

# Package ‘selectiongain’

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**Type** Package

**Title** A Tool for Calculation and Optimization of the Expected Gain  
from Multi-Stage Selection

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**Depends** R (>= 4.1.0), mvtnorm,parallel

**Description** Multi-stage selection is practiced in numerous fields of life and social sciences and particularly in breeding. A special characteristic of multi-stage selection is that candidates are evaluated in successive stages with increasing intensity and effort, and only a fraction of the superior candidates is selected and promoted to the next stage. For the optimum design of such selection programs, the selection gain plays a crucial role. It can be calculated by integration of a truncated multivariate normal (MVN) distribution. While mathematical formulas for calculating the selection gain and the variance among selected candidates were developed long time ago, solutions for numerical calculation were not available. This package can also be used for optimizing multi-stage selection programs for a given total budget and different costs of evaluating the candidates in each stage.

**License** GPL-2

**NeedsCompilation** no

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multistagecor	<i>Function for calculating correlation matrix in a plant breeding context</i>
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### Description

This function is used to calculate the  $(n+1)$ -dimensional correlation matrix  $\Sigma^*$  of  $y$  and  $X$ , where  $y$  is the true value (genotypic value in plant breeding) and  $X = \{X_1, \dots, X_n\}$  are the values of  $y$ 's observations or selection indices, which are linear combinations of the values of observation from each selection stage.

In a plant breeding context, it is assumed that the genetic structure of the candidates to be selected are genetically fixed, e.g., potential cultivars, clones, inbred lines or testcross progenies of inbred lines with the same or different testers in all stages.

### Usage

```
multistagecor(maseff, VGCAandE, VSCA, VLine, ecoweight, rhop,
             T, L, M, Rep, index, indexTrait, covtype, detail,
             VGCAandE2, VSCA2, COVgca, COVsca, maseff2, q12, q22)
```

### Arguments

maseff	is the efficiency of marker-assisted selection (MAS). The default value is NA, which means there is no MAS. If a value between 0 and 1 is assigned to maseff, then the first selection stage will be considered as MAS (Heffner et al., 2010). The value of MAS is recommended to be higher than 0.1 to avoid illshaped correlation matrix.
VGCAandE	is the vector of variance components of genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error. When VSCA is specified, the VGCAandE refers to the general combining ability (hybrid breeding), otherwise it stands for genetic effect (line breeding). The default value is 1,1,1,1,1. Variances types listed in Longin et al. (2007) can be used. For example, VGCAandE="VC2" will set the value as 1,0.5,0.5,1,2.
VSCA	is the vector of variance components for specific combining ability (hybrid breeding). The default value is 0,0,0,0.

VLine	Only to be used if parental and testcross selection are performed in a breeding strategy, For an example see the paper "Wegenast, Longin... 2008. Hybrid maize breeding with doubled haploids. IV". If this strategy is implemented, then Vline correspond to the vector of variance components for the parents (line per se). The default value is 0,0,0,0,0.
ecoweight	is the vector of economic weight. In the case of simultaneous selection of two traits, this vector contains two elements, each corresponding to economical weight of each trait
rhop	is the genetic correlation between line per se performance and GCA
T	is the vector of number of testers at each stage. If there is no tester applied in a certain stage, the value at this stage has to be 1.
L	is the vector of number of locations at each stage.
M	is the vector of tester type, i.e., number of unrelated inbred lines combined in a single tester in stage j.
Rep	is the vector of number of replications at each stage.
index	is the control parameter. If it equals TRUE, the optimum selection index of Longin et al. (2007) will be used in the calculation of correlation matrix without MAS.
indexTrait	is the control parameter for the simultaneous selection of two traits. Possible options are: "Optimum"(default), "Base" and "Restricted" for the implementation of the well known optimum, base and restricted selection indexes in plant breeding.
covtype	is the type of the covariance. Longin's type (covtype=c("LonginII")) is used by default. For the simultaneous selection of two traits possible covtypes are "2traits_PS", "2traits_GS" , "2traits_GS-PS", "2traits_PS-PS", "2traits_GS-PS-PS". If any of these five option is selected the calculation of correlation matrix will use the variance components of the two traits. If the user also require marker assisted selection, the prediction accuracy of MAS for both traits should be also given to the function. Finally, if two traits are selected simultaneously, the desired index have to be defined in indexTrait
detail	is the control parameter to decide if the correlation matrix, optimal selection index and covariance matrix will be returned (=TRUE) or only the correlation matrix (FALSE). The default value is FALSE.
VGCAandE2	In the case of simultaneous selection of two traits (index selection) it is the vector of variance components of genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error for the second trait. When VSCA2 is specified, the VGCAandE refers to the general combining ability, otherwise it stands for genetic effect of the second trait. The default value is 0,0,0,0,0, meaning no simultaneous selection of two traits.
VSCA2	In the case of simultaneous selection of two traits (index selection) it is the vector of variance components for specific combining ability for the second trait. The default value is 0,0,0,0. The default value is 0,0,0,0
COVgca	In the case of simultaneous selection of two traits (index selection) is the vector of covariance components of: genetic effect, genotype $\times$ location interaction,

	genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error.
COVsca	In the case of simultaneous selection of two traits (index selection) is the vector of covariance components of the specific combining ability effects as follows : sca, sca $\times$ location interaction, sca $\times$ year interaction, sca $\times$ location $\times$ year interaction. .
maseff2	is the efficiency of marker-assisted selection (MAS) for the second trait. The default value is NA, which means there is no MAS and there is not simultaneous selection of two traits. If a value between 0 and 1 is assigned to maseff2, then it is assumed that the breeder want to optimize breeding strategies for the simultaneous selection of two traits and also including marker assisted selection. In this case, appropriate options have to be selected in covtype and indexTrait. The value of MAS is recommended to be higher than 0.1 to avoid illshaped correlation matrix.
q12	is the proportion of genetic variance associated with markers for trait 1 as defined by "Dekkers, JCM. 2007. Prediction of response to marker-assited..." This parameter is only needed in the case of simultaneous selection of two traits (index selection)
q22	is the proportion of genetic variance associated with markers for trait 2 as defined by "Dekkers, JCM. 2007. Prediction of response to marker-assited..." This parameter is only needed in the case of simultaneous selection of two traits (index selection)

### Value

The default output is a matrix with dimension  $n+1$  and can be used as input parameter of function multistagegain. When value of detail=TRUE, the correlation matrix, optimal selection index and covariance matrix will be given. If covtype are set to: "2traits\_PS" , "2traits\_GS" , "2traits\_GS-PS" , "2traits\_PS-PS" , or "2traits\_GS-PS-PS" , the output will be a list of seven matrices as follows: (1) correlation matrix for the index, (2) estimates of the relative index weights B (betas) for each trait in each stage, (3) covariance matrix for the index (4) correlation matrix for trait 1, (5) correlation matrix for trait 2, (6) matrix of genotypic covariances and (7) matrix of phenotypic covariances

### Note

no further comment

### Author(s)

Xuefei Mi

### References

- C. Longin, H.F. Utz., J. Reif, T. Wegenast, W. Schipprack and A.E. Melchinger. Hybrid maize breeding with doubled haploids: III. Efficiency of early testing prior to doubled haploid production in two-stage selection for testcross performance. Theor. Appl. Genet. 115: 519-527, 2007.
- E.L. Heffner, A.J. Lorenz, J.L. Jannink, and M.E. Sorrells. Plant breeding with genomic selection: gain per unit time and cost. Crop Sci. 50: 1681-1690, 2010.

