPDE2d' models, the possible elements of the list are fix_sigma_e, start_sigma_e, fix_nu, start_nu, fix_sigma, start_sigma, fix_hx, start_hx, fix_hy, start_hy, fix_hxy, start_hxy. For nonstationary models, we have two options: the first is to pass the starting values as a vector with name start_theta and a vector with the names of the parameters to be fixed with name fix_theta; the second option is to handle the individual parameters, by passing the names thetan where n is the number of the parameter to pass the starting value or to be fixed. For example, to pass the starting value for theta3 one simply pass start_theta3, and to fix theta2, one simply pass fix_theta2.

smoothness_upper_bound
Upper bound for nu, when nu is the smoothness parameter, or for alpha - d/2 when alpha is the smoothness parameter, or for max\{alpha - d/2, beta - d/2) for intrinsic models.

parallel	logical. Indicating whether to use optimParallel or not.
n_cores	Number of cores to be used if parallel is true.
optim_controls	Additional controls to be passed to optim or optimParallel.
improve_hessian	Should a more precise estimate of the hessian be obtained? Turning on might increase the overall time.
hessian_args	List of controls to be used if improve_hessian is TRUE. The list can contain the arguments to be passed to the method.args argument in the numDeriv::hessian function. See the help of the hessian function in numDeriv package for details. Observe that it only accepts the "Richardson" method for now, the method "complex" is not supported.
alpha	**[Deprecated]** Use model_options instead.
nu	**[Deprecated]** Use model_options instead.
beta	**[Deprecated]** Use model_options instead.
starting_values_latent	**[Deprecated]** Use model_options instead.
start_sigma_e	**[Deprecated]** Use model_options instead.
start_alpha	**[Deprecated]** Use model_options instead.
start_nu	**[Deprecated]** Use model_options instead.
start_beta	**[Deprecated]** Use model_options instead.
nu_upper_bound	**[Deprecated]** Use smoothness_upper_bound instead.

Value

A list containing the fitted model.

`simulate.CBrSPDEobj` *Simulation of a fractional SPDE using the covariance-based rational SPDE approximation*

Description

The function samples a Gaussian random field based using the covariance-based rational SPDE approximation.

Usage

```
## S3 method for class 'CBrSPDEobj'
simulate(
  object,
  nsim = 1,
  seed = NULL,
  nu = NULL,
```

```
  kappa = NULL,
  sigma = NULL,
  range = NULL,
  tau = NULL,
  theta = NULL,
  m = NULL,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern.operators()
nsim	The number of simulations.
seed	An object specifying if and how the random number generator should be initialized ('seeded').
nu	If non-null, update the shape parameter of the covariance function.
kappa	If non-null, update the range parameter of the covariance function.
sigma	If non-null, update the standard deviation of the covariance function.
range	If non-null, update the range parameter of the covariance function.
tau	If non-null, update the parameter tau.
theta	For non-stationary models. If non-null, update the vector of parameters.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
...	Currently not used.

Value

A matrix with the n samples as columns.

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
  (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
```

```
  parameterization = "matern"
)

# Sample the model and plot the result
Y <- simulate(op_cov)
plot(x, Y, type = "l", ylab = "u(x)", xlab = "x")
```

`simulate.CBrSPDEobj2d` *Simulation of a fractional SPDE using the covariance-based rational SPDE approximation*

Description

The function samples a Gaussian random field based using the covariance-based rational SPDE approximation.

Usage

```
## S3 method for class 'CBrSPDEobj2d'
simulate(
  object,
  nsim = 1,
  seed = NULL,
  nu = NULL,
  hx = NULL,
  hy = NULL,
  hxy = NULL,
  sigma = NULL,
  m = NULL,
  ...
)
```

Arguments

`object`	The covariance-based rational SPDE approximation, computed using `matern2d.operators()`
`nsim`	The number of simulations.
`seed`	An object specifying if and how the random number generator should be initialized ('seeded').
`nu`	If non-null, update the shape parameter of the covariance function.
`hx`	If non-null, update the hx parameter.
`hy`	If non-null, update the hy parameter.
`hxy`	If non-null, update the hxy parameter.
`sigma`	If non-null, update the standard deviation of the covariance function.
`m`	If non-null, update the order of the rational approximation, which needs to be a positive integer.
`...`	Currently not used.

Value

A matrix with the n samples as columns.

Examples

```
library(fmesher)
n_loc <- 2000
loc_2d_mesh <- matrix(runif(n_loc * 2), n_loc, 2)
mesh_2d <- fm_mesh_2d(loc = loc_2d_mesh, cutoff = 0.03, max.edge = c(0.1, 0.5))
op <- matern2d.operators(mesh = mesh_2d, sigma = 1, nu = 1, hx = 0.1, hy = 0.1, hxy = 0)
u <- simulate(op)
```

---

simulate.intrinsicCBrSPDEobj

Simulation of a fractional intrinsic SPDE using the covariance-based rational SPDE approximation

---

Description

The function samples a Gaussian random field based using the covariance-based rational SPDE approximation.

Usage

```
## S3 method for class 'intrinsicCBrSPDEobj'
simulate(
  object,
  nsim = 1,
  seed = NULL,
  kappa = NULL,
  tau = NULL,
  alpha = NULL,
  beta = NULL,
  integral.constraint = TRUE,
  use_kl = NULL,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using intrinsic.matern.operators()
nsim	The number of simulations.
seed	An object specifying if and how the random number generator should be initialized ('seeded').
kappa	new value of kappa to use
tau	new value of tau to use

alpha	new value of alpha to use
beta	new value of beta to use
integral.constraint	Should the contraint on the integral be done?
use_kl	Simulate based on a KL expansion?
...	Currently not used.

Value

A matrix with the nsim samples as columns.

---

simulate.rSPDEobj *Simulation of a fractional SPDE using a rational SPDE approximation*

Description

The function samples a Gaussian random field based on a pre-computed rational SPDE approximation.

Usage

```
## S3 method for class 'rSPDEobj'
simulate(object, nsim = 1, seed = NULL, ...)
```

Arguments

object	The rational SPDE approximation, computed using fractional.operators(), matern.operators(), or spde.matern.operators().
nsim	The number of simulations.
seed	an object specifying if and how the random number generator should be initialized ('seeded').
...	Currently not used.

Value

A matrix with the n samples as columns.

See Also

simulate.CBrSPDEobj()

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation
op <- matern.operators(
  range = range, sigma = sigma,
  nu = nu, loc_mesh = x, d = 1,
  parameterization = "matern"
)

# Sample the model and plot the result
Y <- simulate(op)
plot(x, Y, type = "l", ylab = "u(x)", xlab = "x")
```

`simulate.rSPDEobj1d` *Simulation of a Matern field using a rational SPDE approximation*

Description

The function samples a Gaussian random field based on a pre-computed rational SPDE approximation.

Usage

```
## S3 method for class 'rSPDEobj1d'
simulate(object, nsim = 1, seed = NULL, ...)
```

Arguments

`object`	The rational SPDE approximation, computed using `matern.rational()`.
`nsim`	The number of simulations.
`seed`	an object specifying if and how the random number generator should be initialized ('seeded').
`...`	Currently not used.

Value

A matrix with the `n` samples as columns.

See Also

matern.rational()

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
range <- 0.2
sigma <- 1
nu <- 0.8

# compute rational approximation
x <- seq(from = 0, to = 1, length.out = 100)
op <- matern.rational(
  range = range, sigma = sigma,
  nu = nu, loc = x
)

# Sample the model and plot the result
Y <- simulate(op)
plot(x, Y, type = "l", ylab = "u(x)", xlab = "x")
```

---

simulate.spacetimeobj *Simulation of space-time models*

Description

Simulation of space-time models

Usage

```
## S3 method for class 'spacetimeobj'
simulate(
  object,
  nsim = 1,
  seed = NULL,
  kappa = NULL,
  sigma = NULL,
  gamma = NULL,
  rho = NULL,
  ...
)
```

Arguments

object	Space-time object created by spacetime.operators()
nsim	The number of simulations.

seed	an object specifying if and how the random number generator should be initialized ('seeded').
kappa	kappa parameter if it should be updated
sigma	sigma parameter if it should be updated
gamma	gamma parameter if it should be updated
rho	rho parameter if it should be updated
...	Currently not used.

Value

A matrix with the simulations as columns.

Examples

```
s <- seq(from = 0, to = 20, length.out = 101)
t <- seq(from = 0, to = 20, length.out = 31)

op_cov <- spacetime.operators(space_loc = s, time_loc = t,
                              kappa = 5, sigma = 10, alpha = 1,
                              beta = 2, rho = 1, gamma = 0.05)
x <- simulate(op_cov, nsim = 1)
image(matrix(x, nrow = length(s), ncol = length(t)))
```

---

spacetime.operators *Space-time random fields*

Description

spacetime.operators is used for computing a FEM approximation of a Gaussian random field defined as a solution to the SPDE

$$du + \gamma(\kappa^2 + \kappa^{d/2}\rho \cdot \nabla - \Delta)^\alpha u = \sigma dW_C.$$

where C is a Whittle-Matern covariance operator with smoothness parameter β and range parameter κ

Usage

```
spacetime.operators(
  mesh_space = NULL,
  mesh_time = NULL,
  space_loc = NULL,
  time_loc = NULL,
  graph = NULL,
  kappa = NULL,
  sigma = NULL,
  gamma = NULL,
```

```
  rho = NULL,
  alpha = NULL,
  beta = NULL,
  graph_dirichlet = TRUE,
  bounded_rho = TRUE,
  bound_rho = NULL
)
```

Arguments

mesh_space	Spatial mesh for FEM approximation
mesh_time	Temporal mesh for FEM approximation
space_loc	Locations of mesh nodes for spatial mesh for 1d models.
time_loc	Locations of temporal mesh nodes.
graph	An optional metric_graph object. Replaces mesh for models on metric graphs.
kappa	Positive spatial range parameter
sigma	Positive variance parameter
gamma	Temporal range parameter.
rho	Drift parameter. Real number on metric graphs and one-dimensional spatial domains, a vector with two number on 2d domains.
alpha	Integer smoothness parameter alpha.
beta	Integer smoothness parameter beta.
graph_dirichlet	For models on metric graphs, use Dirichlet vertex conditions at vertices of degree 1? When bounded_rho = TRUE, the rspde_lme models enforce bounded rho for consistency. If the estimated value of rho approaches the upper bound too closely, we recommend refitting the model with bounded_rho = FALSE. However, this should be done with caution, as it may lead to instability in some cases, though it can also result in a better model fit. The actual bound used for rho can be accessed from the bound_rho element of the returned object.
bounded_rho	Logical. Specifies whether rho should be bounded to ensure the existence, uniqueness, and well-posedness of the solution. Defaults to TRUE. Note that this bounding is not a strict condition; there may exist values of rho beyond the upper bound that still satisfy these properties.
bound_rho	A positive number specifying the bound for rho. If NULL, the default bound will be used.

Value

An object of type spacetimeobj.

Examples