**See Also**

selectiongain()

**Examples**

```
# example for calculating correlation matrix without MAS
multistagecor(VGCAandE=c(1,0.5,0.5,1,2),L=c(2,10),T=c(1,1),Rep=c(1,1))
multistagecor(VGCAandE="VC2",L=c(2,10),T=c(1,1),Rep=c(1,1),index=TRUE)

# example for calculating correlation matrix with MAS in the first stage
VCgca=c(0.40,0.20,0.20,0.40,2.00)
VCsca=c(0.20,0.10,0.10,0.20)
corr.matrix = multistagecor (maseff=0.40, VGCAandE=VCgca,
VSCA=VCsca, T=c(1,1,5), L=c(1,3,8), Rep=c(1,1,1))
```

---

multistagegain

*Function for calculating the expected multi-stage selection gain*

---

**Description**

This is the main function of the package and uses the following equation given by Tallis (1961) for  $y$ , which the true genotypic value is:

$$\frac{\partial m(t)}{\partial t_0} \Big|_{t=0} = E(X_0 = y) = \frac{1}{\alpha} \sum_{k=0}^n \rho_{0,k} \phi_1(q_k) \Phi_n(A_{k,s}; R_k)$$

to calculate the expected selection gain defined by Cochran (1951) for given correlation matrix and coordinates of the truncation points.

**Usage**

```
multistagegain(corr, Q, alg, parallel, Vg)
```

**Arguments**

**corr** is the correlation matrix of  $y$  and  $X$ , which is introduced in the function `multistagecor`. The correlation matrix must be symmetric and positive-definite. If the estimated correlation matrix is negative-definite, it must be adjusted before using this function. Before starting the calculations, it is recommended to check the correlation matrix.

**Q** are the coordinates of the truncation points, which are the output of the function `multistagetp` that we are going to introduce.

Vg	correspond to the genetic variance or variance of the GCA effects. The value entered here is only used during the last multiplication of the expected selection gain times the squared root of the genetic variance or the variance of the GCA effects. The default value is 1, and in this case the breeder is advised to make the multiplication outside the function, as showed in the example by Mi et al 2014 page 1415
alg	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use the Miwa algorithm.
parallel	is a logical variable to decided if the multiple cores can be used for computing, by default is FALSE. The users have to notice that assign cores also cost time. So this procedure can only be efficient if the <code>dim &gt; 5</code> .

### Details

This function calculates the well-known selection gain  $\Delta G$ , which is described by Cochran (1951), for multi-stage selection. For one-stage selection the gain is defined as  $\Delta G = i\delta_y\rho_1$ , where  $i$  is the selection intensity,  $\rho_1$  is the correlation between the true breeding value, which has variance  $\delta_y^2$ , and the selection index (Utz 1969).

### Value

The returned value is the expected gain of selection.

### Note

No further notes

### Author(s)

Xuefei Mi

### References

- A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.
- A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.
- G.M. Tallis. Moment generating function of truncated multi-normal distribution. J. Royal Stat. Soc., Ser. B, 23(1):223-229, 1961.
- H.F. Utz. Mehrstufenselektion in der Pflanzenzuechtung (in German). Doctor thesis, University Hohenheim, 1969.
- W.G. Cochran. Improvement by means of selection. In J. Neyman (ed.) Proc. 2nd Berkeley Symp. on Mathematical Statistics and Probability. University of California Press, Berkeley, 1951.

X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. R Journal, 1:37-39, 2009.

X. Mi, F. Utz, F. Technow and A. E. Melchinger. Optimizing Resource Allocation for Multistage Selection in Plant Breeding with R package selectiongain. Crop Science 54:1413-1418. 2014

### See Also

No link

### Examples

```
Q=c(0.4308,0.9804,1.8603)
corr=matrix( c(1,      0.3508,0.3508,0.4979,
               0.3508, 1,      0.3016,0.5630,
               0.3508, 0.3016,1,      0.5630,
               0.4979, 0.5630,0.5630,1),
             nrow=4
        )

multistagegain(corr=corr,Q=Q, alg=Miwa())

# value 1.227475
```

---

multistagegain.each    *Function for calculating the selection gain in each stage*

---

### Description

In some situations, the user wants to know the increase of  $\Delta G$  in each stage so that it is possible to determine the stage which contributes most to  $\Delta G$ . This function calculates  $\Delta G$  stepwise for each stage.

### Usage

```
multistagegain.each(corr, Q, alg, Vg)
```

### Arguments

**corr** is the correlation matrix of  $y$  and  $X$ , which is introduced in the function multistagecorr. The correlation matrix must be symmetric and positive-definite. If the estimated correlation matrix is negative-definite, it must be adjusted before using this function. Before starting the calculations, it is recommended to check the correlation matrix.

**Q** are the coordinates of the truncation points, which are the output of the function multistagetp that we are going to introduce.

Vg	correspond to the genetic variance or variance of the GCA effects. The default value is 1
alg	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use the Miwa algorithm.

### Details

This function calculates the well-known selection gain  $\Delta G$ , which is described by Cochran (1951), for each stage.

### Value

The output is given as  $(\Delta G_1(y), \Delta G_2(y) - \Delta G_1(y), \Delta G_3(y) - \Delta G_2(y), \dots)$  where  $\Delta G_i(y)$  refers to the total selection gain after the first  $i$  stages of selection.

### Author(s)

Xuefei Mi

### References

- A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.
- A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.
- G.M. Tallis. Moment generating function of truncated multi-normal distribution. J. Royal Stat. Soc., Ser. B, 23(1):223-229, 1961.
- H.F. Utz. Mehrstufenselektion in der Pflanzenzuechtung (in German). Doctor thesis, University Hohenheim, 1969.
- W.G. Cochran. Improvement by means of selection. In J. Neyman (ed.) Proc. 2nd Berkeley Symp. on Mathematical Statistics and Probability. University of California Press, Berkeley, 1951.
- X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. R Journal, 1:37-39, 2009.

### See Also

selectiongain()

### Examples

```
# example 1
corr=matrix( c(1,      0.3508,0.3508,0.4979,
```

```

      0.3508, 1,      0.3016,0.5630,
      0.3508, 0.3016,1,      0.5630,
      0.4979, 0.5630,0.5630,1),
      nrow=4
)

multistagegain.each(Q=c(0.4308,0.9804,1.8603),corr=corr)

# examples 2

alpha1<- 1/24
alpha2<- 1
Q=multistagetp(alpha=c(alpha1,alpha2),corr=corr[2:3,2:3])

corr=matrix( c(1,          0.7071068,0.9354143,
               0.7071068,1,          0.7559289,
               0.9354143,0.7559289,1),
             nrow=3
)

multistagegain.each(Q=Q,corr=corr)

```

---

```
multistageoptimum.grid
```

*Function for optimizing multi-stage selection with grid algorithm for a given correlation matrix*

---

## Description

This function is used to calculate the maximum of  $\Delta G$  for a given correlation matrix by grid search algorithm.