```
s <- seq(from = 0, to = 20, length.out = 101)
t <- seq(from = 0, to = 20, length.out = 31)
```

```
op_cov <- spacetime.operators(space_loc = s, time_loc = t,
                              kappa = 5, sigma = 10, alpha = 1,
                              beta = 2, rho = 1, gamma = 0.05)
Q <- op_cov$Q
v <- rep(0,dim(Q)[1])
v[1565] <- 1
Sigma <- solve(Q,v)

image(matrix(Sigma, nrow=length(s), ncol = length(t)))
```

---

spde.make.A *Observation/prediction matrices for rSPDE models with integer smoothness.*

Description

Constructs observation/prediction weight matrices for rSPDE models with integer smoothness based on inla.mesh or inla.mesh.1d objects.

Usage

```
spde.make.A(
  mesh = NULL,
  loc = NULL,
  A = NULL,
  index = NULL,
  group = NULL,
  repl = 1L,
  n.group = NULL,
  n.repl = NULL
)
```

Arguments

mesh	An inla.mesh, an inla.mesh.1d object or a metric_graph object.
loc	Locations, needed if an INLA mesh is provided
A	The A matrix from the standard SPDE approach, such as the matrix returned by inla.spde.make.A. Should only be provided if mesh is not provided.
index	For each observation/prediction value, an index into loc. Default is seq_len(nrow(A.loc)).
group	For each observation/prediction value, an index into the group model.
repl	For each observation/prediction value, the replicate index.
n.group	The size of the group model.
n.repl	The total number of replicates.

Value

The A matrix for rSPDE models.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("fmesher", quietly = TRUE)) {
  library(fmesher)

  set.seed(123)
  loc <- matrix(runif(100 * 2) * 100, 100, 2)
  mesh <- fm_mesh_2d(
    loc = loc,
    cutoff = 50,
    max.edge = c(50, 500)
  )
  A <- spde.make.A(mesh, loc = loc)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

---

spde.matern.loglike *Parameter-based log-likelihood for a latent Gaussian Matern SPDE model using a rational SPDE approximation*

Description

This function evaluates the log-likelihood function for observations of a Gaussian process defined as the solution to the SPDE

$$(\kappa(s) - \Delta)^{\beta}(\tau(s)u(s)) = W.$$

Usage

```
spde.matern.loglike(
  object,
  Y,
  A,
  sigma.e,
  mu = 0,
  nu = NULL,
  kappa = NULL,
  tau = NULL,
  theta = NULL,
  m = NULL
)
```

Arguments

object	The rational SPDE approximation, computed using spde.matern.operators()
Y	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
A	An observation matrix that links the measurement location to the finite element basis.
sigma.e	IF non-null, the standard deviation of the measurement noise will be kept fixed in the returned likelihood.
mu	Expectation vector of the latent field (default = 0).
nu	If non-null, the shape parameter will be kept fixed in the returned likelihood.
kappa	If non-null, updates the range parameter.
tau	If non-null, updates the parameter tau.
theta	If non-null, updates the parameter theta (that connects tau and kappa to the model matrices in object).
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.

Details

The observations are assumed to be generated as $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using a rational approximation of the fractional SPDE model.

Value

The log-likelihood value.

See Also

rSPDE.loglike().

Examples

```
# this example illustrates how the function can be used for maximum
# likelihood estimation
# Sample a Gaussian Matern process on R using a rational approximation
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51
# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)
tau <- rep(0.5, n.x)
nu <- 0.8
alpha <- nu + 1 / 2
kappa <- rep(1, n.x)
```

```
# compute rational approximation
op <- spde.matern.operators(
  kappa = kappa, tau = tau, alpha = alpha,
  parameterization = "spde", d = 1,
  loc_mesh = x
)
# Sample the model
u <- simulate(op, n.rep)
# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)
# define negative likelihood function for optimization using matern.loglike
mlik <- function(theta) {
  return(-spde.matern.loglike(op, Y, A,
    sigma.e = exp(theta[4]),
    nu = exp(theta[3]),
    kappa = exp(theta[2]),
    tau = exp(theta[1])
  ))
}
#' #The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(1 / sqrt(var(c(Y))), sqrt(8), 0.9, 0.01))
# run estimation and display the results
theta <- optim(theta0, mlik)
print(data.frame(
  tau = c(tau[1], exp(theta$par[1])), kappa = c(kappa[1], exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`spde.matern.operators` *Rational approximations of non-stationary Gaussian SPDE Matern random fields*

Description

`spde.matern.operators` is used for computing a rational SPDE approximation of a Gaussian random fields on R^d defined as a solution to the SPDE

$$(\kappa(s) - \Delta)^{\beta}(\tau(s)u(s)) = W.$$

Usage

```
spde.matern.operators(
```

```
  kappa = NULL,
  tau = NULL,
  theta = NULL,
  B.tau = matrix(c(0, 1, 0), 1, 3),
  B.kappa = matrix(c(0, 0, 1), 1, 3),
  B.sigma = matrix(c(0, 1, 0), 1, 3),
  B.range = matrix(c(0, 0, 1), 1, 3),
  alpha = NULL,
  nu = NULL,
  parameterization = c("spde", "matern"),
  G = NULL,
  C = NULL,
  d = NULL,
  graph = NULL,
  mesh = NULL,
  range_mesh = NULL,
  loc_mesh = NULL,
  m = 1,
  type = c("covariance", "operator"),
  type_rational_approximation = c("brasil", "chebfun", "chebfunLB")
)
```