## Usage

```
multistageoptimum.grid(corr, Vg,
  num.grid, width, Budget, CostProd,
  CostTest,Nf,alg,detail,fig,N.upper, N.lower,alpha.nursery,cost.nursery,vargain)
```

## Arguments

**Vg** is genotypic variance  $\delta_y^2$ . The default value is 1.

**corr** is the correlation matrix of y and X, which is introduced in the function multistagecorr. The correlation matrix must be symmetric and positive-definite. If the estimated correlation matrix is negative-definite, it must be adjusted before using this function. Before starting the calculations, it is recommended to check the correlation matrix.

<code>num.grid</code>	is the number of equally distanced points that divided the axis of $x_1$ into $num.grid_i - 1$ intervals and there are $\prod_i (num.grid_i)$ grids in a $n$ dimensional hyper cube. If $num.grid > N_i$ , then the number of grid points for the $i$ -th axis is $N_i$ . The default value of it is NA.
<code>width</code>	is the width between the equally distanced points. The default value is NA.
<code>Budget</code>	contains the value of total budget.
<code>CostProd</code>	contains the initial costs of producing or providing a candidate in each stage
<code>CostTest</code>	contains a vector with length $n$ reflecting the cost of evaluating a candidate in the tests performed at stage $i$ , $i=1, \dots, n$ . The cost might vary in different stages.
<code>Nf</code>	is the number of finally selected candidates.
<code>detail</code>	is the control parameter to decide if the result of all the grids will be given or only the maximum. The default value is FALSE.
<code>alg</code>	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use Miwa algorithm of this parameter.
<code>fig</code>	is the control parameter to decide if a figure of contour plot will be saved in the default folder of R. The default value is FALSE, which means no figure will be saved.
<code>N.upper</code>	is the vector of upper limits of number of candidates $X$ .
<code>N.lower</code>	is the vector of lower limits of number of candidates $X$ .
<code>alpha.nursery</code>	a value that should be $0 < x < 1$ , preliminary test alpha fraction should be used for the stage 1. it is setted to 1 as default, when no preliminary test "nursery stage".
<code>cost.nursery</code>	a vector of length two $c$ ([cost of producing a DH line],[cost of testing a DH in nursery]). The default value is 0,0.
<code>vargain</code>	is the logical variable to calculate the variance after multi-stage selection. Default is FALSE. Please see more details in the documentation for the function <code>multistagevariance</code> .The default value is FALSE

## Details

for the new added to parameters "alpha.nursery" and "cost.nursery" since v2.0.47:

After producing new DH lines, breeders do NOT go directly for a selection stage in the field, neither for genomic selection. Most of the times, they prefer to make a small field experiment (called "nursery") in which all DH lines are observed and discarded for other traits as disease resistance. That means, all DH lines with poor resistance will be discarded. At the end of the nursery stage only certain amount of DH lines (alpha) advance to the first selection stage (phenotypic or genomic). Specially in maize that makes sense, because in experience around 90 percent of the new DH lines are very weak in terms of per se performance what make them not suitable as new hybrid parents. Then, budget should not be used to make genotyping on or testcrossing with them. Only the alpha fraction should be used for entering the stage 1 of the `multistageoptimum.search` function.

More details are available in the Crop Science and Computational Statistics papers.

**Value**

If `detail = FALSE`, the output of this functions is a vector with the optimal number of candidates in each stage (**N**) and the maximum  $\Delta G$ . Otherwise, the result for all the grid points, which have been calculated, will be exported as a table.

**Note**

no further comment

**Author(s)**

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**References**

- A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.
- A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.
- G.M. Tallis. Moment generating function of truncated multi-normal distribution. J. Royal Stat. Soc., Ser. B, 23(1):223-229, 1961.
- W.G. Cochran. Improvement by means of selection. In J. Neyman (ed.) Proc. 2nd Berkeley Symp. on Mathematical Statistics and Probability. University of California Press, Berkeley, 1951.
- X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. R Journal, 1:37-39, 2009.

**See Also**

selectiongain()

**Examples**

```
corr=matrix( c(1,      0.3508,0.3508,0.4979,
               0.3508 ,1,      0.3016,0.5630,
               0.3508, 0.3016,1      ,0.5630,
               0.4979, 0.5630,0.5630,1),
             nrow=4
           )

Budget=200

multistageoptimum.grid( Vg=1, num.grid=11, corr=corr, Budget=Budget,
  CostProd=c(0.5,0,0), CostTest=c(0.5,1,1), Nf=5,
  N.upper=rep(Budget,3), N.lower=rep(1,3))

multistageoptimum.grid( Vg=1, num.grid=11, corr=corr, Budget=Budget,
  CostProd=c(0.5,0,0), CostTest=c(0.5,1,1), Nf=5,
  N.upper=rep(Budget,3), N.lower=rep(1,3),detail=TRUE,fig=TRUE)
```

---

multistageoptimum.nlm *Function for optimizing n-stage selection with the NLM algorithm for a given correlation matrix*

---

### Description

This function is used to calculate the maximum of  $\Delta G$  with given correlation matrix by non-linear minimization algorithm.

### Usage

```
multistageoptimum.nlm(corr, Vg, ini.value,
  Budget, CostProd, CostTest,
  Nf, iterlim, alg, N.upper, N.lower)
```

### Arguments

corr	is the correlation matrix of y and X, which is introduced in function multistagecorr. The correlation matrix must be symmetric and positive-definite. Before starting the calculations, the user is recommended to check the correlation matrix.
Vg	is genotypic variance $\delta_y^2$ . The default value is 1.
ini.value	is a vector, which stores the number of candidates in each stage for the algorithm to begin with. As default, it will use $N = \{N_1, N_2, \dots, N_n\} = \{a+1, \dots, a+n\}$ , where a is defined as $(N.upper + N.lower)/4$
.	.
Budget	contains the value of total budget.
CostProd	contains the initial costs of producing or providing a candidate in each stage
CostTest	contains a vector with length n reflecting the cost of evaluating a candidate in the tests performed at stage i, i=1,...,n. The cost might vary in different stages.
Nf	is the number of finally selected candidates.
iterlim	is the maximum number of iterations to be executed before the Newton algorithm is terminated. By default it is equal to 20. If the Budget increases 10 times for making the selection, the value of iterlim has to be increased $lg(10)$ times.
alg	is used to switch between two algorithms. If alg = GenzBretz(), which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If alg = Miwa(), the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend the user to use Miwa algorithm of this parameter.

N.upper            is the vector of up limits of number of candidates X.  
 N.lower            is the vector of low limits of number of candidates X.

### Value

The output of this function is a vector similar as in multistageoptimal.grid(). However, the optimal number of candidates in each stage determined by the NLM algorithm is clearly not an integer, because the function uses a numerical algorithm, which depends on derivatives.

### Note

no further comment

### Author(s)

Xuefei Mi

### References

- A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.
- A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.
- G.M. Tallis. Moment generating function of truncated multi-normal distribution. J. Royal Stat. Soc., Ser. B, 23(1):223-229, 1961.
- H.F. Utz. Mehrstufenselektion in der Pflanzenzuechtung (in German). Doctor thesis, University Hohenheim, 1969.
- W.G. Cochran. Improvement by means of selection. In J. Neyman (ed.) Proc. 2nd Berkeley Symp. on Mathematical Statistics and Probability. University of California Press, Berkeley., 1951.
- X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution, R Journal, 1:37-39, 2009.

### See Also

selectiongain()

### Examples

```
VCGCAandError=c(0.40,0.20,0.20,0.40,2.00)
VCSCA=c(0.20,0.10,0.10,0.20)

corr = multistagecor (maseff=0.40,
  VGCAandE=VCGCAandError, VSCA=VCSCA, T=c(1,1,5),
  L=c(1,3,8), Rep=c(1,1,1))

# the time of nlm have to be controled in 5 s, so this example will not be uploaded into cran

#multistageoptimum.nlm( corr=corr, Vg=0.4,
#Budget=1021, CostProd=c(0.5,0,0),CostTest=c(0.5,6,40), Nf=10,
```

```
# N.upper=c(600,120,20), N.lower=rep(5,3))
```

---

```
multistageoptimum.search
```

*Function for optimizing three-stage selection in plant breeding with one marker-assisted selection stage and two phenotypic selection stages*

---

### Description

This function is used to calculate the maximum of  $\Delta G$  based on correlation matrix, which depends on locations, testers and replicates, with a grid search algorithm. The changing correlation matrix of three-stage selection are the testcross progenies of DH lines in one marker-assisted selection (MAS) stage and two phenotypic selection (PS) stages.

### Usage

```
multistageoptimum.search (maseff=0.4, VGCAandE,
  VSCA, CostProd, CostTest, Nf, Budget, N2grid,
  N3grid, L2grid, L3grid, T2grid, T3grid, R2, R3, alg,
  detail, fig,alpha.nursery,cost.nursery,
  t2free,parallel.search)
```

### Arguments

maseff	is the efficiency of MAS.
VGCAandE	is the vector of variance components of genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error. When VSCA is specified, it refers to the general combining ability, otherwise it stands for genetic effect. The default value is 1,1,1,1,1. Variances types listed in Longin et al. (2007) can be used. E.g., VGCAandE="VC2" will set the value as 1,0.5,0.5,1,2.
VSCA	is the vector of variance components for specific combining ability.
CostProd	contains the initial costs of producing or identifying a candidate in each stage.
CostTest	contains a vector with length n reflecting the cost of evaluating a candidate in the tests performed at stage i, i=1,...,n. The cost might vary in different stages.
Nf	is the number of finally selected candidates.
Budget	contains the value of total budget.
N2grid	is the vector of lower and upper limits as well as the grid width of number of candidates in the first field test stage.
N3grid	is the vector of lower and upper limits as well as the grid width of number of candidates in the second field test stage.

L2grid	is the vector of lower and upper limits of number of location as well as the width in the first field test stage.
L3grid	is the vector of lower and upper limits of number of location as well as the width in the second field test stage.
T2grid	is the vector of lower and upper limits of number of tester as well as the width in the first field test stage.
T3grid	is the vector of lower and upper limits of number of tester as well as the width in the second field test stage.
R2	is the number of replications in the first field test stage. By default it is 1.
R3	is the number of replications in the second field test stage. By default it is 1.
alg	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use the Miwa algorithm.
detail	is the control parameter to decide if the result of all the grids will be given (=TRUE) or only the maximum (=FALSE).
fig	is the control parameter to decide if a contour plot will be saved in the default folder of R. The default value is FALSE, which means no figure will be saved.
alpha.nursery	a value that should be $0 < x < 1$ , preliminary test alpha fraction should be used for the stage 1. it is setted to 1 as default, when no preliminary test "nursery stage".
cost.nursery	a vector of length two <code>c([cost of producing a DH line],[cost of testing a DH in nursery])</code> . The default value is 0,0.
t2free	is a logical value. If =FALSE, the cost of using T3 and T2 testers will be accounted seperately. If =TRUE, the cost of using T3 and T2 testers will be accounted according to number of testers, i.e., $CostProd = c(CostProd[1], CostProd[2]*T2, CostProd[3]*(T3 - T2))$
parallel.search	is a logical variable to decided if the multiple cores can be used for computing, by default is FALSE. The users have to notice that assign cores also cost time. So this procedure can only be efficient if the <code>dim &gt; 5</code> .

## Details

for the new added to parameters "alpha.nursery" and "cost.nursery" since v2.0.47:

After producing new DH lines, breeders do NOT go directly for a selection stage in the field, neither for genomic selection. Most of the times, they prefer to make a small field experiment (called "nursery") in which all DH lines are observed and discarded for other traits as disease resistance. That means, all DH lines with poor resistance will be discarded. At the end of the nursery stage only certain amount of DH lines (alpha) advance to the first selection stage (phenotypic or genomic). Specially in maize that makes sense, because in experience around 90 percent of the new DH lines are very weak in terms of per se performance what make them not suitable as new hybrid parents.

Then, budget should not be used to make genotyping on or testcrossing with them. Only the alpha fraction should be used for entering the stage 1 of the `multistageoptimum.search` function.

More details are available in the Crop Science and Computational Statistics papers.

### Value

If `detail = FALSE`, the output of this function is a vector of the optimum allocation i.e., which achieves the maximum  $\Delta G$ . Otherwise, the result for all the grid points, which have been calculated, will be exported as a table in the Rgui.

### Note

no further comment

### Author(s)

Xuefei Mi, Jose Marulanda

### References

A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.

A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. `mvtnorm`: Multivariate normal and t distributions. R package version 0.9-9995, 2013.

E.L. Heffner, A.J. Lorenz, J.L. Jannink, and M.E. Sorrells. Plant breeding with genomic selection: gain per unit time and cost. *Crop Sci.* 50: 1681-1690, 2010.

X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. *R Journal*, 1:37-39, 2009.