Arguments

kappa	Vector with the, possibly spatially varying, range parameter evaluated at the locations of the mesh used for the finite element discretization of the SPDE.
tau	Vector with the, possibly spatially varying, precision parameter evaluated at the locations of the mesh used for the finite element discretization of the SPDE.
theta	Theta parameter that connects B.tau and B.kappa to tau and kappa through a log-linear regression, in case the parameterization is spde, and that connects B.sigma and B.range to tau and kappa in case the parameterization is matern. When both tau and kappa are constant after this transformation, spde.matern.operators() delegates to matern.operators().
B.tau	Matrix with specification of log-linear model for τ. Will be used if parameterization = 'spde'.
B.kappa	Matrix with specification of log-linear model for κ. Will be used if parameterization = 'spde'.
B.sigma	Matrix with specification of log-linear model for σ. Will be used if parameterization = 'matern'.
B.range	Matrix with specification of log-linear model for ρ, which is a range-like parameter (it is exactly the range parameter in the stationary case). Will be used if parameterization = 'matern'.
alpha	smoothness parameter. Will be used if the parameterization is 'spde'.
nu	Shape parameter of the covariance function. Will be used if the parameterization is 'matern'.

parameterization
: Which parameterization to use? `matern` uses range, std. deviation and nu (smoothness). `spde` uses kappa, tau and nu (smoothness). The default is `matern`.

`G`
: The stiffness matrix of a finite element discretization of the domain of interest.

`C`
: The mass matrix of a finite element discretization of the domain of interest.

`d`
: The dimension of the domain. Does not need to be given if `mesh` is used.

`graph`
: An optional `metric_graph` object. Replaces `d`, `C` and `G`.

`mesh`
: An optional inla mesh. `d`, `C` and `G` must be given if `mesh` is not given.

`range_mesh`
: The range of the mesh. Will be used to provide starting values for the parameters. Will be used if `mesh` and `graph` are `NULL`, and if one of the parameters (kappa or tau for spde parameterization, or sigma or range for matern parameterization) are not provided.

`loc_mesh`
: The mesh locations used to construct the matrices C and G. This option should be provided if one wants to use the `rspde_lme()` function and will not provide neither graph nor mesh. Only works for 1d data. Does not work for metric graphs. For metric graphs you should supply the graph using the `graph` argument.

`m`
: The order of the rational approximation, which needs to be a positive integer. The default value is 1.

`type`
: The type of the rational approximation. The options are "covariance" and "operator". The default is "covariance".

`type_rational_approximation`
: Which type of rational approximation should be used? The current types are "brasil", "chebfun" or "chebfunLB".

Details

The approximation is based on a rational approximation of the fractional operator $(\kappa(s)^2 - \Delta)^\beta$, where $\beta = (\nu + d/2)/2$. This results in an approximate model on the form

$$P_l u(s) = P_r W,$$

where $P_j = p_j(L)$ are non-fractional operators defined in terms of polynomials p_j for $j = l, r$. The order of p_r is given by `m` and the order of p_l is $m + m_\beta$ where m_β is the integer part of β if $\beta > 1$ and $m_\beta = 1$ otherwise.

The discrete approximation can be written as $u = P_r x$ where $x \sim N(0, Q^{-1})$ and $Q = P_l^T C^{-1} P_l$. Note that the matrices P_r and Q may be be ill-conditioned for $m > 1$. In this case, the metehods in `operator.operations()` should be used for operations involving the matrices, since these methods are more numerically stable.

Value

`spde.matern.operators` returns an object of class "rSPDEobj. This object contains the quantities listed in the output of `fractional.operators()` as well as the smoothness parameter ν.

See Also

`fractional.operators()`, `spde.matern.operators()`, `matern.operators()`

Examples

```
# Sample non-stationary Matern field on R
tau <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# define a non-stationary range parameter
kappa <- seq(from = 2, to = 20, length.out = length(x))
alpha <- nu + 1 / 2
# compute rational approximation
op <- spde.matern.operators(
  kappa = kappa, tau = tau, alpha = alpha,
  G = fem$G, C = fem$C, d = 1
)

# sample the field
u <- simulate(op)

# plot the sample
plot(x, u, type = "l", ylab = "u(s)", xlab = "s")
```

---

summary.CBrSPDEobj *Summarise CBrSPDE objects*

Description

Summary method for class "CBrSPDEobj"

Usage

```
## S3 method for class 'CBrSPDEobj'
summary(object, ...)

## S3 method for class 'summary.CBrSPDEobj'
print(x, ...)

## S3 method for class 'CBrSPDEobj'
print(x, ...)
```

Arguments

object	an object of class "CBrSPDEobj", usually, a result of a call to matern.operators().
...	further arguments passed to or from other methods.
x	an object of class "summary.CBrSPDEobj", usually, a result of a call to summary.CBrSPDEobj().

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
  (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
  parameterization = "matern"
)

op_cov
```

---

summary.CBrSPDEobj2d *Summarise CBrSPDEobj2d objects*

Description

Summary method for class "CBrSPDEobj2d"

Usage

```
## S3 method for class 'CBrSPDEobj2d'
summary(object, ...)