### See Also

`selectiongain()`

### Examples

```
CostProd =c(0.5,1,1)
CostTest = c(0.5,1,1)
Budget=1021
# Budget is very small here to save time in package checking
# for the example in Heffner's paper, please change it to Budget=10021

VCGCAandError=c(0.4,0.2,0.2,0.4,2)
VCSCA=c(0.2,0.1,0.1,0.2)
Nf=10

multistageoptimum.search (maseff=0.4, VGCAandE=VCGCAandError,
VSCA=VCSCA, CostProd = c(0.5,1,1), CostTest = c(0.5,1,1),
Nf = 10, Budget = Budget, N2grid = c(11, 1211, 30),
N3grid = c(11, 211, 5), L2grid=c(1,3,1), L3grid=c(6,6,1),
```

```
#important note! by Xuefei Mi 2022-02-09
# in the paper L3grid=c(6,8,1) but please do not change it here, otherwise
# due to Budget =1021, the searching room will out of boudry
T2grid=c(1,2,1), T3grid=c(3,5,1), R2=1, R3=1, alg = Miwa(),
detail=TRUE, fig=TRUE, alpha.nursery=1)
```

---

```
multistageoptimum.searchIndexT
```

*Function for optimizing three-stage selection in plant breeding with one marker-assisted selection stage and two phenotypic selection stages*

---

## Description

This function is used to calculate the maximum of  $\Delta G$  based on correlation matrix, which depends on locations, testers and replicates, with a grid search algorithm. The changing correlation matrix of three-stage selection are the testcross progenies of DH lines in one marker-assisted selection (MAS) stage and two phenotypic selection (PS) stages.

## Usage

```
multistageoptimum.searchIndexT (maseff=0.4, VGCAandE, VSCA, CostProd, CostTest,
  Nf, Budget, N2grid, N3grid, L2grid, L3grid, T2grid, T3grid,
  R2, R3, alg, detail, fig, alpha.nursery, cost.nursery,
  t2free,parallel.search, indexTrait, covtype,
  VGCAandE2, VSCA2, COVgca, COVsca, maseff2, q12, q22, ecoweight)
```

## Arguments

maseff	is the efficiency of MAS.
VGCAandE	is the vector of variance components of genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error. When VSCA is specified, it refers to the general combining ability, otherwise it stands for genetic effect. The default value is 1,1,1,1,1. Variances types listed in Longin et al. (2007) can be used. E.g., VGCAandE="VC2" will set the value as 1,0.5,0.5,1,2.
VSCA	is the vector of variance components for specific combining ability.
CostProd	contains the initial costs of producing or identifying a candidate in each stage.
CostTest	contains a vector with length n reflecting the cost of evaluating a candidate in the tests performed at stage i, i=1,...,n. The cost might vary in different stages.
Nf	is the number of finally selected candidates.
Budget	contains the value of total budget.
N2grid	is the vector of lower and upper limits as well as the grid width of number of candidates in the first field test stage.

N3grid	is the vector of lower and upper limits as well as the grid width of number of candidates in the second field test stage.
L2grid	is the vector of lower and upper limits of number of location as well as the width in the first field test stage.
L3grid	is the vector of lower and upper limits of number of location as well as the width in the second field test stage.
T2grid	is the vector of lower and upper limits of number of tester as well as the width in the first field test stage.
T3grid	is the vector of lower and upper limits of number of tester as well as the width in the second field test stage.
R2	is the number of replications in the first field test stage. By default it is 1.
R3	is the number of replications in the second field test stage. By default it is 1.
alg	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use the Miwa algorithm.
detail	is the control parameter to decide if the result of all the grids will be given (=TRUE) or only the maximum (=FALSE).
fig	is the control parameter to decide if a contour plot will be saved in the default folder of R. The default value is FALSE, which means no figure will be saved.
alpha.nursery	a value that should be $0 < x < 1$ , preliminary test alpha fraction should be used for the stage 1. it is setted to 1 as default, when no preliminary test "nursery stage".
cost.nursery	a vector of length two <code>c([cost of producing a DH line],[cost of testing a DH in nursery])</code> . The default value is 0,0.
t2free	is a logical value. If =FALSE, the cost of using T3 and T2 testers will be accounted seperately. If =TRUE, the cost of using T3 and T2 testers will be accounted according to number of testers, i.e., $CostProd=c(CostProd[1],CostProd[2]*T2,CostProd[3]*(T3-T2))$
parallel.search	is a logical variable to decided if the multiple cores can be used for computing, by default is FALSE. The users have to notice that assign cores also cost time. So this procedure can only be efficient if the <code>dim &gt; 5</code> .
indexTrait	is the control parameter for the simultaneous selection of two traits. Possible options are: "Optimum"(default), "Base" and "Restricted" for the implementation of the well known optimum, base and restricted selection indexes in plant breeding.
covtype	is the type of the covariance. Longin's type ( <code>covtype=c("LonginII")</code> ) is used by default. For the simultaneous selection of two traits possible covtypes are "2traits_PS", "2traits_GS", "2traits_GS-PS", "2traits_PS-PS", "2traits_GS-PS-PS". If any of these five option is selected the calculation of correlation matrix will use the variance components of the two traits. If the user also require marker

	assited selection, the prediction accuracy of MAS for both traits should be also given to the function. Finally, if two traits are selected simultaneously, the desired index have to be defined in indexTrait
VGCAandE2	In the case of simultaneous selection of two traits (index selection) it is the vector of variance components of genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error for the second trait. When VSCA2 is specified, the VGCAandE refers to the general combining ability, otherwise it stands for genetic effect of the second trait. The default value is 0,0,0,0,0, meaning no simultaneous selection of two traits.
VSCA2	In the case of simultaneous selection of two traits (index selection) it is the vector of variance components for specific combining ability for the second trait. The default value is 0,0,0,0.
COVgca	In the case of simultaneous selection of two traits (index selection) is the vector of covariance components of: genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error. In case of hybrid breeding strategies it correspond to the covariance of general combining ability effects, while in line breeding strategies it corresponds to the covariance of genetic effects (per se performance).
COVsca	In the case of simultaneous selection of two traits (index selection) is the vector of covariance components of the specific combining ability effects as follows : sca, sca $\times$ location interaction, sca $\times$ year interaction, sca $\times$ location $\times$ year interaction. .
maseff2	is the efficiency of marker-assisted selection (MAS) for the second trait. The default value is NA, which means there is no MAS and there is not simultaneous selection of two traits. If a value between 0 and 1 is assigned to maseff2, then it is assumed that the breeder want to optimize breeding strategies for the simultaneous selection of two traits and also including marker assisted selection. In this case, appropriate options have to be selected in covtype and indexTrait. The value of MAS is recommended to be higher than 0.1 to avoid illshaped correlation matrix.
q12	is the proportion of genetic variance associated with markers for trait 1 as defined by "Dekkers, JCM. 2007. Prediction of response to marker-assited..." This parameter is only needed in the case of simultaneous selection of two traits (index selection)
q22	is the proportion of genetic variance associated with markers for trait 2 as defined by "Dekkers, JCM. 2007. Prediction of response to marker-assited..." This parameter is only needed in the case of simultaneous selection of two traits (index selection)
ecoweight	is the vector of economic weight. In the case of simultaneous selection of two traits, this vector contains two elements, each corresponding to economical weigth of each trait

### Details

for the simultaneous optimization of two traits in multiple stage selection, it is assumed that all locations used during the first round of field trials are also used in the second round of field trials, i.e.,

the second round of field trials uses the same locations of the first round plus some new locations. The same is assumed for testers.

for the parameters "alpha.nursery" and "cost.nursery" since v2.0.47:

After producing new DH lines, breeders do NOT go directly for a selection stage in the field, neither for genomic selection. Most of the times, they prefer to make a small field experiment (called "nursery") in which all DH lines are observed and discarded for other traits as disease resistance. That means, all DH lines with poor resistance will be discarded. At the end of the nursery stage only certain amount of DH lines (alpha) advance to the first selection stage (phenotypic or genomic). Specially in maize that makes sense, because in experience around 90 percent of the new DH lines are very weak in terms of per se performance what make them not suitable as new hybrid parents. Then, budget should not be used to make genotyping on or testcrossing with them. Only the alpha fraction should be used for entering the stage 1 of the multistageoptimum.search function.