## S3 method for class 'summary.CBrSPDEobj2d'
print(x, ...)

## S3 method for class 'CBrSPDEobj2d'
print(x, ...)
```

Arguments

object	an object of class "CBrSPDEobj2d", usually, a result of a call to matern2d.operators().
...	further arguments passed to or from other methods.
x	an object of class "summary.CBrSPDEobj2d", usually, a result of a call to summary.CBrSPDEobj2d().

Examples

```
library(fmesher)
n_loc <- 2000
loc_2d_mesh <- matrix(runif(n_loc * 2), n_loc, 2)
mesh_2d <- fm_mesh_2d(loc = loc_2d_mesh, cutoff = 0.03, max.edge = c(0.1, 0.5))
op <- matern2d.operators(mesh = mesh_2d)
op
```

---

summary.intrinsicCBrSPDEobj *Summary method for class "intrinsicCBrSPDEobj"*

Description

Summary method for class "intrinsicCBrSPDEobj"

Usage

```
## S3 method for class 'intrinsicCBrSPDEobj'
summary(object, ...)

## S3 method for class 'summary.intrinsicCBrSPDEobj'
print(x, ...)

## S3 method for class 'intrinsicCBrSPDEobj'
print(x, ...)
```

Arguments

object	an object of class "intrinsicCBrSPDEobj", usually, a result of a call to intrinsic.operators().
...	further arguments passed to or from other methods.
x	an object of class "summary.intrinsicCBrSPDEobj", usually, a result of a call to summary.intrinsicCBrSPDEobj().

---

summary.rSPDEobj *Summarise rSPDE objects*

Description

Summary method for class "rSPDEobj"

Usage

```
## S3 method for class 'rSPDEobj'
summary(object, ...)

## S3 method for class 'summary.rSPDEobj'
print(x, ...)

## S3 method for class 'rSPDEobj'
print(x, ...)
```

Arguments

object	an object of class "rSPDEobj", usually, a result of a call to fractional.operators(), matern.operators(), or spde.matern.operators().
...	further arguments passed to or from other methods.
x	an object of class "summary.rSPDEobj", usually, a result of a call to summary.rSPDEobj().

---

summary.rSPDEobj1d *Summarise rSPDE objects without FEM*

Description

Summary method for class "rSPDEobj1d"

Usage

```
## S3 method for class 'rSPDEobj1d'
summary(object, ...)

## S3 method for class 'summary.rSPDEobj1d'
print(x, ...)

## S3 method for class 'rSPDEobj1d'
print(x, ...)
```

Arguments

object	an object of class "rSPDEobj1d", usually, a result of a call to matern.rational().
...	further arguments passed to or from other methods.
x	an object of class "summary.rSPDEobj1d", usually, a result of a call to summary.rSPDEobj1d().

summary.rspde_lme *Summary Method for* rspde_lme *Objects.*

Description

Function providing a summary of results related to mixed effects regression models with Whittle-Matern latent models.

Usage

```
## S3 method for class 'rspde_lme'
summary(object, all_times = FALSE, ...)
```

Arguments

object	an object of class "rspde_lme" containing results from the fitted model.
all_times	Show all computed times.
...	not used.

Value

An object of class summary_rspde_lme containing several informations of a *rspde_lme* object.

summary.rspde_result *Summary for posteriors of field parameters for an* inla_rspde *model from a* rspde_result *object*

Description

Summary for posteriors of rSPDE field parameters in their original scales.

Usage

```
## S3 method for class 'rspde_result'
summary(object, digits = 6, ...)
```

Arguments

object	A rspde_result object.
digits	integer, used for number formatting with signif()
...	Currently not used.

Value

Returns a data.frame containing the summary.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)) {
  library(INLA)

  set.seed(123)

  m <- 100
  loc_2d_mesh <- matrix(runif(m * 2), m, 2)
  mesh_2d <- inla.mesh.2d(
    loc = loc_2d_mesh,
    cutoff = 0.05,
    max.edge = c(0.1, 0.5)
  )
  sigma <- 1
  range <- 0.2
  nu <- 0.8
  kappa <- sqrt(8 * nu) / range
  op <- matern.operators(
    mesh = mesh_2d, nu = nu,
    range = range, sigma = sigma, m = 2,
    parameterization = "matern"
  )
  u <- simulate(op)
  A <- inla.spde.make.A(
    mesh = mesh_2d,
    loc = loc_2d_mesh
  )
  sigma.e <- 0.1
  y <- A %*% u + rnorm(m) * sigma.e
  Abar <- rspde.make.A(mesh = mesh_2d, loc = loc_2d_mesh)
  mesh.index <- rspde.make.index(name = "field", mesh = mesh_2d)
  st.dat <- inla.stack(
    data = list(y = as.vector(y)),
    A = Abar,
    effects = mesh.index
  )
  rspde_model <- rspde.matern(
    mesh = mesh_2d,
    nu.upper.bound = 2
  )
  f <- y ~ -1 + f(field, model = rspde_model)
  rspde_fit <- inla(f,
    data = inla.stack.data(st.dat),
    family = "gaussian",
    control.predictor =
      list(A = inla.stack.A(st.dat))
  )
  result <- rspde.result(rspde_fit, "field", rspde_model)
  summary(result)
}
```

```
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`summary.spacetimeobj` *Summarise spacetime objects*

Description

Summary method for class "spacetimeobj"

Usage

```
## S3 method for class 'spacetimeobj'
summary(object, ...)