More details are available in the Crop Science and Computational Statistics papers.

### Value

If detail = FALSE, the output of this function is a vector of the optimum allocation i.e., which achieves the maximum  $\Delta G$ . Otherwise, the result for all the grid points, which have been calculated, will be exported as a table in the Rgui.

### Note

no further comment

### Author(s)

Xuefei Mi, Jose Marulanda

### References

- A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.
- A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.
- E.L. Heffner, A.J. Lorenz, J.L. Jannink, and M.E. Sorrells. Plant breeding with genomic selection: gain per unit time and cost. Crop Sci. 50: 1681-1690, 2010.
- X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. R Journal, 1:37-39, 2009.

### See Also

selectiongain()

### Examples

```
vgv<- c(5.7, 5.19, 0.00, 0.00, 24.37) # from paper Longin 2015
vscav <- c(1.88, 2.94, 0.00, 0.00) # from paper Longin 2015
vlv<-c(0.08,0.02,0,0,0.09) #from paper Zhao 2016
```

```

vscal <- c(0.01, 0.00, 0.00, 0.00) #from paper Zhao 2016
vcovv1<-c(-0.235,0,0,0,0) #come from Y. Zhao's email communication on June 20/2016
vcovs1<-c(-0.011,0,0,0) #testing value on Dic 07/2016

a1<-17.2 # economic weight for yield
a2<-4.5 # economic weight for protein

multistageoptimum.searchIndexT(
  maseff=0.3, maseff2=0.36, q12=0.85, q22=0.85,
  VGCAandE=vgv, VSCA=vscav, VGCAandE2=vlv, VSCA2=vscal,
  COVgca=vcovv1, COVsca=vcovs1,
  CostProd = c(0,4,4), CostTest = c(2,1,1), Budget = 1000,
  alpha.nursery=0.25,cost.nursery=c(1,0.3), Nf = 5,
  N2grid = c(5, 100, 10), N3grid = c(5, 40, 5),
  L2grid=c(7,8,1), L3grid=c(9,10,1),
  T2grid=c(1,2,1), T3grid=c(2,3,1), t2free= TRUE,
  R2=1,R3=1, alg = Miwa(),detail=FALSE,fig=FALSE,
  covtype=c("2traits_GS-PS-PS"),indexTrait=c("Optimum"),ecoweight=c(a1,a2))

```

---

```
multistageoptimum.searchThreeS
```

*Function for optimizing four-stage selection in plant breeding with one marker-assisted selection stage and three phenotypic selection stages*

---

## Description

This function is used to calculate the maximum of  $\Delta G$  based on correlation matrix, which depends on locations, testers and replicates, with a grid search algorithm. The changing correlation matrix of four-stage selection are the testcross progenies of DH lines in one marker-assisted selection (MAS) stage and three phenotypic selection (PS) stages.

## Usage

```

multistageoptimum.searchThreeS (maseff=0.4, VGCAandE,
  VSCA, CostProd, CostTest, Nf, Budget, N2grid,
  N3grid, N4grid, L2grid, L3grid, L4grid, T2grid,
  T3grid, T4grid, R2, R3, R4, alg,
  detail, fig,alpha.nursery,cost.nursery,
  t2free,parallel.search,saveresult)

```

## Arguments

maseff	is the efficiency of MAS, if set to NA no marker assisted selection or genomic selection is developed in the first stage
VGCAandE	is the vector of variance components of genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and

the plot error. When VSCA is specified, it refers to the general combining ability, otherwise it stands for genetic effect. The default value is 1,1,1,1,1. Variances types listed in Longin et al. (2007) can be used. E.g., VGCAandE="VC2" will set the value as 1,0.5,0.5,1,2.

VSCA	is the vector of variance components for specific combining ability.
CostProd	contains the initial costs of producing or identifying a candidate in each stage, then the vector should be of length four.
CostTest	contains a vector with length n reflecting the cost of evaluating a candidate in the tests performed at stage i, $i=1,\dots,n$ . The cost might vary in different stages. For this function $n=4$
Nf	is the number of finally selected candidates.
Budget	contains the value of total budget.
N2grid	is the vector of lower and upper limits as well as the grid width of number of candidates in the first field test stage.
N3grid	is the vector of lower and upper limits as well as the grid width of number of candidates in the second field test stage.
N4grid	is the vector of lower and upper limits as well as the grid width of number of candidates in the third field test stage.
L2grid	is the vector of lower and upper limits of number of location as well as the width in the first field test stage.
L3grid	is the vector of lower and upper limits of number of location as well as the width in the second field test stage.
L4grid	is the vector of lower and upper limits of number of location as well as the width in the third field test stage.
T2grid	is the vector of lower and upper limits of number of tester as well as the width in the first field test stage.
T3grid	is the vector of lower and upper limits of number of tester as well as the width in the second field test stage.
T4grid	is the vector of lower and upper limits of number of tester as well as the width in the third field test stage.
R2	is the number of replications in the first field test stage. By default it is 1.
R3	is the number of replications in the second field test stage. By default it is 1.
R4	is the number of replications in the third field test stage. By default it is 1.
alg	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use the Miwa algorithm.
detail	is the control parameter to decide if the result of all the grids will be given (=TRUE) or only the maximum (=FALSE).

<code>fig</code>	is the control parameter to decide if a contour plot will be saved in the default folder of R. The default value is FALSE, which means no figure will be saved.
<code>alpha.nursery</code>	a value that should be $0 < x < 1$ . The alpha fraction, or amount of genotypes preliminary selected in nurseries, correspond to the fraction entering stage 1 (when MAS is used) or stage 2 (when there is no MAS). It is setted to 1 as default, i.e. no preliminary test "nursery stage".
<code>cost.nursery</code>	a vector of length two $c$ ([cost of producing a DH line],[cost of testing a DH in nursery]). The default value is 0,0.
<code>t2free</code>	is a logical value. If =FALSE, the cost of using T4, T3 and T2 testers will be accounted separately. If =TRUE, the cost of using T4, T3 and T2 testers will be accounted according to number of testers, i.e., $CostProd = c(CostProd[1], CostProd[2]*T2, CostProd[3]*(T3-T2), CostProd[4]*(T4-T3))$
<code>parallel.search</code>	is a logical variable to decided if the multiple cores can be used for computing, by default is FALSE. The users have to notice that assign cores also cost time. So this procedure can only be efficient if the $dim > 5$ .
<code>saveresult</code>	is a logical variable to save resultfile in saveresult.csv.

## Details

Some breeding programs require more than two phenotypic selection stages. In this programs, a large number of genotypes are assessed for the target trait only in few locations in the first stage and strong selection pressure is applied. The second and third stages of phenotypic selection are developed in a large number of locations including only a reduced number of genotypes. Even if this strategy could lead to a reduced selection gain, it could be of major advantage when breeding programs have biological or operative restrictions to conduct large experiments in a large number of locations. This function allows breeders to estimate the possible increase or reduction of selection gain when moving from two stages of phenotypic selection to three stages and also when a restricted number of genotypes and locations in each of the three stages of phenotypic selection is used.

for the new added to parameters "alpha.nursery" and "cost.nursery" since v2.0.47:

After producing new DH lines, breeders do NOT go directly for a selection stage in the field, neither for genomic selection. Most of the times, they prefer to make a small field experiment (called "nursery") in which all DH lines are observed and discarded for other traits as disease resistance. That means, all DH lines with poor resistance will be discarded. At the end of the nursery stage only certain amount of DH lines (alpha) advance to the first selection stage (phenotypic or genomic). Specially in maize that makes sense, because in experience around 90 percent of the new DH lines are very weak in terms of per se performance what make them not suitable as new hybrid parents. Then, budget should not be used to make genotyping on or testcrossing with them. Only the alpha fraction should be used for entering the stage 1 of the multistageoptimum.search function.

More details are available in the Crop Science and Computational Statistics papers.

## Value

If `detail = FALSE`, the output of this function is a vector of the optimum allocation i.e., which achieves the maximum  $\Delta G$ . Otherwise, the result for all the grid points, which have been calculated, will be exported as a table in the Rgui.

**Note**

no further comment

**Author(s)**

Jose Marulanda, Xuefei Mi

**References**

- A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.
- A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.
- E.L. Heffner, A.J. Lorenz, J.L. Jannink, and M.E. Sorrells. Plant breeding with genomic selection: gain per unit time and cost. Crop Sci. 50: 1681-1690, 2010.
- X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. R Journal, 1:37-39, 2009.

**See Also**

selectiongain()

**Examples**

```

VCGCAandError=c(0.4,0.2,0.2,0.4,2)
VCSCA=c(0.2,0.1,0.1,0.2)

#Budget is reduced to 1000 to save computation time

multistageoptimum.searchThreeS(maseff=NA, VGCAandE=VCGCAandError, VSCA=VCSCA,
  alpha.nursery = 0.25, cost.nursery = c(1,0.3), CostProd=c(0,4,4,4), CostTest=c(0,1,1,1),
  Nf=3, Budget=1000, N2grid=c(50,200,50),N3grid=c(10,50,5), N4grid=c(10,20,5),
  L2grid=c(1,2,1), L3grid=c(2,3,1), L4grid=c(4,5,1),
  T2grid=c(1,2,1), T3grid=c(2,3,1), T4grid=c(4,5,1),
  R2=1, R3=1, R4=1, alg=Miwa(), detail=FALSE, fig= FALSE, t2free=TRUE)

```

---

multistagetp

*Function for calculating the truncation points*

---

**Description**

This function calculates the coordinates of the truncation points  $Q$  for given selected fractions  $\vec{\alpha} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$  and correlation matrix of  $X$ . The R function `uniroot` in core package `stats` is called internally to solve the truncation point equations.

**Usage**

```
multistagetp(alpha, corr, alg)
```

**Arguments**

alpha	is probability vector $\vec{\alpha}$ for random variable X. In plant breeding, it is also called the selected fraction.
corr	is the correlation matrix of y and X, which is introduced in the function multistagecorr. The correlation matrix must be symmetric and positive-definite. If the estimated correlation matrix is negative-definite, it must be adjusted before using this function. Before starting the calculations, it is recommended to check the correlation matrix.
alg	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use the Miwa algorithm.

**Details**

This function calculates the non-equi coordinate quantile vector  $Q = \{q_1, q_2, \dots, q_n\}$  for a multivariate normal distribution from a given  $\vec{\alpha}$ . It can be compared with the function `qmvnorm()` in R-package **mvtnorm**, which calculates only the equi coordinate quantile  $q$  for multi-variate normal distribution from a given  $\vec{\alpha}$ . The function `multistagetp` is used by function `multistagegain` to calculate the expected gain.

**Value**

The output is a vector of the coordinates.

**Note**

When a  $\vec{\alpha}$  is given, the quantiles are calculated consecutively to satisfy the given  $\vec{\alpha}$ . The calculation from other direction to  $-\infty$  of the integral is also possible for `qmvnorm()`.

**Author(s)**

Xuefei Mi

**References**

- A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.
- A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.
- X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. R Journal, 1:37-39, 2009.

**See Also**

selectiongain(), qnorm()

**Examples**

```
# first example

VCGCAandError=c(0.40,0.20,0.20,0.40,2.00)
VCSA=c(0.20,0.10,0.10,0.20)

corr.matrix = multistagecor(maseff=0.40, VGCAandE=VCGCAandError,
VCSA=VCSA, T=c(1,1,5), L=c(1,3,8), Rep=c(1,1,1))

N1=4500;N2=919;N3=45;Nf=10

Q=multistagetp(c(N2/N1,N3/N2,Nf/N3), corr=corr.matrix)
```

---

multistagevariance      *Expected variance after selection after k stages selection*

---

**Description**

This function uses the algorithm described by Tallis (1961) to calculate the variance after multi-stage selection. The variance among candidates of  $y$  in the selected area  $\mathbf{S}_Q$  is defined as the second central moment,  $\psi_n(y) = E(Y^2|\mathbf{S}_Q) - [E(Y|\mathbf{S}_Q)]^2$ , where  $E(Y^2|\mathbf{S}_Q) = \alpha^{-1} \int_{-\infty}^{\infty} \int_{q_1}^{\infty} \dots \int_{q_n}^{\infty} y^2 \phi_{n+1}(\mathbf{x}^*; \Sigma^*) d\mathbf{x}^*$

**Usage**

```
multistagevariance(Q, corr, alg, Vg)
```

**Arguments**

Q	are the coordinates of the truncation points, which are the output of the function multistagetp that we are going to introduce.
corr	is the correlation matrix of $y$ and $\mathbf{X}$ , which is introduced in the function multistagecorr. The correlation matrix must be symmetric and positive-definite. If the estimated correlation matrix is negative-definite, it must be adjusted before using this function. Before starting the calculations, it is recommended to check the correlation matrix.
alg	is used to switch between two algorithms. If <code>alg = GenzBretz()</code> , which is by default, the quasi-Monte Carlo algorithm from Genz et al. (2009, 2013), will be used. If <code>alg = Miwa()</code> , the program will use the Miwa algorithm (Mi et al., 2009), which is an analytical solution of the MVN integral. Miwa's algorithm has higher accuracy (7 digits) than quasi-Monte Carlo algorithm (5 digits). However, its computational speed is slower. We recommend to use the Miwa algorithm.
Vg	correspond to the genetic variance or variance of the GCA effects. The default value is 1

**Value**

The output is the value of  $\psi_n(y|\mathbf{S}_Q)$ .