## S3 method for class 'summary.spacetimeobj'
print(x, ...)

## S3 method for class 'spacetimeobj'
print(x, ...)
```

Arguments

`object`	an object of class "spacetimeobj", usually, a result of a call to `spacetime.operators()`.
`...`	further arguments passed to or from other methods.
`x`	an object of class "summary.spacetimeobj", usually, a result of a call to `summary.spacetimeobj()`.

`transform_parameters_anisotropic`
Transform Anisotropic SPDE Model Parameters to Original Scale

---

Description

This function takes a vector of transformed parameters and applies the appropriate transformations to return them in the original scale for use in anisotropic SPDE models.

Usage

```
transform_parameters_anisotropic(theta, nu_upper_bound = NULL)
```

Arguments

theta	A numeric vector of length 4 or 5, containing the transformed parameters in this order: **lhx** The logarithmic representation of hx. **lhy** The logarithmic representation of hy. **logit_hxy** The logit-transformed representation of hxy. **lsigma** The logarithmic representation of sigma. **lnu (optional)** The logarithmic representation of nu. If not provided, nu is not returned.
nu_upper_bound	(optional) A numeric value representing the upper bound for the smoothness parameter nu. This is only used, and must be provided, if lnu is provided.

Value

A named list with the parameters in the original scale:

hx The original scale for hx (exponential of lhx).

hy The original scale for hy (exponential of lhy).

hxy The original scale for hxy (inverse logit transformation of logit_hxy).

sigma The original scale for sigma (exponential of lsigma).

nu (optional) The original scale for nu (using the forward_nu transformation). Only included if lnu is provided.

Examples

```
# With lnu
theta <- c(log(0.1), log(0.2), log((0.3 + 1) / (1 - 0.3)), log(0.5), log(1))
nu_upper_bound <- 2
transform_parameters_anisotropic(theta, nu_upper_bound)

# Without lnu
theta <- c(log(0.1), log(0.2), log((0.3 + 1) / (1 - 0.3)), log(0.5))
transform_parameters_anisotropic(theta)
```

---

transform_parameters_spacetime

Transform Spacetime SPDE Model Parameters to Original Scale

Description

This function takes a vector of transformed parameters and applies the appropriate transformations to return them in the original scale for use in spacetime SPDE models.

Usage

```
transform_parameters_spacetime(theta, st_model)
```

Arguments

theta	A numeric vector containing the transformed parameters in this order: **lkappa** The logarithmic representation of kappa. **lsigma** The logarithmic representation of sigma. **lgamma** The logarithmic representation of gamma. **logit_rho (optional)** The logit-transformed representation of rho, if drift = 1. **logit_rho2 (optional)** The logit-transformed representation of rho2, if drift = 1 and d = 2.
st_model	A list containing the spacetime model parameters: **d** The dimension (e.g., 1 or 2). **bound** The bound for rho and rho2. **is_bounded** A logical value indicating if rho and rho2 are bounded. **drift** A logical value indicating if drift is included in the model.

Value

A named list with the parameters in the original scale:

kappa The original scale for kappa (exponential of lkappa).

sigma The original scale for sigma (exponential of lsigma).

gamma The original scale for gamma (exponential of lgamma).

rho (optional) The original scale for rho.

rho2 (optional) The original scale for rho2, if d = 2.

---

update.CBrSPDEobj *Update parameters of CBrSPDEobj objects*

Description

Function to change the parameters of a CBrSPDEobj object

Usage

```
## S3 method for class 'CBrSPDEobj'
update(
  object,
  nu = NULL,
  alpha = NULL,
  kappa = NULL,
  tau = NULL,
  sigma = NULL,
  range = NULL,
  theta = NULL,
  m = NULL,
```

```
  mesh = NULL,
  loc_mesh = NULL,
  graph = NULL,
  range_mesh = NULL,
  compute_higher_order = object$higher_order,
  parameterization = NULL,
  type_rational_approximation = object$type_rational_approximation,
  return_block_list = object$return_block_list,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern.operators()
nu	If non-null, update the shape parameter of the covariance function. Will be used if parameterization is 'matern'.
alpha	If non-null, update the fractional SPDE order parameter. Will be used if parameterization is 'spde'.
kappa	If non-null, update the parameter kappa of the SPDE. Will be used if parameterization is 'spde'.
tau	If non-null, update the parameter tau of the SPDE. Will be used if parameterization is 'spde'.
sigma	If non-null, update the standard deviation of the covariance function. Will be used if parameterization is 'matern'.
range	If non-null, update the range parameter of the covariance function. Will be used if parameterization is 'matern'.
theta	For non-stationary models. If non-null, update the vector of parameters.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
mesh	An optional inla mesh. Replaces d, C and G.
loc_mesh	The mesh locations used to construct the matrices C and G. This option should be provided if one wants to use the rspde_lme() function and will not provide neither graph nor mesh. Only works for 1d data. Does not work for metric graphs. For metric graphs you should supply the graph using the graph argument.
graph	An optional metric_graph object. Replaces d, C and G.
range_mesh	The range of the mesh. Will be used to provide starting values for the parameters. Will be used if mesh and graph are NULL, and if one of the parameters (kappa or tau for spde parameterization, or sigma or range for matern parameterization) are not provided.
compute_higher_order	Logical. Should the higher order finite element matrices be computed?
parameterization	If non-null, update the parameterization. Only works for stationary models.

type_rational_approximation
: Which type of rational approximation should be used? The current types are "chebfun", "brasil" or "chebfunLB".

return_block_list
: Logical. For type = "covariance", should the block parts of the precision matrix be returned separately as a list?

...
: Currently not used.

Value

It returns an object of class "CBrSPDEobj. This object contains the same quantities listed in the output of matern.operators().

See Also

simulate.CBrSPDEobj(), matern.operators()

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op_cov <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2,
  parameterization = "matern"
)
op_cov

# Update the range parameter of the model:
op_cov <- update(op_cov, kappa = 20)
op_cov
```

---

update.CBrSPDEobj2d *Update parameters of CBrSPDEobj2d objects*

Description

Function to change the parameters of a CBrSPDEobj object

Usage

```
## S3 method for class 'CBrSPDEobj2d'
update(
  object,
  hx = NULL,
  hy = NULL,
  hxy = NULL,
  sigma = NULL,
  nu = NULL,
  m = NULL,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern2d.operators()
hx	If non-null, update the hx parameter.
hy	If non-null, update the hy parameter.
hxy	If non-null, update the hxy parameter.
sigma	If non-null, update the standard deviation of the covariance function.
nu	If non-null, update the shape parameter of the covariance function. Will be used if parameterization is 'matern'.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
...	Currently not used.

Value

It returns an object of class "CBrSPDEobj2d.

See Also

simulate.CBrSPDEobj2d(), matern2d.operators()

Examples

```
library(fmesher)
n_loc <- 2000
loc_2d_mesh <- matrix(runif(n_loc * 2), n_loc, 2)
mesh_2d <- fm_mesh_2d(loc = loc_2d_mesh, cutoff = 0.03, max.edge = c(0.1, 0.5))
op <- matern2d.operators(mesh = mesh_2d)
op <- update(op, nu = 0.5)
```

`update.intrinsicCBrSPDEobj` *Update parameters of intrinsicCBrSPDEobj objects*

Description

Function to change the parameters of a intrinsicCBrSPDEobj object

Usage

```
## S3 method for class 'intrinsicCBrSPDEobj'
update(object, kappa = NULL, tau = NULL, alpha = NULL, beta = NULL, ...)
```

Arguments

`object`	Model object created by `intrinsic.matern.operators()`
`kappa`	kappa value to be updated.
`tau`	tau value to be update.
`alpha`	alpha value to be updated.
`beta`	beta value to be updated. .
`...`	currently not used.

Value

An object of type intrinsicCBrSPDEobj with updated parameters.

Examples

```
if (requireNamespace("RSpectra", quietly = TRUE)) {
  x <- seq(from = 0, to = 10, length.out = 201)
  beta <- 1
  alpha <- 1
  kappa <- 1
  op <- intrinsic.matern.operators(
    kappa = kappa, tau = 1, alpha = alpha,
    beta = beta, loc_mesh = x, d = 1
  )
op <- update(op, beta = 1.1, alpha = 0.9)
}
```

update.rSPDEobj *Update parameters of rSPDEobj objects*

Description

Function to change the parameters of a rSPDEobj object

Usage

```
## S3 method for class 'rSPDEobj'
update(
  object,
  nu = NULL,
  alpha = NULL,
  kappa = NULL,
  sigma = NULL,
  range = NULL,
  tau = NULL,
  theta = NULL,
  m = NULL,
  mesh = NULL,
  loc_mesh = NULL,
  graph = NULL,
  range_mesh = NULL,
  parameterization = NULL,
  ...
)
```

Arguments

object	The operator-based rational SPDE approximation, computed using matern.operators() with type="operator"
nu	If non-null, update the shape parameter of the covariance function.
alpha	If non-null, update the fractional order.
kappa	If non-null, update the range parameter of the covariance function.
sigma	If non-null, update the standard deviation of the covariance function.
range	If non-null, update the range parameter of the covariance function.
tau	If non-null, update the parameter tau.
theta	If non-null, update the parameter theta, that connects tau and kappa to the model matrices.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
mesh	An optional inla mesh. Replaces d, C and G.

loc_mesh	The mesh locations used to construct the matrices C and G. This option should be provided if one wants to use the rspde_lme() function and will not provide neither graph nor mesh. Only works for 1d data. Does not work for metric graphs. For metric graphs you should supply the graph using the graph argument.
graph	An optional metric_graph object. Replaces d, C and G.
range_mesh	The range of the mesh. Will be used to provide starting values for the parameters. Will be used if mesh and graph are NULL, and if one of the parameters (kappa or tau for spde parameterization, or sigma or range for matern parameterization) are not provided.
parameterization	If non-null, update the parameterization. Only works for stationary models.
...	Currently not used.

Value

It returns an object of class "rSPDEobj. This object contains the same quantities listed in the output of matern.operators().