**Note**

No further notes

**Author(s)**

Xuefei Mi

**References**

A. Genz and F. Bretz. Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195, Springer-Verlag, Heidelberg, 2009.

A. Genz, F. Bretz, T. Miwa, X. Mi, F. Leisch, F. Scheipl and T. Hothorn. mvtnorm: Multivariate normal and t distributions. R package version 0.9-9995, 2013.

G.M. Tallis. Moment generating function of truncated multi-normal distribution. J. Royal Stat. Soc., Ser. B, 23(1):223-229, 1961.

X. Mi, T. Miwa and T. Hothorn. Implement of Miwa's analytical algorithm of multi-normal distribution. R Journal, 1:37-39, 2009.

**See Also**

No link

**Examples**

```
# first example

Q =c(0.4308,0.9804,1.8603)

corr=matrix( c(1,          0.3508,0.3508,0.4979,
               0.3508, 1,          0.3016,0.5630,
               0.3508, 0.3016,1,      0.5630,
               0.4979, 0.5630,0.5630,1),
             nrow=4
           )

multistagevariance(Q=Q,corr=corr,alg=Miwa)

# time comparsion

var.time.miwa=system.time (var.miwa<-multistagevariance(Q=Q,corr=corr,alg=Miwa))

var.time.bretz=system.time (var.bretz<-multistagevariance(Q=Q,corr=corr))
```

```

# second examples

Q= c(0.9674216, 1.6185430)
corr=matrix( c(1,          0.7071068, 0.9354143,
              0.7071068, 1,          0.7559289,
              0.9354143, 0.7559289, 1),
            nrow=3
          )

multistagevariance(Q=Q,corr=corr,alg=Miw)

var.time.miw=system.time (var.miw<-multistagevariance(Q=Q, corr=corr, alg=Miw))

var.time.bretz=system.time (var.bretz<-multistagevariance(Q=Q, corr=corr))

# third examples

alpha1<- 1/(24)^0.5
alpha2<- 1/(24)^0.5
Q=multistagep(alpha=c(alpha1,alpha2),corr=corr)

corr=matrix( c(1,          0.7071068,0.9354143,
              0.7071068, 1,          0.7559289,
              0.9354143, 0.7559289,1),
            nrow=3
          )

multistagevariance(Q=Q, corr=corr, alg=Miw)

```

---

SDselectiongain

*Function for calculating the standard deviation of selection gain*


---

### Description

This function is used to calculate the standard deviation of sel gain according to longin 2015

### Usage

```
SDselectiongain(Ob, maseff, VGCAandE, VSCA, VLine, years, Genotypes)
```

**Arguments**

Ob	matrix object produced by the function <code>multistageoptimum.search</code> or <code>multistageoptimum.grid</code>
maseff	is the efficiency of marker-assisted selection (MAS). The default value is NA, which means there is no MAS. If a value between 0 and 1 is assigned to maseff, then the first selection stage will be considered as MAS (Heffner et al., 2010). The value of MAS is recommended to be higher than 0.1 to avoid illshaped correlation matrix.
VGCAandE	is the vector of variance components of genetic effect, genotype $\times$ location interaction, genotype $\times$ year interaction, genotype $\times$ location $\times$ year interaction and the plot error. When VSCA is specified, the VGCAandE refers to the general combining ability, otherwise it stands for genetic effect. The default value is 1,1,1,1,1. Variances types listed in Longin et al. (2007) can be used. For example, <code>VGCAandE="VC2"</code> will set the value as 1,0.5,0.5,1,2.
VSCA	is the vector of variance components for specific combining ability. The default value is 0,0,0,0.
VLine	is the vector of variance components for line per se. The default value is 0,0,0,0,0.
years	Duration of the breeding scheme in years, it is used only to compute the annual selection gain
Genotypes	character vector to indicate the function which variance components we are using. Possible values are "Hybrids" if we are using GCA and SCA variance components or "Lines" if we are using line per se variance components

**Details**

for the new added to parameters "alpha.nursery" and "cost.nursery" since v2.0.47:

After producing new DH lines, breeders do NOT go directly for a selection stage in the field, neither for genomic selection. Most of the times, they prefer to make a small field experiment (called "nursery") in which all DH lines are observed and discarded for other traits as disease resistance. That means, all DH lines with poor resistance will be discarded. At the end of the nursery stage only certain amount of DH lines (alpha) advance to the first selection stage (phenotypic or genomic). Specially in maize that makes sense, because in experience around 90 percent of the new DH lines are very weak in terms of per se performance what make them not suitable as new hybrid parents. Then, budget should not be used to make genotyping on or testcrossing with them. Only the alpha fraction should be used for entering the stage 1 of the `multistageoptimum.search` function.

More details are available in the Crop Science and Computational Statistics papers.

**Value**

The output is equivalent to the matrix object produced by the functions `multistageoptimum.search` or `multistageoptimum.grid` but with two columns added, one for the values of the annual selection gain and the second for the standard deviation of selection gain

**Note**

no further comment

**Author(s)**

Jose Marulanda

**References**

C. Longin, X. Mi and T. Wuerschum. Genomic selection in wheat: optimum allocation of test resources and comparison of breeding strategies for line and hybrid breeding. *Theoretical and Applied Genetics* 128: 1297-1306. 2015.

C. Longin, H.F. Utz., J. Reif, T. Wegenast, W. Schipprack and A.E. Melchinger. Hybrid maize breeding with doubled haploids: III. Efficiency of early testing prior to doubled haploid production in two-stage selection for testcross performance. *Theor. Appl. Genet.* 115: 519-527, 2007.

E.L. Heffner, A.J. Lorenz, J.L. Jannink, and M.E. Sorrells. Plant breeding with genomic selection: gain per unit time and cost. *Crop Sci.* 50: 1681-1690, 2010.

**See Also**

selectiongain()

**Examples**

```
CostProd =c(0.5,1,1)
CostTest = c(0.5,1,1)
Budget=1021
# Budget is very small here to save time in package checking
# for the example in Heffner's paper, please change it to Budget=10021

VCGCAandError=c(0.4,0.2,0.2,0.4,2)
VCSCA=c(0.2,0.1,0.1,0.2)
Nf=10
maseff=0.4
years=7
# this breeding scheme takes 7 years from the initial cross to the final field testing.
# See references for more details

Ob<-multistageoptimum.search (maseff=maseff, VGCAandE=VCGCAandError,
VSCA=VCSCA, CostProd = CostProd, CostTest = CostTest,
Nf = Nf, Budget = Budget, N2grid = c(11, 1211, 30),
N3grid = c(11, 211, 5), L2grid=c(1,1,1), L3grid=c(6,6,1),
T2grid=c(1,2,1), T3grid=c(3,5,1), R2=1, R3=1, alg = Miwa(),
detail=TRUE, fig=FALSE, t2free=TRUE)

SDselectiongain(Ob=Ob,maseff=maseff,VGCAandE=VCGCAandError,VSCA=VCSCA,
years=years,Genotypes="Hybrids")
```

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