See Also

simulate.rSPDEobj(), matern.operators()

Examples

```
# Compute the operator-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8
range <- sqrt(8 * nu) / kappa

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op <- matern.operators(
  loc_mesh = x, nu = nu,
  range = range, sigma = sigma, d = 1, m = 2, type = "operator",
  parameterization = "matern"
)
op

# Update the range parameter of the model:
op <- update(op, kappa = 20)
op
```

update.rSPDEobj1d *Update parameters of rSPDEobj1d objects*

Description

Function to change the parameters of a rSPDEobj1d object

Usage

```
## S3 method for class 'rSPDEobj1d'
update(
  object,
  nu = NULL,
  alpha = NULL,
  kappa = NULL,
  tau = NULL,
  sigma = NULL,
  range = NULL,
  theta = NULL,
  m = NULL,
  loc = NULL,
  graph = NULL,
  parameterization = NULL,
  type_rational_approximation = object$type_rational_approximation,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern.rational()
nu	If non-null, update the shape parameter of the covariance function. Will be used if parameterization is 'matern'.
alpha	If non-null, update the fractional SPDE order parameter. Will be used if parameterization is 'spde'.
kappa	If non-null, update the parameter kappa of the SPDE. Will be used if parameterization is 'spde'.
tau	If non-null, update the parameter tau of the SPDE. Will be used if parameterization is 'spde'.
sigma	If non-null, update the standard deviation of the covariance function. Will be used if parameterization is 'matern'.
range	If non-null, update the range parameter of the covariance function. Will be used if parameterization is 'matern'.
theta	For non-stationary models. If non-null, update the vector of parameters.
m	If non-null, update the order of the rational approximation, which needs to be a positive integer.

loc	The locations of interest for evaluating the model.
graph	An optional metric_graph object.
parameterization	If non-null, update the parameterization.
type_rational_approximation	Which type of rational approximation should be used? The current types are "chebfun", "brasil" or "chebfunLB".
...	Currently not used.

Value

It returns an object of class "rSPDEobj1d". This object contains the same quantities listed in the output of `matern.rational()`.

See Also

`simulate.rSPDEobj1d()`, `matern.rational()`

Examples

```
s <- seq(from = 0, to = 1, length.out = 101)
kappa <- 20
sigma <- 2
nu <- 0.8
r <- sqrt(8*nu)/kappa #range parameter
op_cov <- matern.rational(loc = s, nu = nu, range = r, sigma = sigma, m = 2,
parameterization = "matern")
cov1 <- op_cov$covariance(ind = 1)
op_cov <- update(op_cov, range = 0.2)
cov2 <- op_cov$covariance(ind = 1)
plot(s, cov1, type = "l")
lines(s, cov2, col = 2)
```

---

update.spacetimeobj *Update parameters of spacetimeobj objects*

Description

Function to change the parameters of a spacetimeobj object

Usage

```
## S3 method for class 'spacetimeobj'
update(object, kappa = NULL, sigma = NULL, gamma = NULL, rho = NULL, ...)
```

Arguments

object	Space-time object created by spacetime.operators()
kappa	kappa value to be updated.
sigma	sigma value to be updated.
gamma	gamma value to be updated.
rho	rho value to be updated.
...	currently not used.

Value

An object of type spacetimeobj with updated parameters.

Examples

```
s <- seq(from = 0, to = 20, length.out = 101)
t <- seq(from = 0, to = 20, length.out = 31)

op_cov <- spacetime.operators(space_loc = s, time_loc = t,
                              kappa = 5, sigma = 10, alpha = 1,
                              beta = 2, rho = 1, gamma = 0.05)
op_cov <- update(op_cov, kappa = 4,
                              sigma = 2, gamma = 0.1)
```

---

variogram.intrinsic.spde

Variogram of intrinsic SPDE model

Description

Variogram $\gamma(s_0, s)$ of intrinsic SPDE model

$$(-\Delta)^{\beta/2}(\kappa^2 - \Delta)^{\alpha/2}(\tau u) = \mathcal{W}$$

with Neumann boundary conditions and a mean-zero constraint on a square $[0, L]^d$ for $d = 1$ or $d = 2$.

Usage

```
variogram.intrinsic.spde(
  s0 = NULL,
  s = NULL,
  kappa = 0,
  alpha = 0,
  beta = NULL,
  tau = 1,
  L = NULL,
```

```
  N = 100,
  d = NULL,
  semi = FALSE
)
```

Arguments

s0	The location where the variogram should be evaluated, either a double for 1d or a vector for 2d
s	A vector (in 1d) or matrix (in 2d) with all locations where the variogram is computed
kappa	Range parameter.
alpha	Smoothness parameter.
beta	Smoothness parameter.
tau	Precision parameter.
L	The side length of the square domain.
N	The number of terms in the Karhunen-Loeve expansion.
d	The dimension (1 or 2).
semi	Compute the semi variogram? Default FALSE.

Details

The variogram is computed based on a Karhunen-Loeve expansion of the covariance function.

See Also

intrinsic.matern.operators()

Examples

```
if (requireNamespace("RSpectra", quietly = TRUE)) {
  x <- seq(from = 0, to = 10, length.out = 201)
  beta <- 1
  alpha <- 1
  kappa <- 1
  op <- intrinsic.matern.operators(
    kappa = kappa, tau = 1, alpha = alpha,
    beta = beta, loc_mesh = x, d = 1
  )
  # Compute and plot the variogram of the model
  Sigma <- op$A[,-1] %*% solve(op$Q[-1,-1], t(op$A[,-1]))
  One <- rep(1, times = ncol(Sigma))
  D <- diag(Sigma)
  Gamma <- 0.5 * (One %*% t(D) + D %*% t(One) - 2 * Sigma)
  k <- 100
  plot(x, Gamma[k, ], type = "l")
  lines(x,
    variogram.intrinsic.spde(x[k], x, kappa, alpha, beta, L = 10, d = 1),
```

```
    col = 2, lty = 2
  )
}
```

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